Buckling and free vibrations behaviour through differential quadrature method for foamed composites

Dasari Duryodhana\textsuperscript{a}, Sunil Waddar\textsuperscript{b}, Dileep Bonthu\textsuperscript{a}, Jeyaraj Pitchaimani\textsuperscript{a}, Satvasheel Powar\textsuperscript{c,d,}\textsuperscript{*}, Mrityunjay Doddamani\textsuperscript{c}

\textsuperscript{a} Department of Mechanical Engineering, National Institute of Technology Karnataka, Surathkal, India
\textsuperscript{b} Department of Mechanical Engineering, MVJ College of Engineering, Bangalore, India
\textsuperscript{c} School of Mechanical and Materials Engineering, Indian Institute of Technology Mandi, Mandi, 175075, Himachal Pradesh, India
\textsuperscript{d} School of Technology and Business Studies, Energy Technology, Dalarna University, Falun 791 31, Sweden

\textbf{ARTICLE INFO}

\textbf{Keywords:}
Syntactic foam
Differential quadrature method (DQM)
Axial varying loads

\textbf{ABSTRACT}

The current work focuses on predicting the buckling and free vibration frequencies ($f_n$) of cenosphere reinforced epoxy based syntactic foam beam under varying loads. Critical buckling loads ($N_{cr}$) and $f_n$ are predicted using the differential quadrature method (DQM). $N_{cr}$ and $f_n$ have been calculated for beams of varying cenosphere volume fractions subjected to axial load under clamped-clamped (CC), clamped-simply (CS), simply-simply (SS), and clamped-free (CF) boundary conditions (BC’s). Upon increasing the cenosphere volume fraction, $N_{cr}$ and $f_n$ of syntactic foam composites increases. These numerical outcomes are compared with the theoretical values evaluated through the Euler-Bernoulli hypothesis and further compared with experimental outcomes. Results are observed to be in precise agreement. The results of the DQM numerical analysis are given out for the different BC’s, aspect ratios, cenosphere volume fractions, and varying loads. It is perceived that depending on the BC’s, the type of axial varying loads and aspect ratios has a substantial effect on the $N_{cr}$ and $f_n$ behaviour of the syntactic foam beams. A comparative study of the obtained results showed that the beam subjected to parabolic load under CC boundary conditions exhibited a higher buckling load.

1. Introduction

Composite materials are naturally available and artificially manufactured by combining two or more chemically dissimilar phases separated by a distinct interface. These possess significant properties of both constituent materials \cite{1}. Typically, these composites consist of reinforcing materials like flakes, fibers, and particles embedded in a matrix (ceramic, metal, and polymer). The reinforcement material improves the mechanical properties, while the matrix holds the reinforcement material in the desired shape. These composites developed by reinforcing hollow fillers in a matrix are syntactic foam \cite{2,3}. The significant advantages of these foams are their low density \cite{4,5}. These composites find their application in various industries \cite{6}. For example, in aerospace applications, syntactic foam produces rocket engine castings, antenna baffles, landing gear doors, nacelles, stabilizers, wing boxes, aircraft wings, etc. \cite{7}. As the components developed by these composite materials are subjected to various loads or combinations of loads, it is necessary to study the component’s response to them.

The different loads or their combinations may not always be uniform. For example, loads placed on the wings of aircraft, the stiffening beams of ship structures, or the slabs passing through multi-storey buildings are often non-uniform \cite{8}. The stiffness, strength, ultimate displacement capacity, and all hysteresis properties of reinforced concrete columns can be considerably altered by fluctuating axial load during earthquakes \cite{9}. The inelastic response of the beam is substantially impacted by changes in the intensity of the axial load \cite{10-12}. Among the most probable structural failure, buckling is caused due to application of axial load \cite{13-16}. Therefore, a comprehensive determination of the maximum sustainable buckling load is necessary to avoid this failure. This can be done using experimental, analytical, and numerical techniques. M.A. Hamed et al. \cite{17} presented the optimization of sandwich functional graded beams static stability under the axial load. M.A. Eltaher et al. \cite{18} studied $N_{cr}$ and $f_n$ the behaviour of composite orthotropic symmetric and un-symmetric laminated beams under six

\textsuperscript{*} Corresponding author. School of Mechanical and Materials Engineering, Indian Institute of Technology Mandi, Mandi, 175075, Himachal Pradesh, India.
\textit{E-mail addresses: spw@du.se (S. Powar), mrityunjay@iitmandi.ac.in (M. Doddamani).}

https://doi.org/10.1016/j.rineng.2023.100894

Received 28 November 2022; Received in revised form 12 January 2023; Accepted 12 January 2023
Available online 14 January 2023

2590-1230/© 2023 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).
parameter-based Eigen value problem by discretizing and substituting behaviour of tapered columns loaded axially with end restriction of Bernoulli, combined with the nonlocal elasticity theory, can predict sets of axially varying loads.

