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Task design with a focus on conceptual and creative challenges

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Tasks are an important part of the education in mathematics. In an ongoing study, an analytic framework for identifying challenges in students mathematical task solving has been developed, and the conceptual and the creative challenge has been defined. Preliminary results indicate that considerations are needed to include these challenges in mathematical tasks. This paper takes off from there to describe a structure for selection and (re)design of tasks. The aim is to be able to discuss the basis for the structure. A further aim is to develop a support for teachers, test designers, textbook authors and others, in creating tasks with specific learning goals.

Keywords: Task design, challenges, mathematical concept.

Introduction

Tasks are a central aspect of the teaching and learning of mathematics. They may offer students an opportunity to learn and to gain understanding. One way of defining understanding is to manage different mathematical tasks and thereby also overcome obstacles (Sierpinska, 1994). Sriraman, Haavold and Lee (2013) however describe the concept of task difficulty as hard to define and to operationalize. One way of gaining insight into this may be to regard a task as possibly including different types of difficulties, with unique definitions. This also supports the idea of having clear intentions of what learning goals may be reached by solving a task, and to include criterion for success in line with these goals as important aspects of teaching (Brousseau, 1997; Brousseau & Warfield, 2014; Simon, 1995). The opportunities to learn depends on the design of the task and the cumulative effect of certain kinds of tasks is important to consider (Stein & Lane, 1996). Different learning goals imply the use of different kinds of tasks. The definition of two challenges, the conceptual and the creative challenge has, in an ongoing study, been developed from Lithner (2017), and an analytic framework has been developed to be able to identify and characterize these challenges. With a better understanding of tasks, their solutions, and the challenges included, a nuanced discussion of students’ opportunities to learn is in focus. Central to the study has been to make it possible to make a distinction between the two challenges. However, the results also indicate that the two challenges exist together and possibly also interplay. One of the important ideas of the framework is that discrepancy between students prior understanding and experiences and what a task requires them to do, is what creates a challenge. Earlier research has indicated that students benefit from working on tasks where they are challenged to go beyond using routine procedures (Jonsson et al., 2014), solving mathematical problems, and struggling with mathematical ideas (Hiebert & Grouws, 2007). However, unfortunately, students in their task solving often use imitative approaches rather than deal with challenges (Sidenvall, Lithner, Jäder, 2015). Preliminary results from the ongoing study indicate that the sum of challenges, conceptual and creative ones, affects students’ approach to the task solving. The students in the study, when overcoming
conceptual challenges also overcame creative ones. There are however tasks with creative challenges, but no conceptual challenges.

This paper therefore, and supported by the framework and definitions previously developed, suggests a task design structure to include conceptual challenges to a greater extent in tasks, and specifically mathematical problems.

Background

Tasks used as artifacts in the teaching and learning of mathematics

A task in this paper is seen as an exercise intended to support the students’ learning (Halldén, Scheja & Haglund, 2008). There is a clear distinction between mathematical problem solving, and other work on mathematical tasks, where a problem is a task to which the individual has no obvious method for solution (Schoenfeld, 1985). In contrast to mathematical problems, are tasks of routine character where the solution method is available to the solver as she identifies the type of question and what is a suitable, corresponding solution method. The use of mathematical tasks in the teaching and learning of mathematics highlights the relationship between task, student, teacher and the subject matter, mathematics. In this paper, the focus is on the design of the task. Brousseau (1997) suggests the arrangement of adidactical learning situations, where students get to connect with specific mathematical ideas according to the learning goals of the task. The teacher, for a while, delegates the responsibility of taking actions on the task, to the student. So, the teacher acts on the student through an activity, in this case a mathematical task. It has been shown that a textbooks and its tasks is important when teachers structure their teaching (Mullis, Martin, Foy, & Arora, 2012). It seems like a common way of approaching the textbook and its tasks, at least in a Swedish context, is by working from the beginning of each chapter and finish up when time has run out (Sidenvall et al., 2015). However, this may lead to students meeting few challenging tasks (Jäder, Lithner & Sidenvall, 2015). Simon and Tzur (2004) describe a relation between an activity, and the effects of performing the activity, to explain how thoughtfully designed tasks can contribute to the development of a student’s conceptual understanding. To actively address specific learning goals, the selection, evaluation and possibly redesign of tasks is important.

