A STUDY OF STUDENTS' ABILITY TO SOLVE TEXT-BASED MATHEMATICAL PROBLEMS WITH IRRELEVANT OR SUPERFLUOUS INFORMATION

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A limited quantitative survey has been performed in order to study the capability of students to solve text-based mathematical problems containing irrelevant or superfluous information, as well as their ability to scrutinize text and sort out the relevant information. The students took a written test with text-based problems of different amount of irrelevant or superfluous information. The capability of understanding such problems was investigated versus degree of maturity and gender. Obtained results indicate a noticeable correlation between the occurrence of irrelevant or superfluous information and ability of solving the problems. Furthermore, the results show that the degree of maturity, expressed in terms of age, has a clear influence on the results. This study has not revealed any significant difference between genders as regards ability of sorting out the relevant information.

Introduction

Mathematics and problem-solving does not seem to be a favorite subject among Swedish students. Confusion can occur when students have to deal with problem-solving. An anecdote regarding this phenomenon is given by Curcio (1987, p. 39):

You see, daddy: I am very good in arithmetic at school. I can do addition, subtraction, multiplication, division, anything you like, very quickly and without mistakes. The trouble is, often I don't know which of them to use.

Many authors have been dealing with problem-solving techniques, skills and methods (Pólya, 1957; Curcio 1987; Malmer, 1990; Emanuelsson, 1991; Björk, 2000). They have proposed strategies to facilitate for students in mathematical problem-solving. The most well-known strategy is proposed by Pólya (1957) and contains four principles: Understand the problem, Devise a plan, Carry out the plan, and Review/Extend (Pólya, 1957, p. 26-35).

Möllehed (2001) presents 16 factors which can affect students' ability of solving problems: Understanding of text, Visual understanding, Conception of the reality, Attention, Separation, Relations between a whole and parts of a whole, Ability to do combinations, Logic, Proportional understanding, Consistency, Mathematical concepts, Understanding of numbers, Ability to calculate, Relations between different quantities, Relations between different units and Accuracy (Möllehed, 2001, p. 62-63). On the other hand, Ahlberg (2001) mentions other factors which can affect students' ability of problem-solving. He specifically names the amount of words and sentences in a problem, the degree of difficulty of the chosen words, the grammatical complexity, the amount of statements in a formulation of a problem and the structure of a problem and access to material. (Ahlberg, 2001, p. 41)

In an attempt to explain the way students think when they work with problem-solving, Arfwedson & Arfwedson (2002) claim that an incorrect answer to a problem is not due to incorrect reasoning by the students or misconceptions. This is rather due to alternative frameworks of the students, to their alternative theories or to their comprehension/understanding (Arfwedson & Arfwedson, 2002, p. 23-24). Malmer (1992) claims that students do not give an incorrect answer to a question. She believes that students rather give an answer to another question. Students can sometimes experience difficulties to follow the reasoning in mathematical texts, which is due to a compressed content that is not comprehensible (Malmer, 1992, p. 28).

Nowadays, it is common to introduce irrelevant or superfluous information in problem-solving in mathematics as well as in related subjects. This reflects the reality, and is supposed to prepare students for life. However, this is not a new concept. In USA, this method was applied in 1923 in the "Standford Achievement Test" (Paradis, 1968, p. 125), and during the 30ies the importance of such problems was significantly emphasized (Buyse, 1935, p. 410 and Goumy, 1933, p. 128). Goumy (1933) has noted that many students have the bad habit of "unconsciously" use the irrelevant or superfluous information included in the formulation of a problem.

An extreme example of this phenomenon, i.e. problems with irrelevant, superfluous or even missing information, was published by researchers of IREM (1980) in France. Sarrazy (1995) refers to IREM's work and quotes the following problem given to students of age between 9 and 10 years. "*In a boat there are 26 sheep and 10 goats. How old is the captain?*" More than three quarters of the students calculated the age of the captain through adding the numerical information in the problem (Sarrazy, 1995, p. 17). This phenomenon is known as "l'âge du capitaine", which means "the age of the captain".