Armagan Karamanli et al. [22] found the method could give accurate results even considering the low number of initial and boundary value problems. They also mentioned that this numerical technique named DQM, which can be applied to solving make prediction easier [20]. In 1971, Bellman et al. [21] developed a

Mohamed Taha et al. [26] used DQM to analyze the stability and that DQM is a sound technique that can converge results accurately.


differential equations of the beam under different explored much. In the present study, DQM is applied to solve governing

<table>
<thead>
<tr>
<th>Load type</th>
<th>Load symbol</th>
<th>( a_2 )</th>
<th>( a_1 )</th>
<th>( a_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant load</td>
<td>N1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Linear load-zero from left side</td>
<td>N2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Linear load-zero from right side</td>
<td>N3</td>
<td>0</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Parabolic load-zero from left side</td>
<td>N4</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Parabolic load-zero from right side</td>
<td>N5</td>
<td>3</td>
<td>-6</td>
<td>3</td>
</tr>
<tr>
<td>Symmetric parabolic load</td>
<td>N6</td>
<td>-6</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

sets of axially varying loads.

Different theories like Levinson, Reddy, Timoshenko, and Euler-Bernoulli, combined with the nonlocal elasticity theory, can predict this buckling load and free vibrations [19]. The complexity to predict \( N_{cr} \) and \( f_n \) exhibited increasing trend with respect to different number of variations in load, hence numerical approaches are mostly preferred to make prediction easier [20]. In 1971, Bellman et al. [21] developed a numerical technique named DQM, which can be applied to solving initial and boundary value problems. They also mentioned that this method could give accurate results even considering the low number of grid points. Armagan Karamanli et al. [22] found the \( N_{cr} \) for laminated composites and sandwich beams under six sets of axially varying loads using the Ritz method, shear-based displacement fields, and normally deformable beam theory. The study reveals a conclusion that the axial varying loads remarkably affect the \( N_{cr} \) of the beams depending on the BC's. Gurkan Sakar [23] studied \( f_n \) analysis of beam subjected axial load using an Euler beam assumption and the FEM. Zhangxian Yuan et al. [24] have done the pre and post-buckling behaviour of the extensible beam-column. They used the DQ method to evaluate \( N_{cr} \) and post-convergent buckling solutions at different loads. Moreover, they observed that the DQ numerical integration method outperformed Runge-Kutta method of fourth-order in terms of efficiency for higher application loads. H. Haftchenari et al. [25] presented \( f_n \) analysis of composite cylindrical shells with various BC's using DQM. Results show that DQM is a sound technique that can converge results accurately. Mohamed Taha et al. [26] used DQM to analyze the stability and \( f_n \) behaviour of tapered columns loaded axially with end restriction of rotation and translation. They converted entire problem in to two parameter-based Eigen value problem by discretizing and substituting the BC's in the governing differential equation using DQM. In this one Eigen value will give the buckling result and other gives the natural frequency. Mohamed Nassar et al. [27] have been used DQM to validate the \( f_n \) the behaviour of a functionally graded cracked cantilever beam supported on a Winkler-Pasternak elastic foundation. Laxmi Behera et al. [28] used DQM to predict the \( f_n \) of nanobeams, combined with nonlocal elasticity theory, using various beam theories such as Timoshenko, Euler-Bernoulli, Reddy and Levinson. Apart from these, there few research works done on predicting the \( N_{cr} \) and \( f_n \) using FEM [29,30].

From the above literature, many researchers have worked on static and dynamic analysis of plates, shells, and non-uniform cross-section beams using DQM. Adopting the DQM method among the available finite element methods helps predict accurate results by considering the minimum number of grid points. This method helps solve boundary condition-based problems with less computational time and complexity. It is also observed that better convergence is found in DQM compared to the other numerical approaches. Several analytical, numerical, and experimental investigations have been conducted to analyze the \( N_{cr} \) and \( f_n \) the behaviour of the beams, but the influence of variable axial load on the free vibration behaviour of syntactic foam composites are not explored much. In the present study, DQM is applied to solve governing differential equations of the beam under different BC's (CC, CS, SS, and CF) under six sets of axially varying loads. The differential equation governing the motion of beams is derived from the Euler-Bernoulli beam theory. Influence of volume percentage of cenosphere on the \( N_{cr} \) and \( f_n \) characteristics are analyzed.