Conceptual challenges

Mathematical concepts can be regarded as cognitive objects with different pieces of information attached to them. An ongoing encapsulation develops an elaborated picture of the concept within a person (Tall & Vinner, 1981). Conceptual understanding can be defined as the relationship between different pieces of information into an internal, mental network (Hiebert & Carpenter, 1992). Thus, the development of a conceptual understanding must have a student’s current understanding as a basis (Simon & Tzur, 2004). Relationships are based on similarities and differences and, in a hierarchical way, on general and special cases, where a piece of information may include parts of another piece of information (Hiebert & Carpenter, 1992).

Tall and Vinner (1981) describe a difference between a person’s concept image, and the formal concept definition. A person’s concept image is the cognitive structure associated with a specific concept. It is a mental picture of properties and processes related to the concept (Tall & Vinner,
The concept image develops over time, and with new experiences. The concept image, to a great deal, depends on the specific examples of the concept that a person has been presented to. Thus, a possible discrepancy between concept image and concept definition may occur with, for example, tasks covering only a limited part of the aspects related to the concept (Niss, 2006). The part of the concept image that a student activates in a specific situation can be called the evoked concept image (Tall & Vinner, 1981). At times, a student’s concept image will be challenged in some way. It may be either by meeting a situation where the mental picture is not covering enough or is not clear enough in relation to the demands of the situation. The new ideas are thus assimilated (Piaget, 1952) with what the student already knows, adding to the concept image. It may also be that the concept image is in contrast with new experiences, and requires the student to change the mental picture in some way to incorporate the new ideas. This is what Piaget (1952) describes as accommodation, which may be seen as a two-step process, where the first step is the adjustment of the existing concept image, and the second step is an assimilation of the new ideas.

For the purpose of this paper, a conceptual challenge is to make the necessary considerations of, and to properly use the (non-trivial) mathematical concepts, required in order to solve the task. For a task to include a conceptual challenge, there needs to be some kind of discrepancy between students’ current (and evoked) concept image and a suitable concept image for the task.

**Creative challenges**

Creativity is defined as “the use of the imagination or original ideas to create something” (Creativity, n.d.). In a school setting, and relating to mathematical problem solving, mathematical creativity can be defined as a process resulting in what is, to the solver, a new solution (Sriraman et al., 2013). Hence, the solver, rather than using an available template has to create her new solution method. The definition does not contradict the idea that problem solving skills can be achieved at a basic level, as well as at a more advanced level. However, as problem solving does not mean following a given algorithm, the problem solving process is hard to predict. Explorations, and hypothesizing may well be part of a problem solving process. To argue that a familiar method is reasonable to apply in a new situation may also be defined as creating a substantial part of a solution method. The characteristics of tasks are important in catering for a learning environment where problem solving is made easily available to the students (e.g. Jonsson et al., 2014; Stein & Lane, 1996).

The definition of a creative challenge used in this research, is that the challenge is involved in the creation of what is to students, a new solution method, or the selection and modification of a familiar method in a new situation where the choice of method is not obvious. This can be either the creation of an overall method, sequencing a number of well-known and/or new sub-methods. Or it can be the creation of a new sub-method, as part of an overall solution method. Thus, for it to be a creative challenge, there needs to be a discrepancy between students’ prior experiences and a conceivable method.

