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points. The muffin-tin basis set included s , p , d and f orbitals. The exchange-correlation effects were treated within the generalized gradient approximation.²⁹ The paramagnetic (PM) state was simulated by the disordered local moments (DLM) model.³⁰ The validity of this theoretical approach has been verified against observations including the case of FeCrCoNi-based HEAs.^{31,32}

The relative phase stability at ambient pressure and as a function of temperature is investigated from the free energies computed for various structures. Here we decompose the free energy as $F = E + F^{\text{conf}} + F^{\text{mag}} + F^{\text{vib}} + F^{\text{el}}$, where E is the internal energy, and F^{conf} , F^{mag} , F^{vib} , and F^{el} reflect the additional temperature-dependent contributions for the configurational, magnetic, vibrational, and electronic free energies, respectively. For an ideal solid-solution, we have $F^{\text{conf}} = k_B T \sum_i c_i \ln c_i$, where c_i is the concentration of atom i , T is the temperature, and k_B is the Boltzmann constant. For the paramagnetic state, the magnetic free energy is estimated within the mean-field approximation as $F^{\text{mag}} = -k_B T \sum_i c_i \ln(1 + \mu_i)$, where μ_i is the local magnetic moment of the i th alloying element. This expression corresponds to a completely disordered paramagnetic state.³³ The vibrational free energy is derived from the Debye-Grüneisen model $F^{\text{vib}} = 9k_B \theta_D / 8 + 3k_B T \ln(1 - e^{-\theta_D/T}) - k_B T D(\theta_D/T)$, where θ_D is the Debye temperature, and D is the Debye integral.³⁴ Finally, the electronic free energy is defined as $F^{\text{el}} = E^{\text{el}} - TS^{\text{el}}$, where E^{el} and S^{el} are the electronic energy and entropy, respectively, which are obtained directly from the EMTO calculations using the finite-temperature Fermi distribution.³⁵

Figure 1 shows the calculated temperature-dependent free energies of FeCrCoNiGa for the fcc and bcc structures at both ferromagnetic (FM) and paramagnetic (PM) states. For clarity, all free energies are plotted with reference to the bcc FM state. It is found that, irrespectively to the crystal structure, the FM state is energetically stable at low temperature, and the PM state becomes favorable at a high temperature. The magnetic transition temperatures are estimated from the present FM and PM free energies to be 643 K and 691 K for the fcc and bcc structures, respectively. The Curie temperature (T_C) obtained from the thermomagnetic measurements is around 649 K. The very good parallelism between the theoretical and experimental T_C values supports the accuracy of the present theoretical scheme. As indicated in Fig. 1, the fcc and bcc phases in the FM state arrive at equilibrium around room temperature where the free energy

difference vanishes. Namely, the present system is expected to form a thermodynamic stable fcc-bcc duplex phase near room temperature. This prediction is consistent with experimental observation.²¹ When comparing all four free energies, we find that the FM bcc phase is the most favorable phase at low temperature (cryogenic conditions) and the PM fcc at high temperature. It should be mentioned that samples synthesized in experiments always possess complex microstructures. For example, FeCr-rich precipitates embedded in a NiAl-rich matrix were observed in as-cast FeCrCoNiAl system.³⁶ In the present theoretical model, to better isolate the crystal structure effect, we assumed that the HEAs have perfect composition and display ideal solid solution phase with fcc and bcc lattices, i.e., the ordering/segregation effects were completely neglected. Nevertheless, the good agreement between theory and experiment indicates that the present theoretical approach can capture the structural energy differences and trace energy changes related to the temperature variation in the HEAs.