2. Problem definition

Currently, equipment for experimentation on beams under axial varying loads (Fig. 1) and different boundary conditions like (CS, SS, and CF) have not been sufficiently available. Sunil et al. [31] have experimentally and analytically evaluated \( N_{cr} \) values and \( f_n \) of nanobeams, combined with nonlocal elasticity theory, using various beam theories such as Timoshenko, Euler-Bernoulli, Reddy and Levinson. Apart from these, there few research works done on predicting the \( N_{cr} \) and \( f_n \) using FEM [29,30].

From the above literature, many researchers have worked on static and dynamic analysis of plates, shells, and non-uniform cross-section beams using DQM. Adopting the DQM method among the available finite element methods helps predict accurate results by considering the minimum number of grid points. This method helps solve boundary condition-based problems with less computational time and complexity. It is also observed that better convergence is found in DQM compared to the other numerical approaches. Several analytical, numerical, and experimental investigations have been conducted to analyze the \( N_{cr} \) and \( f_n \) the behaviour of the beams, but the influence of variable axial load on the free vibration behaviour of syntactic foam composites are not explored much. In the present study, DQM is applied to solve governing differential equations of the beam under different BC's (CC, CS, SS, and CF) under six sets of axially varying loads. The differential equation governing the motion of beams is derived from the Euler-Bernoulli beam theory. Influence of volume percentage of cenosphere on the \( N_{cr} \) and \( f_n \) characteristics are analyzed.

2. Problem definition

Currently, equipment for experimentation on beams under axial varying loads (Fig. 1) and different boundary conditions like (CS, SS, and CF) have not been sufficiently available. Sunil et al. [31] have experimentally and analytically evaluated \( N_{cr} \) values and \( f_n \) of syntactic foam beams under end compressive axial load under clamped-clamped BC’s. These beams are developed by reinforcing different volume fractions of cenosphere in epoxy resins (E00, E20, E40, and E60), here, E represents epoxy, and numbers represent filler volume fraction. In the present work using the numerical based DQ method, \( N_{cr} \) and \( f_n \) values are determined for syntactic foam beam exposed to axial compressive load under CC, BC’s. These numerical results are validated with experimental as well as
theoretical results. MATLAB code has been developed for finding results using DQM. In addition, this code is used for the prediction of $N_{cr}$ and $f_n$ by changing loading coefficients (Table 1) and respective boundary condition (BC)'s weighted coefficients. Material properties considered in the present work are listed in Table 2.

### Table 4

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Sample</th>
<th>Critical buckling loads $N_{cr}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N1</td>
<td>N2</td>
</tr>
<tr>
<td>CC</td>
<td>E00</td>
<td>247.409</td>
</tr>
<tr>
<td></td>
<td>E20</td>
<td>286.770</td>
</tr>
<tr>
<td></td>
<td>E40</td>
<td>332.054</td>
</tr>
<tr>
<td></td>
<td>E60</td>
<td>385.232</td>
</tr>
<tr>
<td>CS</td>
<td>E00</td>
<td>121.275</td>
</tr>
<tr>
<td></td>
<td>E20</td>
<td>140.573</td>
</tr>
<tr>
<td></td>
<td>E40</td>
<td>162.770</td>
</tr>
<tr>
<td></td>
<td>E60</td>
<td>188.837</td>
</tr>
<tr>
<td>SS</td>
<td>E00</td>
<td>59.2570</td>
</tr>
<tr>
<td></td>
<td>E20</td>
<td>68.6858</td>
</tr>
<tr>
<td></td>
<td>E40</td>
<td>79.5319</td>
</tr>
<tr>
<td></td>
<td>E60</td>
<td>92.2687</td>
</tr>
<tr>
<td>CF</td>
<td>E00</td>
<td>14.6450</td>
</tr>
<tr>
<td></td>
<td>E20</td>
<td>16.9753</td>
</tr>
<tr>
<td></td>
<td>E40</td>
<td>19.6559</td>
</tr>
<tr>
<td></td>
<td>E60</td>
<td>23.8037</td>
</tr>
</tbody>
</table>

Fig. 2. Critical buckling loads under CC boundary condition for (a) 100 (b) 50 and (c) 20 aspect ratios.