Another awkward phenomenon related to problem-solving is brought forward by Löwing & Kilborn (2002) as a serious obstacle to learning. Students try to "see through" the construction of a problem in order to guess the right answer without understanding the problem itself (Löwing & Kilborn, 2002, p. 263). The authors have performed a test where Swedish students were asked to solve problems from a Japanese and a Portuguese text book. The first problem to solve was the following: "*De um livro que tem 148 páginas já li 84 páginas. Quantas tenho ainda para ler ?*" (Löwing & Kilborn, 2002, p. 264). And the second problem was: "*Tenho 24 berlindes guardados em 3 caixas. Cada caixa tem o mesmo número de berlindes. Em cada caixa estao...*" (Löwing & Kilborn, 2002, p. 264). The authors found it quite remarkable that the students, despite insufficient or non-existent understanding of the text, actually were able to guess that the first problem was about addition or subtraction. Since the larger number appeared first, presumably subtraction should be the case. In the second problem, the students noticed that the numeral 24 easily can be divided by 3. Conclusion: Division should be the way to calculate (Löwing & Kilborn, 2002, p. 264).

Bergsten (1997) claims that students sometimes manipulate the given information to reach an answer of a problem. In order to compensate the lack of understanding, students may memorize rules and procedures (Bergsten, 1997, p. 51). Sometimes they even try to concretize a situation through guessing an answer without reasoning or thinking. Hagland et al. (2005) as well as Bergsten (1997) emphasize the importance of students being able to translate a problem from an ordinary everyday language to a mathematical language of symbols. Curcio (1987) writes that "it not only is possible but also seems necessary to show *explicitly* how to translate the written words of a problem into a mathematical statement" (Curcio, 1987, p. 107). But do the students really have this ability? How will they behave if they are faced with problems containing irrelevant or superfluous information? Will they be capable of translating from an ordinary language to a mathematical language?

To sum up, it is an established fact that there is an extensive need of giving students knowledge, adequate tools, and courage to reflect in a mathematical way, as well as looking into, managing and solving text-based mathematical problems. According to Sarrazy (2003), it is possible to achieve this by encouraging students to take certain measures before they hand in a task (Sarrazy, 2003, p. 10):

- Search for information in a text-based mathematical problem and exclusively sort out the relevant information.
- Scrutinize if all given information is necessary in the formulation of the problem.
- Examine if the given information is sufficient.
- Organize the information.
- Make use and work on the relevant material in order to solve the problem.

Gender

It is becoming increasingly popular to problematize studies and compare results with respect to gender. Many authors are focusing on this aspect, e.g. Mendick (2006), Björnsson (2005), Gallagher & Kaufman (2005), Skolverket (1996), Hyde, Fennema & Lamon (1990) and Petterson (1990). Gender differences are studied regarding mathematics in general and problem-solving in particular.

In a study performed by The Swedish Agency for Education (Skolverket), the results showed that girls have less mathematical skills than boys (Skolverket, 1996, p. 1). Björnsson (2005) made a study on Swedish students and noticed that mathematics is a subject where girls are having difficulties. Regarding attitudes to the subject, girls have a bad self-confidence and their self-image in this respect is remarkably negative. (Björnsson, 2005, p. 23, 37). Furthermore, Hyde et al. (1990) state that girls can perform better in tests containing simple calculus, whereas boys perform better in tests of problem-solving. Pettersson (1990) claims that there is a correlation between gender and the cognitive level of the problems to solve. Mendick (2006) and Björnsson (2005) emphasize that mathematics is associated with masculinity (Mendick, 2006, p. 87; Björnsson, 2005, p. 23, 47). Gallagher and Kaufman (2005) explain that boys perform better in formal test situations, whereas girls perform better in the classroom and get higher grades in informal test situations (Gallagher & Kaufman, 2005, p. 301).

Aim and method

The aim of this study is to evaluate the capability of the students to extract relevant information and ignore the irrelevant or superfluous information presented in the formulation of text-based mathematical problems. The main purpose is to compare the ability of students to understand such problems with respect to gender and degree of maturity expressed in terms of age.

This study included 244 Swedish students, of which 96 students belong to upper secondary school (age 16-19) and 148 belong to higher compulsory school (age 13-16). The students have been divided in 6 groups, according to which class they attend. The selection was systematic in such a way that all the students took identical written tests with 10 text-based problems of different amount of irrelevant or superfluous information. In addition, one extra problem contained only a simple first-degree equation without text in order to check the mathematical capability (See Appendix 1). Furthermore the students wrote comments on the test. All the students accepted voluntarily to participate in the study.

A limited quantitative method has been applied, with a large statistical insecurity due to the limited number of students participating in the study. However, despite the large statistical insecurity, 224 participants should give sufficient material to make comparisons leading to relatively good conclusions.