According to the present *ab initio* calculations, the lattice vibrations play an important role in the phase stabilization. As the key parameter in the Debye-Grüneisen model, the Debye temperature θ_D can be obtained from the average sound velocity v_m by the relations $\theta_D = (\hbar/k_B)(6\pi^2/V)^{1/3} v_m$ with $v_m = [(1/v_L^3 + 2/v_T^3)/3]^{-1/3}$, where \hbar is the Planck constant, V is the volume, v_L and v_T are the longitudinal and transverse sound velocities, respectively. The sound velocities are given by the polycrystalline bulk modulus B , shear modulus G , and density ρ , viz., $v_L = \sqrt{(B + 4G/3)/\rho}$ and $v_T = \sqrt{G/\rho}$. It should be mentioned that the so derived Debye temperatures are consistent with the elastic Debye temperatures obtained by solving the Christoffel equation and the following results are not sensitive to the actual choice.^{25,37} Figure 2 shows the calculated values of θ_D , ρ , v_L , v_T , and v_m for the fcc and bcc FeCrCoNiGa at both FM and PM states. Our present experimental results are also plotted for comparison. It is found that the theoretical ρ at static conditions (0 K) is the smallest in the bcc FM state ($\sim 8.26 \text{ g/cm}^3$) and the largest in the fcc PM state ($\sim 8.38 \text{ g/cm}^3$). These values are only 0.7%–2.2% larger than the experimental value ($\sim 8.20 \text{ g/cm}^3$) measured at room temperature. For sound velocity, the calculated v_L and v_T are about 5.37–5.71 km/s and 2.69–2.84 km/s, respectively, which compare well with the corresponding experimental values of 5.22 km/s and 2.82 km/s. Using the computed densities and

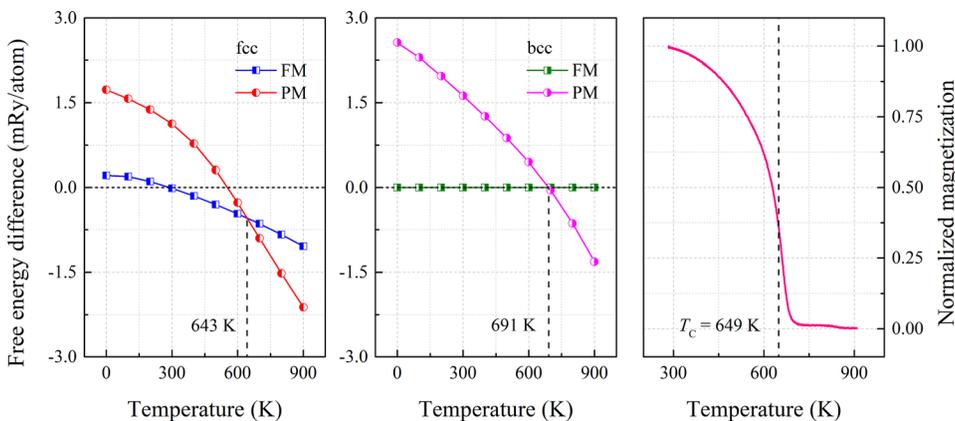


FIG. 1. Temperature-dependent free energies (left and middle panels) of the FeCrCoNiGa high-entropy alloy for the fcc and bcc structures with the ferromagnetic (FM) and paramagnetic (PM) states, respectively. All energies are plotted with respect to the corresponding energy of FM bcc. The right panel shows the measured magnetization after normalization as a function of temperature for the as-cast FeCrCoNiGa alloy. The dashed line denotes the magnetic transition temperature.

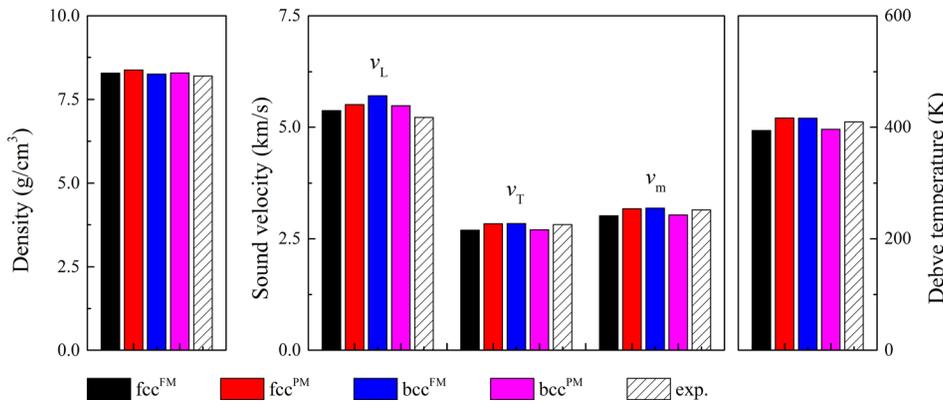


FIG. 2. Density (left panel), sound velocity (middle panel), and Debye temperature (right panel) of the FeCrCoNiGa high-entropy alloy for the fcc and bcc structures with the ferromagnetic (FM) and paramagnetic (PM) states, respectively. The experimental results are shown by the shaded bars. The experimental error bars for the density and sound velocities are 0.2% and 2.5%, respectively.

sound velocities, for the Debye temperature in the fcc FM (PM) state and bcc FM (PM) state, we get 394 K (417 K) and 416 K (396 K), respectively. These values are very close to 410 K estimated from measurements.