3. Buckling

Fourth order Euler-Bernoulli beam governing differential equation \[32\] of motion has been considered to determine the $N_{cr}$ values of various syntactic beam configurations

\[
EI \left( \frac{d^4w}{dx^4} \right) + \frac{d}{dx} \left( N \frac{dw}{dx} \right) = 0
\]  
\[
N_{\text{max}}(x) = N_e \left( a_2 \left( x + \frac{L}{2} \right)^2 + a_1 \left( x + \frac{L}{2} \right) + a_0 \right)
\]  
Here $E$ is the modulus of elasticity, $I$ is the area moment of inertia, and $N$ is the axially varying load defined as $N_{\text{max}}(x)$. The values of these coefficients are mentioned in Table 1.
EI \left( \frac{\partial^4 w}{\partial x^4} \right) + \frac{\partial}{\partial x} \left( N_0 \left[ a_2 \left( x + \frac{L}{2} \right)^2 + a_1 \left( x + \frac{L}{2} \right) + a_0 \right] \frac{\partial w}{\partial x} \right) = 0 \tag{3}

Consider
\begin{align*}
CC &= \begin{bmatrix}
C_c(1) & 0 & \ldots & 0 \\
0 & C_c(2) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & C_c(N)
\end{bmatrix}
\quad \text{and} \quad RR = \begin{bmatrix}
R_c(1) & 0 & \ldots & 0 \\
0 & R_c(2) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & R_c(N)
\end{bmatrix}
\end{align*}
\tag{4}

where,
\begin{align*}
C_c(i) &= a_2 \left[ x(i) + \frac{L}{2} \right]^2 + a_1 \left[ x(i) + \frac{L}{2} \right] + a_0 \\
R_c(i) &= 2 \times a_2 \left( x + \frac{L}{2} \right) + a_1 \\
x(i) &= -\left[ \frac{L}{2} + \frac{L}{2} \right] \times \left[ 1 - \cos \left( \frac{(i-1)\pi}{N-1} \right) \right] \quad i = 1, 2, \ldots, N
\end{align*}

After substituting Equation-4 in Equation-3, the governing differential is transformed as
\begin{align*}
EI \left( \frac{\partial^4 w}{\partial x^4} \right) + \frac{\partial}{\partial x} \left( N_0 \left[ CC \left( \frac{\partial^2 w}{\partial x^2} \right) + RR \left( \frac{\partial w}{\partial x} \right) \right] \right) = 0 \tag{5}
\end{align*}

4. DQM formulation

DQM is a FE method for solving differential equations. The main fundamental concept behind the DQ approach is performing a function derivative with respect to a given specific discrete point in space, approximating a weighted linear sum of all discrete points in the variable domain [33]. Let w(x) is the deflection of the beam per unit length with N discrete grid points,
\begin{equation}
w_i = \left( \frac{\partial w}{\partial x} \right)_i = A_j w_j \quad i = 1, 2 \ldots N
\end{equation}

where \( A_j \) are the weighted coefficients of the 1\textsuperscript{st} derivative, and replaced index \( j \) represents the summation of all values of that index (i.e., 1 to \( N \)). The derivative of the higher order can be found in the consonantly, using weighting coefficients of the 2\textsuperscript{nd}, 3\textsuperscript{rd} and 4\textsuperscript{th} derivatives, denoted by \( B_j \), \( C_j \) and \( D_j \) as shown below
\begin{align*}
w_i'' &= \left( \frac{d^2 w}{dx^2} \right)_i = B_j w_j \tag{7} \\
w_i''' &= \left( \frac{d^3 w}{dx^3} \right)_i = C_j w_j \tag{8} \\
w_i'''' &= \left( \frac{d^4 w}{dx^4} \right)_i = D_j w_j \tag{9}
\end{align*}

5. Evaluation of weighted coefficients

The calculation of the weighting coefficient plays an important role in DQM. In this investigation, Quan and Chang’s approach [34] Chebyshev Guass Lobotto non-uniform grid points approach have been considered, which is expressed as
\begin{align*}
x(i) &= -\left( \frac{L}{2} \right) + \left( \frac{L}{2} \right) \times \left[ 1 - \cos \left( \frac{(i-1)\pi}{N-1} \right) \right] \quad i = 1, 2, 3, \ldots, N \quad (10)
\end{align*}

Using Quan and Chang’s method, the 1\textsuperscript{st} order derivative weighting coefficients \( A_i = A_0 \) can be computed by following the below mentioned method.