The results have been quantified in order to be able to elaborate the answers in a numerical way. The individual results of the test were weighted, so that an incorrect answer to a problem due to irrelevant or superfluous information corresponds to 1 point, an incorrect answer which is not due to irrelevant or superfluous information corresponds to 2 points, and a correct answer corresponds to 3 points.

The chosen problems relate to one level of difficulty, namely the linguistic content. They contain an increasing amount of irrelevant or superfluous information and can all be solved using a simple first-degree equation (See Appendix 1). The main idea here is not to analyze the specific mathematical skills, but rather to analyze the way students reflect and do their reasoning. However, the total amount of text in each problem was kept fairly constant. Consequently, whether or not the students manage to solve the problem depends on their reading comprehension skills and not on their specific mathematical skills.

Reliability & Validity of the study

Despite all measures taken to keep a high level of reliability and validity in this study, some factors should be pointed out specifically. For reliability, no group of students was favored. Regardless of age, class and gender, all the students took the same written test and were not allowed to use calculators or help. Although a high level of validity was desired, the results may have been affected by a number of factors, e.g. limitation in time for the test, the limited number of participants in the study, students' cheating and the attitude of the teachers who, sometimes, tried to help their students. These factors may not be quantified, but should nevertheless be kept in mind.

Results

Three questions have been investigated in this study.

- Is it confirmed that there is a high correlation between the occurrence of irrelevant or superfluous information in a problem formulation and the ability of students to solve text-based mathematical problems?
- Is it confirmed that the degree of maturity, expressed in terms of age, has a clear influence on the results?
- Is mathematics a masculine subject? Is it correlated to gender or is it gender-neutral?

When reviewing the obtained results, it appears that the students had a higher performance when the problems contained less irrelevant or superfluous information. The percentage of irrelevant or superfluous information is reflected in the frequency of the amount of incorrect answers, since the tendency is increasing, however not strictly increasing (See Figure 1, Appendix 2).

Maturity, here expressed in terms of age, reflects the same increasing tendency of incorrect answers with an increasing amount of irrelevant or superfluous information. In other words, older students performed better than younger ones, and the percentage of their incorrect answers was correlated to the amount of irrelevant or superfluous information. This phenomenon is due to the fact that older students may have higher skills in reading comprehension than younger students. However, the obtained results show that for certain problems it was not obvious that older students performed better than younger ones (See Table 1, Appendix 2). For example in the last problem that contains the most irrelevant or superfluous information, it was the oldest students (last year students of upper secondary school) that performed the worst among all the students in this study. Such cases will be discussed individually in the Discussion section (See Table 1, Appendix 2).

Obtained results show no significant correlation to gender. An interesting observation when comparing the results is that both genders showed the same tendency concerning problem-solving. It is impossible to assert that one gender is superior in comprehension or problem-solving skills over the other. Consequently, it is impossible to conclude a tendency, since the frequency of incorrect answers is equally divided between genders. Obtained results show that sometimes female students and sometimes male students performed better.

The students were encouraged to write their comments on the test. Many students thought that the test was difficult, despite the fact that the solution consisted only of a simple first-degree equation. Below is a sample of some interesting answers.

- It contained unnecessary information.
- For certain problems, I did not understand what I was supposed to calculate.
- I did not understand all the problems, the test is written in a somewhat complicated way.
- A lot of information which is not needed to solve the problems was given, which made it a little confusing.
- The problems were strange.

Discussion

It is logical to presume that all students, regardless of age and maturity level, should perform in a similar way, i.e. to have more incorrect answers as the amount of the irrelevant or superfluous information increases in the problems. However, this was not case. One can imagine an increasing function illustrating this tendency. Although it is clear that there is an increasing tendency, yet it is not possible to obtain a strictly increasing trend (See Figure 1, Appendix 2). Some factors that may have affected the results are discussed below, as well as some reflections worthy of mentioning.

