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Daily Calls Volume Forecasting

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Abstract

A massive amount has been written about forecasting but few articles are written about the development of time series models of call volumes for emergency services. In this study, we use different techniques for forecasting and make the comparison of the techniques for the call volume of the emergency service Rescue 1122 Lahore, Pakistan. For the purpose of this study data is taken from emergency calls of Rescue 1122 from 1st January 2008 to 31 December 2009 and 731 observations are used. Our goal is to develop a simple model that could be used for forecasting the daily call volume. Two different approaches are used for forecasting the daily call volume Box and Jenkins (ARIMA) methodology and Smoothing methodology. We generate the models for forecasting of call volume and present a comparison of the two different techniques.

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1. Introduction

There are many organizations working in the developed world for providing Emergency service, consisting of pre-hospital medical care and transport to medical facility. We got the idea for this study when we were comparing Pakistani Emergency Services Rescue 1122 with the emergency services of the developed countries. We observed that the developed countries planned their future needs and they even have master plan for each coming year. On the basis of forecasting they decide how they will utilize their present resources and how much more resources they need to, not only this, they can plan their resources on daily basis.

In this study we examine the forecasting of emergency incoming calls to Emergency Services centres Rescue 1122 Lahore Pakistan. These centres are responsible for answering the incoming emergency requests that need immediate emergency help. Data for this study is taken from emergency calls of Rescue 1122 from 1st January 2008 to 31 December 2009 and 731 total observations used. The two different approaches used for forecasting the daily call volume include Box and Jenkins (ARIMA) methodology and Smoothing methodology. Both methods are smoothing methods. Our objective is to use past data to develop a forecasting model for the closest days to come. We will to this end use data from emergency calls Rescue 1122 to

1. Develop different time series models for daily call volume.
2. Make comparison of different forecasting techniques to suggest the better one.

Our hope is that our findings will help to improve the efficiency of Rescue 1122.

2. Review of Literature

In this section we summarize some different research articles concerning the number of calls to (emergency) call centers.

2.1 Improving Forecasting For Telemarketing Centers by ARIMA Modeling With Intervention

The incoming calls to telemarketing centers was analyzed for the purposes of planning and budgeting by Lisa Bianchi, Jeffrey Jarrett and R. Choudary Hanumara (1998).In their publication they used Box–Jenkins (ARIMA) modeling with intervention analysis and additive and multiplicative versions of Holt–Winters (HW) exponentially weighted moving average models. With aid of these models they forecasted the daily call volumes. The data used for analysis was from March 1, 1991 to June 26, 1991.

Their first model was the ARIMA(p,d,q)$(P,D,Q)^7$ .Their second model was the multiplicative Holt-Winter model

$$Y(t) = (a(t) + b)\, s(t) + e(t)$$

When seasonal variation is constant over time an additive seasonal factor model is appropriate. Hence the third additive model used was

$$Y(t) = (a(t) + b) + s(t) + e(t)$$

The Root Mean Square Error (RMSE) is used to compare different model forecasts performance. It was found that ARIMA models with intervention analysis provided better forecasts for planning and control.

A complete version of this study can be found from paper [1].

2.2 Wireless Traffic Modeling and Prediction

In this article Yantai Shu, Minfang Yu, and Jiakun Liu (2003) studied wireless traffic.In their study to predict traffic, seasonal an ARIMA model with two periodicities was used. The hourly traffic data from 0:00 June 1 2001 (Friday) to 0:00 April 27 2002(Saturday) was measured. A total of 330 days from the dial-up access network of China net-Tianjin. To trace the daily traffic the model ARIMA $(1,0,1)(1,1,0)_7$ was found and for the hourly traffic ARIMA(0,1,1)$(1,1,0)_{24}(1,0,0)_{168}$.For estimating the model the first 300 daily data was used. The last 30 days to evaluate the model. An adjusted traffic prediction method is proposed using seasonal ARIMA models. The comparison is repeated with many prediction experiments on the actual measured GSM traces of China Mobile of Tianjin. It is founded that the relative error between the actual values and forecasting values are all less than 0.02.Their study showed that the seasonal ARIMA model is a good traffic model capable of capturing the properties of real traffic.

A complete version of this study can be found from paper [2].

2.3 The application of forecasting techniques to modeling emergency medical system calls

The emergency medical system calls of major Canadian city Alberta was analyzed by Nabil Channouf, Pierre L Ecuyer (2006).In their analysis two different methods was used, autoregressive model of data obtained after eliminating the trend, seasonality, special day effect and a doubly-seasonal ARIMA model with special day effect. Then the comparison of the both models is presented. For the purpose of analysis the data for emergency medical calls was obtained from January 1, 2000 to March 16, 2004 including call priority, and the geographical zone where the call originated. The modeling is done on the first 1096 observations and the remaining 411 observation is used for evaluation.