For \( i \neq j \)

\begin{align*}
w_i &= A_i w_i \quad i = 1, 2 \ldots N
\end{align*}
\[ A_{ij} = \frac{1}{x_j - x_i} \prod_{k 
eq i,j} \frac{x_k - x_i}{x_j - x_i} \quad \text{where } i, j = 1, 2, \ldots, N \] (11)

For \( i = j \)

\[ A_{ii} = \prod_{k \neq i} \frac{1}{x_k - x_i} \quad i = 1, 2, \ldots, N \] (12)

After computing first-order derivative weighting coefficients, one can obtain the weighting coefficients of higher order derivatives using straightforward matrix multiplication.

\[ B = B_{ij} = \sum_{k=1}^{N} A_{ik} A_{kj} \] (13)

\[ C = C_{ij} = \sum_{k=1}^{N} A_{ik} B_{kj} \] (14)

\[ D = D_{ij} = \sum_{k=1}^{N} A_{ik} C_{kj} = \sum_{k=1}^{N} B_{ik} B_{kj} \] (15)

Now substitute the respective order weighting coefficients in the equation-5.

\[ \text{EI} \sum_{j=1}^{N} D_{ij} \frac{\partial^4 w}{\partial x^4} + \rho A \left( \frac{\partial^2 w}{\partial t^2} \right) \left( \frac{\partial w}{\partial x} \right) = 0 \] (16)

where \( w = w(x, t) \) represents transverse displacement, \( E \) represents elastic modulus, \( I \) represents a moment of inertia of the area, \( \rho \) stands for the material density, and \( A \) for the cross-sectional area. The procedure

\[ \{ \text{EI} [D] + N_e [CC][B] + [RR][AA] \} \{ w \} = 0 \] (17)

The system admits a non-trivial solution. Thus, the coefficient matrix determinant will be equal to zero.

\[ \{ \text{EI} [D] + N_e [CC][B] + [RR][AA] \} \{ w \} = 0 \] (18)

By solving the above equation-18, one may obtain the generalized Eigen value problem. Here the Eigen value represents the \( N_{cr} \). In the present study, four classical boundary conditions are taken into consideration. Those are SS, CS, CC, and CF. Subrat Kumar Jena et al. [35] clearly mentioned the weighting coefficients for higher order derivatives to four classical boundary conditions. By substituting respective BC’s weighted coefficients in buckling characteristic equation-18, the \( N_{cr} \) values of beam subjected to different axially varying loads for various syntactic foam beam (E00, E20, E40, and E60) can be calculated. The MATLAB code has been used to solve equation (18).

6. Free Vibrations

It is assumed that syntactic foam will behave like a linearly elastic material. The shape of the fly ash cenosphere is spherical. Thereby, syntactic foam beam can behave as isotropic material. The governing differential for motion of the beam [36] (Bokaian A 1988) subjected to an axially compressive load by ignoring the effects of rotational inertia and shear deformation is mentioned in equation (8).

\[ \text{EI} \left( \frac{\partial^4 w}{\partial x^4} \right) + \frac{\partial}{\partial t} \left( \rho A \frac{\partial^2 w}{\partial x^2} \right) + \rho A \left( \frac{\partial w}{\partial x} \right) = 0 \] (19)

Fig. 4. Critical buckling loads under SS boundary condition for (a) 100 (b) 50 and (c) 20 aspect ratios.

D. Duryodhana et al.
mentioned below represents the substitution of Equation (2) in Equation (19). Equations (20)–(22) are related to free vibration analysis of the beam under different types of variable axial loads based on the Euler-Bernoulli beam assumptions. Solving Equation (23) as the Eigen value problem using DQM results in the natural frequencies at different intensity of applied variable axial load. Equations (20)–(22) are used to obtain Equation (23).

\[
EI \left( \frac{d^4 w}{dx^4} \right) + \frac{d}{dx} \left\{ N_0 \left( a_1 \left[ x + \frac{L}{2} \right] + a_2 \left[ x + \frac{L}{2} + a_3 \right] \right) \frac{dw}{dx} \right\} + \rho A \left( \frac{d^2 w}{dt^2} \right) = 0
\]

(20)

Substitute loading coefficient matrices from equation (4) in above equation (20)

\[
\rho A \left( \frac{d^2 w}{dt^2} \right) + \left\{ EI \right\} x \sum_{j=1}^{N} D_j + N_0 \left\{ [CC] \left( \sum_{j=1}^{N} B_j \right) + [RR] \left( \sum_{j=1}^{N} A_j \right) \right\} \{ w \} = 0
\]

(21)

And apply DQM to equation (20) to obtain Equation (21).

\[
\rho A \left( \frac{d^2 w}{dt^2} \right) + \left\{ EI[D] + N_0([CC][B] + [RR][AA]) \right\} \{ w \} = 0
\]

(22)
Considering \( w(x, t) = W(x) \cos(\omega t) \) the above Equation (22) becomes Equation (23). Here \( \omega \) represents the natural frequency for beams normal mode oscillation.