- The formulation of a problem has a big influence on the students' capability of mathematical problem-solving. An adapted context is always in the favor of the students, especially if the problem formulation is taken out of their own reference framework and particularly related to their personal interests. In this study, it was clear that the problem dealing with feeding a puppy was much easier for students to solve than the problem dealing with timing of skiers, even if the latter problem contained less irrelevant or superfluous information than the former (See Appendix 1).
- The choice of schools was done arbitrarily, however located in the same city. When comparing the results of two corresponding classes in two different schools, the results diverged significantly. This indicates that the choice of schools can influence the results to a high degree due to socio-economical conditions, and that it is rather questionable if the same results can be obtained when a similar study is performed elsewhere.
- The presence of immigrant students in the Swedish schools may be a factor that should be taken in consideration when analyzing the obtained results. In fact, the understanding of text-based mathematical problems is highly related to the understanding of the language. Regardless if immigrant students -from certain countries- may have better skills in mathematics, their capacities are significantly reduced when they are confronted with text-based mathematical problems due to the language barrier.
- The oldest students performed worse than the younger ones in the last problem, which contained the most irrelevant or superfluous information. Since the problem was dealing with lottery issues, the older students tried to complicate their answers using advanced mathematical concepts, i.e. probability theory, instead of unveiling the problem as a simple equation (See Appendix 1). The younger students had an advantage since the concept of probability was not yet introduced in the mathematics curriculum and thus they solved the problem the simple way.

– According to the teachers of the participating classes, the test contains too much text. It should be pointed out that this study did not include students with special educational needs. Based on the teachers' opinion and on the comments of the students, it is obvious that students dislike or are untrained to solve text-based mathematical problems. This phenomenon is reflected by a few comments made by the students:
  -Difficult test, I hate problems with reading.
  -One is forced to read carefully in order to calculate the right thing.
  -One was forced to read the question 2-3 times to avoid misunderstanding it.
  -For some problems one should read again and again in order to understand.
– Some comments given below made it clear that students have tried to "see through" the construction of a problem in order to guess the right answer without having to understand the problem itself. This is a typical unwanted behavior when students are solving mathematical problems.
  -I added all together.
  -I added a little here and there.

Conclusion

In this study it is claimed that there is an obvious relation between the occurrence of irrelevant or superfluous information in text-based mathematical problems and the students' solving ability. Obtained results illustrate an increasing tendency of incorrect answers with an increasing content of irrelevant or superfluous information (See Figure 1, Appendix 2). Furthermore it is shown that the maturity level, expressed in terms of age, has a significant influence on the results. Older students performed better than the younger (See Figure 1, Appendix 2). This study has not revealed any significant difference between genders as regards ability of sorting out the relevant information.

Further researches related to this paper is suggested to examine if the theoretical strategies presented by Pólya (1957), Curcio (1987), Malmer (1990), Emanuelsson (1991) and Björk (2000) can optimize the mathematical performance of students and contribute to a better capability in problem-solving. An approach to such investigations is called *Learning studies* which include observations and iterates teaching (Marton et al, 2004). Another idea for further researches is to study the phenomenon related to the oldest students' attempt to complicate the answers with advanced mathematics as soon as they come across an ambiguity. Finally, a necessity for future educational development of school mathematics is to understand students' lack of motivation and commitment towards the subject. Without this knowledge, any pedagogical effort appears to be meaningless.

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Appendix 1: Test

1. Solve this equation: x+54=876
2. The duration of a football match is 165 minutes. After 92 minutes Hanna falls and hurts herself badly. The coach decides to exchange players. Cecilia is the one to replace Hanna. How long time did Cecilia get to play?
3. Kalle and his friend Fredrik are in the same school. Kalle's way to the school is 876 m. Fredrik must walk 54 m. less than Kalle in order to arrive to school. How long is Fredrik's way to the school?
4. Before the winter season it is important to have the right ski equipment. Erik bought ski boots in size 42. He had 876 crowns in his wallet. When he went out of the shop he had 54 crowns left. How much did the boots cost?
5. Some friends are comparing how tall they are. Lisa is 165 cm tall, Pelle is 180 cm tall and Kalle is 175 cm tall. How much taller is the tallest person than the shortest person?

6. A visit to Marc-Philippe's gym costs 80 crowns for youth and 100 crowns for adults. A season card costs 900 crowns for youth and 1200 crowns for adults. 15 years old Hasse and his dad buy each a season card. How much do they have to pay?
7. The distance between Borås and Falun is 421 km, between Falun and Gävle 90 km, between Gävle and Karlstad 315 km, and between Gävle and Örebro 229 km. How long is the distance between Falun and Karlstad?
8. You download 3 mp3 tunes from Internet. The tunes are 4 min, 3 min and 6 min long, respectively. You pay 100 crowns for the membership and each tune costs 5 additional crowns to download. How long can you listen to your tunes if you play them once each?
9. Per weighs 89 kg. He is skiing down a slope with an average speed of 12 m/s. His time is eight seconds. His friend Jonas weighs 90 kg and is skiing down the same slope in 2 seconds shorter time. His average speed is 16 m/s. How long time does it take for Jonas?
10. A puppy weighs 5 kg and eats 0,4 kg dry food and 0,3 kg of meat and drinks 2 litres of water each day. A bag of dry food weighs 20 kg and costs 100 crowns. The meat costs 50 crowns per kg. How much food does the puppy eat during a day?
11. Pia draws 16 lots in a lottery where the lots are numbered from 1 to 100. All the lots ending with 77 give a prize of 100 crowns and those ending with 3 give a prize of 10 crowns. Pia gives away 4 lots to her sister and keeps the rest. How many chances has Pia to win?