The corresponding theoretical elastic moduli for the present system are summarized in Table I, along with the available experimental data. Here, the bulk modulus B and its pressure derivative B' at equilibrium volume are extracted from the equation of state described by an exponential Morse-type function³⁴ fitted to the *ab initio* total energies for a series of volumes. The two cubic shear modulus $C' = (C_{11} - C_{12})/2$ and C_{44} are derived from the volume-conserving orthorhombic and monoclinic deformations.²⁵ The polycrystalline shear modulus G is estimated via the arithmetic Hill average $G = (G_V + G_R)/2$, where G_V and G_R are the Voigt and Reuss bounds, respectively.³⁸ The ratio $A_{VR} = (G_V - G_R)/(G_V + G_R)$ gives an alternative measure of the elastic anisotropy (for isotropic crystal A_{VR} is zero), which is independent of the crystal structure and thus can be used to compare anisotropies of various structures. The Young's modulus E and Poisson ratio ν are connected to B and G by the relations $E = 9BG/(3B + G)$ and $\nu = (3B - 2G)/(6B + 2G)$, respectively. As shown in Table I, the G and E values for FeCrCoNiGa are predicted to be 59.9–67.4 GPa and 159.8–178.2 GPa, respectively, which are in line with the corresponding experimental data.²¹ It is interesting to notice that, the present system is expected to be ductile because its B/G ratios in all considered phases are well above the critical value of 1.75.³⁹ Furthermore, the PM fcc FeCrCoNiGa is predicted to possess similar ductile/brittle characteristics as pure Ni based on the Gilman's line (C_{44}/C_{12} vs G/B).⁴⁰ The Grüneisen parameter γ describes the anharmonic effects and gives the volume dependence of the Debye temperature by $\theta_D(V) = \theta_D(V_0)(V_0/V)^\gamma$. We

notice that γ can be expressed as $\gamma = -f + B'/2$, and for pure metals the factor $f = 1/2$ gives a good agreement with the experimental data.^{25,34}

Figure 3 displays the theoretical lattice parameter L and thermal expansion coefficient α for FeCrCoNiGa in the temperature range of 0–900 K. The values of L at 300 K are 3.637 (3.627) Å and 2.885 (2.884) Å for the FM (PM) fcc and bcc phases, respectively, and they increase by 1.11 (1.33)% and 0.68 (1.08)% when the temperature reaches 900 K. It is evident that the temperature gives a larger influence in the fcc phase than in the bcc phase. Further insight can be gained by considering the thermal expansion coefficient defined as $\alpha = (1/L)dL/dT$. As shown in Fig. 3, we find that the calculated α for all considered phases increases rapidly at low temperatures and gradually turn towards a linear trend at high temperatures. For the highest temperature region considered here, the propensity of increment becomes moderate, especially for the FM bcc. We recall that according to experiments the present system possesses a duplex fcc/bcc structure.²¹ Hence, the actual mean α should be estimated by averaging over the individual phases. For example, the fcc and bcc phases in the FM state are in equilibrium at room-temperature. Assuming equal fractions for the two phases, we get the thermal expansion coefficient $\alpha_{300\text{K}} \approx 13.3 \times 10^{-6} \text{K}^{-1}$, which agrees well with the experimental data shown in Fig. 3. The present theoretical value is also close to that of the other HEAs such as FeCrCoNi ($\sim 14 \times 10^{-6} \text{K}^{-1}$) and FeCrCoNiMn ($\sim 15 \times 10^{-6} \text{K}^{-1}$).^{41,42} It is of particular interest to highlight that the experimental α increases sharply around the Curie temperature. We notice that theory predicts α values which increase in the order of FM bcc, PM bcc, FM fcc, and PM fcc in the entire temperature range. As a consequence, the fcc/bcc duplex structure of FeCrCoNiGa in the FM state results in a small α derived as