**Task selection and (re)design to reach learning goals**

The discrepancy between task requirements and students’ concept image and previous experiences is here used as an indicator of the challenges of a task. Considering (reasonable) task challenges to
be valuable to students’ learning (Hiebert & Grouws, 2007; Jonsson et al., 2014), the discrepancy also refers to the opportunities to learn. Depending on the selection and the (re)design of tasks, different learning goals may be possible to obtain (Stein & Lane, 1996). Specific learning goals, and students’ current state of knowledge are important factors for teachers when making informed teaching decisions (Simon & Tzur, 2004). The decisions made in a selection and (re)design process can gain strength if informed by possible discrepancies and students’ approaches to tasks.

**Purpose**

The purpose of this paper is to present and discuss a structure for the selection and (re)design of tasks, to include conceptual as well as creative challenges in a task. The aim of this is to empirically study the effects of the design structure with both students and teachers. A further aim is thus that the structure will be of help to teachers, but possibly also test designers, textbook authors and others participating in the selection and (re)design of tasks used for the teaching and learning of mathematics.

**A structure for task selection and (re)design**

This study, in line with the development of the framework for identifying conceptual and creative challenge, has a focus on discrepancies between students’ concept image and their experiences with different methods, and what is required of them to solve a task. The discrepancy is a way to describe the opportunities to learn. By selecting and (re)designing tasks to fit a discrepancy to a specific learning goal, the opportunities to learn can be enhanced. There are assumptions about a student’s knowledge, concept image, mathematical background and way of approaching a task, but may as well be assumptions of a (rather homogenous) group of students (Simon, 1995). The effects of a solution process depend on the learners’ goal with the activity, possibly not tightly related to a learning goal, but for example to solve the task. Considering the importance of textbook tasks in mathematics classrooms, the structure makes use of existing tasks, gives support to an evaluation and suggests a direction for modification to these tasks.

1. The starting point of the structure is the learning goal. A well-defined learning goal is important to create a suitable task (Brousseau, 1997; Brousseau & Warfield, 2014, Simon, 1995). The **learning goal is here described as aspects of a desired concept image.** The description is focused on the concept(s) included in the learning goal, but also assumes that the learning goal is a clear widening of the understanding of a concept or the addition of a new concept (or property thereof). 
   **Example:** The learning goal is to develop a concept image including that unknowns can be represented as variables, and two unknowns can be related to each other through different relations and represented algebraically. An unknown can be part of several relations. *Through such a system of relations each of the unknowns can possibly be determined.*

2. The student’s current state of knowledge (Simon & Tzur, 2004) is described in terms of a concept image and of experiences with solution methods linked to the mathematics in focus. 
   **Example:** The student’s concept image is assumed to include that unknowns can be determined through the use of equations, where the unknown is represented by a variable
and related to constants and coefficients in an equality. Methods for solving equations algebraically are familiar, as are trial and error approaches to tasks.

3. A candidate for a task in the area of the learning goal is found in the textbook. Example: A group of friends visit a coffee bar. Some order coffee at the price of 16 kr per cup and some order café latte at the price of 24 kr per cup. When the bill arrives the total cost is 296 kr. How many had a cup of coffee and how many had café latte if there were 15 friends?

To be able to perform an evaluation of the suggested task a conceivable solution method and the possible creative and conceptual challenges related to these solutions are needed. The evaluation is based on the definitions of the challenges presented above.

4. Conceivable solution methods are proposed. Conceivable in this sense makes it necessary to also consider that the learners’ goal is possibly not primarily the initially defined learning goal, but rather solving the task. Example: The task can be solved in several way. Here, two example are presented. a) The task can be solved by setting up and solving a system of equations b) The task can also be solved by a trial and error approach.