The model found was an ARIMA decomposed model with two seasonal cycles.

$$Y_t = N_t + \omega_1 H_{t,1} + \omega_2 H_{t,2}$$

The ARIMA model with two seasonal cycles, weekly s1 and annual cycle s2 is suggested. They found that this model performed poorly when forecasting more than two weeks into the future.

A complete version of this study can be found from paper [3].

2.4 Forecasting Police Calls during Peak Times for the City of Cleveland USA

The police service calls during peak times for the city Cleveland, US was presented by the police department of the city. Professor John P.Holcomb, Jr (2007) used autoregressive integrated moving average (ARIMA) modeling technique, Multiple Regression and different smoothing methods to analyze data. As a first step the data of call volume (per hour) is obtained and it was divided into 10 important categories. This provided 24,000 data points across all kinds of calls, further the calls are divided priority wise, priority 1 calls being the most important. Priority 1 calls are the calls where crime is in progress: such as robbery or domestic violence. The researcher used different methodologies for building models. For model evaluation, the mean absolute percent error (MAPE) is used. He suggested that multiple regression approach have difficulty. The final ARIMA $(1,0,0)(5,1,0)_8$ model is used. This model produced an improved MAPE over the Holt-Winters method approximately 12%. A complete version of this study can be found from paper [4].

2.5 Predicting call arrivals in call centre

The daily call volume of car damage insurance claims at Vrije University, Netherlands was analyzed by Koen Van Den Bergh (2006).In this publication he discussed four different methods,ARIMA modeling, modeling by Dynamic Regression, modeling thorough Exponential smoothing and modeling by Regression. These four techniques are applied to the daily call center data to forecast the daily call volume. The models used for forecasting are given below.

The ARIMA model is

$$y_t = \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \ldots\ldots.+\emptyset_p y_{t-p} + \varepsilon_t + \theta_1\, \varepsilon_{t-1} + \ldots\ldots+\theta_q\, \varepsilon_{t-q}$$

The dynamic regression model is

$$y_t = \alpha + v_0 x_t + v_1 x_{t-1} + v_2 x_{t-2} + \ldots + v_k x_{t-k} + n_t$$

$$y_t = v(L)x_t + n_t$$

The single exponential smoothing model is

$$\hat{y}_t = w_0 y_{t-1} + w_1 y_{t-2} + w_2 y_{t-3} + \ldots,$$

$$= \sum_{i=0}^{inf} w_i y_{t-i+1},$$

The regressions model is

$$Y_t = S_t + T_t + R_t + \sum_{i=1}^{n} b_i\, X_i$$

It is presented that all of this methodology can at least deal with Randomness. Single Exponential smoothing is not good enough to deal with seasonality and trend pattern but this methodology can handle the random part which is a least result which a forecasting technique can give. The Dynamic Regression model and Regression model can deal with the interventions as well, where the ARIMA models can not deal with intervention.

A complete version of this study can be found from paper [5]

3. Methodology

3.1 Necessity of Forecasting

Uncertainty means that no clarity about future may be achieved. When uncertain decisions are made upon historical experiences. Historical data can be smoothed in different ways. But the scientific approach is essential to make decision. The forecasting is one of the major scientific approaches that help in process of making decision in condition of uncertainty. Forecasting is based on the assumption that the past patterns and behaviour of a variable will continue into the future. The objective is to use past data to develop a forecasting model for the future periods. To reach our goal of forecasting daily call volume of rescue 1122, the sophisticated forecasting techniques known as ARIMA (Auto Regressive Integrated Moving Average) and Smoothing Methodology are applied.

3.2 Assumptions of Time Series Analysis

A major assumption in time series analysis is the stationarity of the series, this mean that the average value and the variation of the series should be constant with respect to time. If the series is not stationary then we make it stationary by the different transformations the most commonly used transformations are log and first difference.

3.2.1 Stationarity Tests

There are different tests for checking the stationarity of data, one of them is the
unit root test by Dicky and Fuller.