TABLE I. Theoretical single-crystal elastic constants (C_{11} , C_{12} , and C_{44} , in units of GPa) and polycrystalline elastic modulus (B , G , and E , in units of GPa), pressure derivative of the bulk modulus (B'), Poisson's ratio (ν), Zener anisotropy ratio (A_Z), Pugh ratio (B/G), and the elastic anisotropy (A_{VR}) for the FeCrCoNiGa high-entropy alloy for the fcc and bcc structures and for the ferromagnetic (FM) and paramagnetic (PM) states, respectively. All theoretical results refer to 0 K. The room-temperature experimental shear modulus and Young's modulus are taken from Ref. 21 and they have error bar of 5%.

Str.		C_{11}	C_{12}	C_{44}	B	G	E	B'	ν	A_Z	B/G	A_{VR}
fcc	FM	184.7	146.9	123.3	159.5	59.9	159.8	5.6	0.333	6.5	2.66	0.36
	PM	196.2	148.8	131.2	164.6	67.4	178.0	6.3	0.320	5.5	2.44	0.31
bcc	FM	215.1	162.7	122.3	180.2	66.7	178.2	4.5	0.335	4.7	2.70	0.26
	PM	193.9	156.4	125.4	168.9	60.5	162.2	5.7	0.340	6.7	2.79	0.37
Exp.						65.0	168.0					

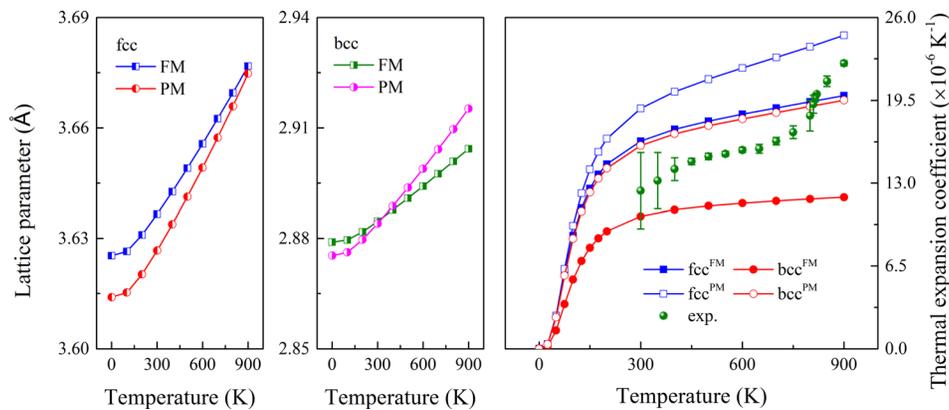


FIG. 3. Lattice parameter (left and middle panels) and thermal expansion coefficient (right panel) of the FeCrCoNiGa high-entropy alloy for the fcc and bcc structures with the ferromagnetic (FM) and paramagnetic (PM) states, respectively. The experimental results are depicted by circles with error bars.

the mean value below the critical temperature. Furthermore, the experimental α is close to the theoretical value obtained for the FM bcc phase at cryogenic conditions and to that for the PM fcc phase at high temperatures. Hence, the sizable difference of the calculated α values between the FM and PM states can explain the observed anomalous thermal expansion behavior.

In summary, our combined experimental and theoretical results demonstrate that the FeCrCoNiGa HEA exhibits an anomalous thermal expansion behavior. From the theoretical free energies, a mixture of the fcc and bcc phases is predicted around room-temperature, in good agreement with experiment. The FM bcc phase is energetically the lowest one among the considered phases at cryogenic conditions and the PM fcc phase at high temperatures. The predicted ferro- to paramagnetic transition critical temperatures are close to the measured Curie temperature. In the entire temperature range, the smallest and the largest thermal expansion coefficient occur for the FM bcc phase and PM fcc phase, respectively. The difference between the mean theoretical thermal expansion coefficients below and above the Curie point explains the observed anomalous thermal expansion behavior. The present findings demonstrate that engineering the magnetic and structure transitions provides rich opportunities for designing and optimizing new HEAs with interesting thermophysics properties. Finally, we notice that the revealed strong coupling between these two degrees of freedom brings the present and similar HEAs into the focus of magnetocaloric applications as well.

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