5. Possible creative and conceptual challenges are visualized. The discrepancy between the proposed methods and students’ prior experiences gives indications of creative challenges. The discrepancy between a, for each solution method suitable concept image, and an assumed concept images indicates possible conceptual challenges included in the task. Example, creative challenge: a) Setting up a system of equations, and solving this is not familiar to the student and will thus be a creative challenge. b) The student has experience with using trial and error to find the value of an unknown. In other context she has also experienced two unknowns, such as when suggesting a format for a rectangle a specific area. This will not be a creative challenge to the student. Example, conceptual challenge: a) Not included in the student’s concept image is how two variables can be represented in two relations. Further, the contexts, with relations based on two different quantities (number of cups and cost) requires the student to use the concept of unknowns and their relation to each other in a more flexible way. Thus, this is a conceptual challenge b) When using a trial and error approach to solve the task, the student does not have to consider the relations of the two unknowns and how to represent them algebraically to the same extent. The approach is deemed to include no (or a low) conceptual challenge.

6. The possible challenges of the task are compared to the learning goal. The comparisons indicate in what way a task can be redesigned. If there are more than one conceivable solution method to the task, it is also important to consider all of these. I) If the solution method avoids addressing the learning goal and the desired concept image, the task design should make the approach less favorable. Similarly, if the challenges is too small to address the learning goal, the design needs to consider this. II) If the challenge related to the solution method is deemed to be too high, modifications to the task can be made if the learning goal can still be reached.
Example: I) The use of the method of trial and error in this case might not contribute to a conceptual challenge in a desired way, and might be possible prevent.

II) It is possible that the conceptual challenge, including a flexible use of unknowns related to two different quantities is too high. It might be possible to redesign the task so that this aspect is not included, however, still including a conceptual challenge addressing the learning goal.

7. Suggestions for adjustments of the task to meet the requests of the evaluations.

Example: II) Change the requirements to possibly lower the conceptual challenge of setting up and solving a system of equations: “Two families visit a coffee bar. Some order coffee and some order café latte. The Simpsons order 3 coffee and 2 latte, and the total cost is 96 kr, while the Bundys order 1 coffee and 3 latte, and their total cost is 88 kr. How much does a cup of coffee cost, and how much does a cup of café latte cost?”

I) Change the numbers of the task (and therefore also the context) so that trial and error is made less attractive to use: “Two families are building a shed each. For the exterior they need panel boards of different dimensions. They need thin panel boards and wide panel boards. The Simpsons buy 171.5 meters of thin panel boards and 39.5 meters of wide panel boards, and the total cost is 3692 kr, while the Bundys buy 48.2 meters of thin panel boards and 149.7 meters of wide panel boards, and their total cost is 4364 kr. How much does each of the different panel boards cost per meter?”

A redesign of the task takes into consideration all the information from the evaluation. A process of evaluating and redesigning may iterate until a desirable task where the discrepancy between the suitable concept image and the current concept image coincides with the learning goal.

Discussion

Following this theoretical paper are empirical studies of how the proposed structure for task design works in relation to both students and teachers. The next step is to try the structure, and to redesign a number of tasks starting with a specific learning goal. Yet another aspect to examine is the way teachers make use of the structure, and the way they understand the included terminology to enhance the opportunities for learning for students through task solving. There may be aspects where the theoretical construct does not oblige to the circumstances. On the other hand, the structure may possibly serve as a foundation for task design and for further elaboration on how this can be done.

One aspect to consider is how tasks work in a social context. Solving tasks and overcoming challenges together with others implies that, along with reconfiguring one’s concept image, it may prove necessary to also create meaning within the group (Hiebert & Grouws, 2007). Included in such work should be considerations regarding how homogenous the concept images of a group of students really are.

The reflexive relationship between, for example a teacher’s considerations of a student’s work with a task and the design of the task, is likely to continue as the task is in use. As well as redesigning the task after the lesson, a teacher may also act in the moment to answer to a new evaluation of the situation. For example, as a teacher, it may not at all times be possible to fully capture what understanding of a concept implies. It is therefore important to let the task design structure and the evaluation phase continue during and after the task is used in the classroom. As one way of looking
at understanding is to consider the difficulties a student encounters, the discrepancy between the suitable and evoked concept image may be possible to reformulate as the task is used in the classroom (Sierpinska, 1994; Simon, 1995).

References