Unit Root Test

The unit root presence can be illustrated as follows by using a first order autoregressive process:

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \qquad (1)$$

The basic Dickey- Fuller test examines whether $\rho < 1$

After subtracting y_{t-1} from both sides in equation above,

$$\Delta y_t = \mu + (\rho - 1) y_{t-1} + \varepsilon_t$$

$$\Delta y_t = \mu + \theta\, y_{t-1} + \varepsilon_t$$

(2)

H_0: θ = 0 (there is a unit root iny_t)

H_1 : θ < 0

Equation (1) and (2) are the simplest case where the residual is white noice.In general, there is serial correlation in the residuals and Δy_t can be represented as an autoregressive process:

$$\Delta y_t = \mu + \theta\, y_{t-1} + \sum_i^p \emptyset_i\, \Delta y_{t-i} + \varepsilon_t \qquad (3)$$

Corresponding to equation (3), Dickey-Fuller procedure becomes the Augmented Dickey-Fuller test. We can also include a deterministic trend in equation (2).Altogether, there are four test specification with regard to the combination of an intercept and a deterministic trend. [6]

3.3 Box-Jenkins modelling

The methodology introduced 1970 by Box and Jenkins assumes that the data is dependent on itself. And the very first thing to decide on is the number of lags. Then a number of parameters are estimated, the residuals are checked and finally a forecast is made. The general model looks like.

$$y_t = \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \ldots\ldots + \emptyset_p y_{t-p} + \varepsilon_t + \theta_1\, \varepsilon_{t-1} + \ldots\ldots + \theta_q\, \varepsilon_{t-q}$$

3. 3.1 Auto-Regressive (AR) Model

In the pure AR (p) autoregressive with p lags model, we have

$$Y_t = U_t + \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \ldots + \emptyset_{1p} Y_{t-p} + \varepsilon_t$$

that is the series depend on itself up to p lags. The simplest and most widely used model with serial correlation is the first order autoregressive model of first order. The AR (1) model is specified by:

$$Y_t = U_t + \emptyset_1 Y_{t-1} + \varepsilon_t$$

The higher order autoregressive model or autoregressive model of order ‘p’ denoted by AR (p) is given:

$$Y_t = U_t + \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \ldots + \emptyset_{1p} Y_{t-p} + \varepsilon_t$$

Where$\emptyset_1$, $\emptyset_2 \ldots$ $\emptyset_p$are the parameters of the model, μ_t is constant with respect to t and ε_t is white noise. Many authors omit the constant term.

3. 3.2 Moving Average (MA) model

The moving average model models the error terms, which are not observed. The moving average model is defined as:

$$Y_t = U_t + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_p \varepsilon_{t-q}$$

Where θ_1, $\theta_2 \ldots$ θ_q are the parameters of the model, μ_t is a constant with respect to t and ε_t is white noise. Many authors omit the constant term.

3. 3.3 Auto-Regressive Integrated Moving Average (ARIMA) model

ARIMA (p, d, q) (P, D, Q) where (p is the order of AR process, 'q' is the order of MA process and'd' is the order of differencing) is a regular model and (P, D, Q) are seasonal elements.
The ARIMA models are generalization of the simple AR model that uses three tools for modeling series correlation in the disturbance. The first tool is the auto-regressive terms. The second tool is the integrated (difference) terms. A first order integrated component means that the forecasting model is designed for the first difference of the original series. A second order component difference of the original series and so on. The third tool is the moving average terms. A moving average forecasting model uses lagged values of the forecasted errors. A first order moving average term uses the forecasted errors from the two most recent periods, and so on [7].

3. 4 Exponential Smoothing

There are several exponential smoothing methods. The major ones used are Single Exponential Smoothing, Holt's Linear Model (1957) and Holt-Winters' Trend and Seasonality Model.

3.4.1 Single Exponential Smoothing

The simplest form of exponential smoothing is single exponential smoothing, which may be used when data is without any systematic trend or seasonal components. Given such a time serie, a logical approach is to take a weighted average of past values. So for a series $y_1, y_2, \ldots, y_{t-1}$, the estimate of the value of Y_t, given the information available up to time t, is

$$\hat{Y}_t = w_0 Y_{t-1} + w_1 Y_{t-2} + w_2 Y_{t-3} + \ldots$$

Where $w_i = \alpha(1-\alpha)^i$ are the weights given to the past values of the series and they sum to one. Here the 'α' lies between '0' and '1'. Since the most recent observations of the series are also the most relevant, it is logical that these forecasting observations should be given more weight than the observations further in the past. This is done by giving declining weights to the series. These decrease by a constant ratio.

3.4.2 Holt's Linear Model

Holt's linear model is an extension of single exponential smoothing. This method allowed forecasting data with trends.

For a time series $y_1, y_2, y_3, \ldots$ the estimate of the value of y_{t+k}, is given by the next formula:

$$\hat{y}_{t+k} = m_t + b_t k \qquad where \;\; k=1,2,3,\ldots.$$

Where m_t denotes an estimate of the level of the series at time t and b_t denotes an estimate of the slope of the series at time t.