Appendix 2: Results

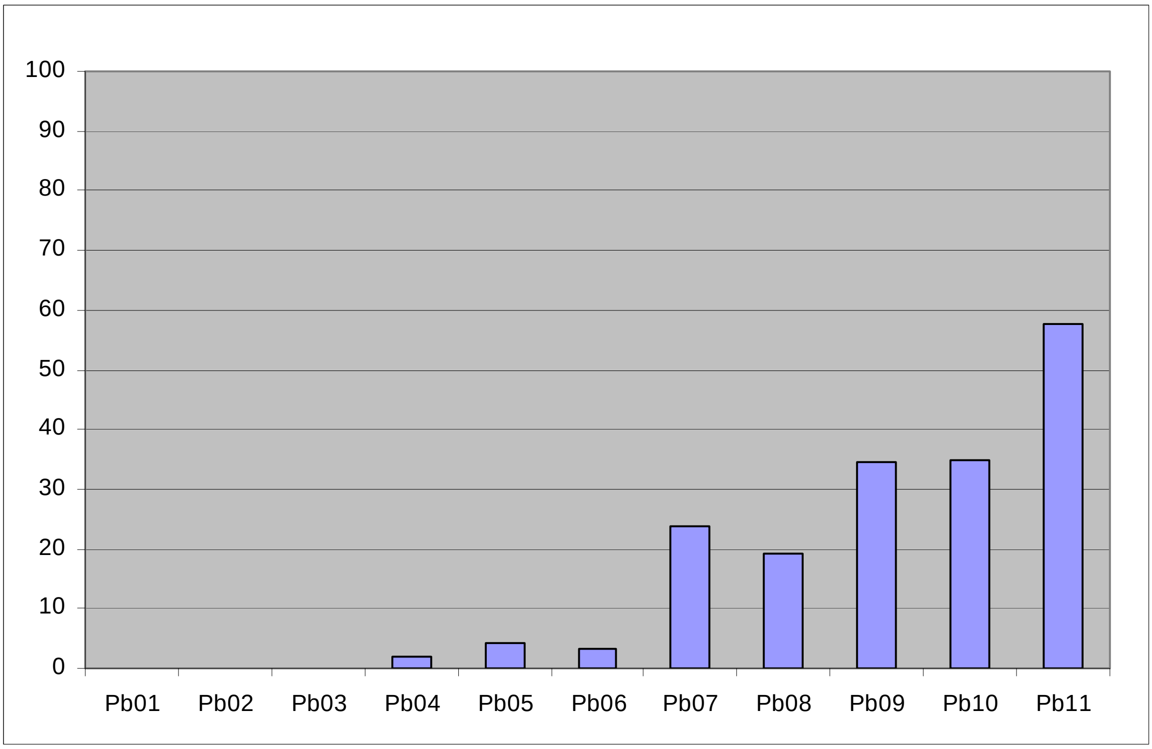

Figure 1. Frequency in percents of incorrect answers due to irrelevant/superfluous information for all the students participating in the study.

	13-14 years old	14-15 years old	15-16 years old	16-17 years old	17-18 years old	18-19 years old
Pb04	4.3	3.2	0	0	2.8	0
Pb05	6.5	4.8	5	2.4	0	5.3
Pb06	8.7	6.4	0	0	0	0
Pb07	17.4	30.6	45	9.8	22.2	5.3
Pb08	37	25.8	15	9.8	11.1	0
Pb09	41.3	51.6	35	24.4	19.4	10.5
Pb10	52.2	43.5	32.5	12.2	27.8	31.6
Pb11	60.9	50	60	63.4	52.8	68.4

Table 1. Frequency in percents of incorrect answers due to irrelevant/superfluous information for all the students participating in the study, divided according to degree of maturity, expressed in terms of age.