\[
\left\{ [EI] \left[ D \right] + N_0 \left[ \left[ CC \right] \left[ B \right] + \left[ RR \right] \left[ AA \right] \right] \right\} \omega^2 = \alpha \rho A I \frac{[w]}{0} = 0 \quad (23)
\]

The system admits a non-trivial solution. Thus, for non-trivial solutions, the determinant of the coefficient matrix is null.

\[
\left\{ [EI] \left[ D \right] + N_0 \left[ \left[ CC \right] \left[ B \right] + \left[ RR \right] \left[ AA \right] \right] \right\} \omega^2 = \alpha \rho A I = 0 \quad (24)
\]

From the above, one may obtain a generalized Eigen value problem. \( f_n \) values of various cenosphere volume fraction samples (E00, E20, E40, and E60) under various boundary conditions and subjected to axially varying loads can be calculated by substituting different boundary conditions and respective load parameters. MATLAB code has been used for solving natural frequency characteristic Equation (23).

7. Results and discussions

In this section, the results of studies on variations in \( N_{cr} \) and \( f_n \) the behaviour of syntactic foam beams under different boundary conditions and loads is briefly discussed.

7.1. Buckling

The \( N_{cr} \) values are determined numerically by solving Euler-Bernoulli’s beam governing differential equation using DQM. The characteristic equation-17 is solved by using MATLAB code to obtain the \( N_{cr} \) values. Sunil Waddar et al. [31] have determined \( N_{cr} \) graphically from the experimentally acquired load-deflection data of syntactic beams for different configurations under CC boundary conditions subjected to axially compressive load. They have also determined theoretical results and validated their experimental results with theoretical results. The current numerical based DQM results are validated with these experimental and theoretical data taken from previous works [31]. The code developed using MATLAB, for numerical based DQM has validated the results obtained at CC boundary conditions for end compressive load. Thus, this code was applied to predict variation in the buckling behaviour for remaining axially varying loads. For all the samples, it is observed that the \( N_{cr} \) shows an increasing trend with respect to filler content (Table 3). A higher cenosphere loading in the matrix (increasing the volume fraction of the cenosphere) increases the modulus, and therefore the global stiffness of the syntactic foam increases. \( N_{cr} \) also depends on the relative difference between the modulus of constituents. The modulus of the cenosphere is nearly 19 times greater than that of the epoxy matrix. For pure epoxy, the \( N_{cr} \) value is 237.56 N. The increase in the \( N_{cr} \) value is in the range of 15.9–55.71\% using DQM. A comparison study showed a fair agreement between all specimens’ experimental, theoretical, and DQM buckling load results (Table 3). Numerical methods are approximate methods. If the deviation is less than 5\%, then the results are very good for validation. In the current work, the DQM results are very near to the theoretical and experimental results, and for all cases, the deviation is less than 5\%, as shown in Table 3. In this work, the \( N_{cr} \) values are also evaluated for the different volume fractions of cenosphere samples subjected to six other axially varying loads for different BC’s (CC, CS, SS, and CF). All these results are numerically obtained by solving the characteristic equation-17 using MATLAB code, and the outcomes are mentioned in Table 4. The maximum \( N_{cr} \) is acquired under parabolic varying load (N4) conditions. As the load varies along the length of the beam, the beam’s effective length varies with loading type. The effective length is the least for parabolic load, which varies from zero at one end to a maximum at the other end. The effective length becomes less if the loading direction changes from lower to higher. Therefore, the slenderness ratio is highest and lowest for N5 and

![Fig. 6. Natural frequency comparison between theoretical, experimental and Differential Quadrature Methods.](image)
N4 loading profiles. The $N_{cr}$ exhibited inverse relation with respect to the square of the slenderness ratio. The slenderness ratio is the ratio of the effective length to the least radius of gyration of the cross-section area. Therefore, minimum and maximum $N_{cr}$ obtained under N5 and N4 loading profiles, respectively, as shown in Table 4. The same trend was observed for the other BC's, as shown in Table 4. Moreover, it is observed that the uniformly distributed axial compressive load ($N_1$) is higher than parabolic axial variable load ($N_6$) as shown in Table 4. In the current section, the $N_{cr}$ values of beams for different types of axially varying loads, aspect ratios, and the different volume fractions of cenosphere samples and BCs were investigated. It can be seen from Fig. 2 - Fig. 5 that $N_{cr}$ exhibited a directly proportional relation with respect to aspect ratio. In CC beams, the influence of aspect ratio on $N_{cr}$ is more pronounced. The maximum and minimum $N_{cr}$ is observed for beams under CC and CF, BC's, respectively. It can see from Fig. 2 that for all aspect ratios, the maximum and minimum $N_{cr}$ obtained for N5 and N6 types, respectively. $N_{cr}$ obtained under uniformly distributed load ($N_1$) is higher than the parabolic load type ($N_6$). It may be important to notice that a reduction in the beam’s stiffness may result from the increased density of the axial load in the center of the beam. It is also noted that with a decrease in aspect ratio, the $N_{cr}$ value started increasing. The slenderness ratio decreases with the decrease in the aspect ratio. This is one of the major parameters that significantly influence the beam’s buckling tendency. From the numerical results at the CC boundary condition, it is perceived that upon decreasing the aspect ratio from 100 to 50, 20 the $N_{cr}$ increases in the range of 1400–1600%, as shown in

Fig. 7. Influence of axial varying loads (a) N1 (b) N2 (c) N3 (d) N4 (e) N5 and (f) N6 on natural frequency for clamped-clamped boundary condition.
A similar trend is observed in the remaining all other BCs (CS, SS, and CF), as shown in Fig. 3, Fig. 4, and Fig. 5. The type of boundary support is a crucial factor in predicting the $N_{cr}$ value of a beam, along with other factors such as aspect ratio. Four sets of BCs, namely CC, SS, CS, and CF, are considered in this study. It is perceived that for all the boundary cases the $N_{cr}$ increases with the decrease in the aspect ratio, but the rate of increase in $N_{cr}$ depends on whether the beam is CC, CS, SS, or CF supported. For all six sets of axial varying loads, the minimum and maximum $N_{cr}$ is observed under CF and CC support, respectively, as shown in Table 5. The reason behind the sample exhibiting higher $N_{cr}$ load at CC support is due to its rigidity.

7.2. Free vibration under axial varying loads

In this work, the natural frequency of different configurations of syntactic foam beam (E00, E20, E40, and E60) under axially varying...
loads for four sets of BC’s is determined using DQM. Sunil et al. [31] used Frequency Response Function (FRF) to predict the experimental \( f_n \) corresponding to initial three modes shapes using software named DEWE soft. The numerical DQM results obtained for CC boundary conditions are validated with experimental and theoretical natural frequencies, as shown in Fig. 6. The comparative studies show that all results are in fair agreement. The natural frequency of the syntactic foam based beam decreased initially with an increase in load, and post buckling, the \( N_{cr} \) trend reverses (Fig. 6). This might be due to the gain in stiffness in the post-buckling geometric deformation. The present numerical analysis is based on linear structural behaviour. The post-buckling geometric deformation is not considered, leading to deviation between the numerical and experimental results. From the result, it is also observed that for all six sets of varying loads under CC boundary conditions natural frequency of the syntactic foam beam exhibited an increasing trend corresponding to an increase in filler
percentage, as shown in Fig. 7. The natural frequency decreases for all beams upon gradually increasing the axial varying compressive load. The same trend is observed for all six sets of loads. Theoretically, $f_n$ becomes minimum when the beam is under axial compressive load, which is nearly equal to $N_{cr}$ (Fig. 6). The gradual loss in the stiffness in the beam with an increase in the load results in the phenomenon of a decrease in $f_n$. Similar behaviour is observed for remaining BC’s (CS, SS and CF) as shown in Fig. 8, Figs. 9 and 10.

7.3. Effect of BC’s on natural frequency

In the present section, the variation of $f_n$ corresponding to various support conditions (CC, CS, SS, and CF) under each of the six sets of axially varying loads have been studied using DQM. The results acquired under the case of CC are validated accurately with existing theoretical and experimental results. The comparative study of $f_n$ under different BCs (CC, CS, SS, and CF) are represented in Table 6. From this, it is

![Influence of axial varying loads](image)
observed that all configurations of beams exhibited the highest and lowest natural frequency at CC and CF BCs. Moreover, the natural frequency of E00 sample at (N1) under load at CC BC’s is 33.83%, 57.43% and 77.81% higher than the natural frequency under CS, SS and CF boundary conditions. A similar range of enhancements in $f_n$ is observed for remaining loading conditions as well.

<table>
<thead>
<tr>
<th>Sample</th>
<th>BC’s</th>
<th>Axially varying load types</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N1</td>
<td>N2</td>
</tr>
<tr>
<td>E00</td>
<td>CC</td>
<td>176.816</td>
</tr>
<tr>
<td></td>
<td>SS</td>
<td>75.176</td>
</tr>
<tr>
<td></td>
<td>CF</td>
<td>39.227</td>
</tr>
<tr>
<td>E20</td>
<td>CC</td>
<td>194.783</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>128.835</td>
</tr>
<tr>
<td></td>
<td>SS</td>
<td>82.750</td>
</tr>
<tr>
<td></td>
<td>CF</td>
<td>43.040</td>
</tr>
<tr>
<td>E40</td>
<td>CC</td>
<td>220.271</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>141.986</td>
</tr>
<tr>
<td></td>
<td>SS</td>
<td>91.156</td>
</tr>
<tr>
<td></td>
<td>CF</td>
<td>47.280</td>
</tr>
<tr>
<td>E60</td>
<td>CC</td>
<td>237.212</td>
</tr>
<tr>
<td></td>
<td>CS</td>
<td>156.834</td>
</tr>
<tr>
<td></td>
<td>SS</td>
<td>160.648</td>
</tr>
<tr>
<td></td>
<td>CF</td>
<td>52.074</td>
</tr>
</tbody>
</table>

**7.4. Influence of aspect ratio on natural frequency**

The effect of aspect ratio on the natural frequency of syntactic foam beams (E00, E20, E40 and E60) subjected to six sets of axially varying loads has been studied in the present study. A close look at the beam subjecting axial compressive load (N1) under CC boundary condition...
reveals that natural frequency enhanced with the decreasing aspect ratio, as shown in Fig. 11(a). As the aspect ratio increases (20–50, 50–100), the natural frequency decreases by 60% and 49% for pure epoxy samples (E00). The major reason for this is that when the aspect ratio increases, the slenderness ratio increases, which decreases the beam stiffness. Thereby decrease in the frequency is observed. Even for other syntactic foam samples (E20, E40 and E60), the natural frequency decreases in the same proportion as pure epoxy. Additionally, it is perceived that by increasing the value of the aspect ratio, the $f_n$ for other five axially varying loads is also increased. The natural frequency of the syntactic foam beam under CS boundary conditions also decreases with the increase in aspect ratio, as shown in Fig. 11 (b). By applying the (N1) load and increasing the aspect ratio of the beam from 20 to 50 and 50 to 100, the natural frequency decreases by 60% and 49%, respectively. A similar range of frequency variation is observed for the remaining five axially varying loads. A comparative study of natural frequency among different samples at different boundary conditions for all configurations is presented in Fig. 11.

8. Conclusions

Buckling and free vibrations characteristics of syntactic foam based beams developed by varying the volume fractions of cenosphere (E00, E20, E40 and E60) are investigated numerically under four BC’s (CC, CS, SS, and CF) and compared with analytical and experimental results. The DQ numerical method has been used to investigate $N_{cr}$ and $f_n$ the behaviour of syntactic foam beam on Euler-Bernoulli beam theory. $N_{cr}$ and $f_n$ are evaluated for all six sets of axial varying loads. Based on the acquired results, one can conclude that the present method for the application of BC’s is very simple and easy to use. Even it is capable enough to yield excellent results for all the cases, with minimum grid points.

- It is observed that the $N_{cr}$ and $f_n$ increased with an increase in the volume percentage of the cenosphere.

- The syntactic foam beam has the highest $N_{cr}$ under CC BC’s subjected to (N4) load. The beam subjected to (N6) load under the CF boundary condition exhibited the lowest $N_{cr}$.

- The $N_{cr}$ enhanced with a decrease in the aspect ratio and $f_n$ decreases with an increase in the aspect ratio. This is observed for all axially varying load types.

- In all syntactic foam samples, $f_n$ exhibited a decreasing trend with an increase in load.

- At the site of $N_{cr}$, the fundamental frequency numerically reaches its lowest value. At CC and CF BC, the beam showed the highest and lowest $f_n$ for all six sets of loads.

The change in the aspect ratio of syntactic foam constantly affects its trend in natural frequencies for all BCs. The syntactic foam beams registered better stiffness compared to the pure epoxy beams for all types of loads. The present work has observed fair agreement among the numerical, experimental, and theoretical results.

Author statement

Dasari Duryodhana, Investigation, Formal analysis, Writing – original draft, Sunil Waddar, Resources, Writing – review & editing, Supervision, Dileep Bonthu, Writing – review & editing, Jeyaraj Pitchaimani Resources, Writing – review & editing, Visualization, Supervision, Satvasheel Powar, Resources, Writing – review & editing, Visualization, Mritunjay Doddamani, Conceptualization, Resources, Writing – review & editing, Visualization, Supervision

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

data is part of the ongoing research work

References