Where

$$m_t = \alpha_o y_t + (1-\alpha_o)(m_{t-1} + b_{t-1})$$

$$b_t = \alpha_1(m_t + m_{t-1}) + (1-\alpha_1)b_{t-1}$$

$$with\ 0 < \alpha_o < 1\ and\ 0 < \alpha_1 < 1$$

3.5 Model Selection Criteria

Here we discuss the few criteria we used in the study when selecting the best model among the competing models. Several criteria can be used for this purpose, here we discuss Akaike information criterion (AIC) and the Schwarz information criterion (SIC). These criterions are used for measuring the goodness of fit of the model. These criterion are minimized over the choice of repressors, it will be minimum when the model is good fit and less complex. In comparing two or more models, the best model is the one having the least AIC and BIC values.

In a regression setting, the estimates of the β_I based on least squares and the maximum likelihood estimates are identical. The difference comes from estimating the common variance σ^2 of the normal distribution for the errors around the true means. We have been using the best unbiased estimator of σ^2, $\hat{\sigma}^2 = \mathrm{RSS}/(\mathrm{n}-\mathrm{p})$ where there are p parameters for the means (p different β_I parameters) and RSS is the residual sum of squares. This estimate does not tend to be too large or too small on average. The maximum likelihood estimate, on the other hand, is RSS/n. This estimate has a slight negative bias, but also has a smaller variance. Putting all of this together, we can write -2 times the log-likelihood to be

$$\mathrm{n} + \mathrm{n}\log(2\pi) + \mathrm{n}\log(\mathrm{RSS/n}).$$

In a regression setting. Now, AIC is defined to be -2 times the log-likelihood plus 2 times the number of parameters. If there are p different β_I parameters, there are a total of p+1 parameters if we also count σ^2. The correct formula for the AIC for a model with parameters $\beta_0, \dots, \beta_{p-1}$ and σ^2 is

$$AIC = n + nlog2\pi + \mathrm{nlog}\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + 2(\mathrm{p}+1)$$

and the correct formula for BIC is

$$BIC = n + nlog2\pi + \mathrm{nlog}\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + (\log \mathrm{n})(\mathrm{p}+1)$$

3.6 Measurements of Forecasting Accuracy

Before the forecasting results can be given, some measurements of forecasting accuracy must be determined. This section captures the equations of the most widely applied measurement methods. The following list of methods shall be utilized for assessing the accuracy of forecasts

3.6.1 Mean Absolute Percentage Error

$$MPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{e_t}{y_t}\right| \times 100$$

3.6.2 Mean Square Error

$$MSE = \sum_{t=1}^{n}\frac{e_t^{\,2}}{n}$$

3.6.3 Root Mean Square Error

$$RMSE = \sqrt{\sum_{t=1}^{n} \frac{e_t^{\ 2}}{n}}$$

3.6.4 Mean Absolute Error

$$MAE = \frac{1}{n} \sum_{t=1}^{n} |e_t|$$

Where e_t is the error at time t or $e_t = y_t - \hat{y}_t$ (for all measurements).

3.7 Residual Analysis

Residuals are the difference between the predicted output from the model and the original values (data). Residuals basically represent the portion of the data not explained by the model. Residual analysis may be regarded to consist of two tests: Whiteness Test and Normality Test.

3.7.1 Normality Test

A good model is the one for which the residuals fulfil the assumption of normality. The histogram of the residuals gives a good idea about the normality. The Normal probability graph is also used to assess that the data set is approximately normally distributed. In a normal probability graph the data is plotted against the theoretical normal distribution in such a way that it makes a straight line. If the points depart from straight line, we have a departure from normality.

The Anderson Darling test is one of the three generally known tests for the normality.. It is the modified form of Kolmogorov-Smirnov test and gives more weight to the tails as compared to the Kolmogorov-Smirnov test. In the Kolmogorov-Smirnov test the critical values do not depend on the specific distribution being tested but the Anderson Darling test use the specific distribution for calculating the critical value. The test statistic of the test is given below:

$$A^2 = -N - S$$

$$S = \sum \frac{(2i-1)}{N}[\log F(Y_i) + \log\{1 - F(Y_{N=1-i})\}]$$

Where F is the cumulative distribution function of interest.

3.7.2 Whiteness Test

The purpose of this test is to analyze the correlation between the residuals at different lags. According to the whiteness test criteria all autocorrelation should be zero.

3.7.3 Ljung–Box test

This is an objective way to test the null hypothesis that there is no autocorrelation. The Q-statistic at lag k is a test statistic for the null hypothesis that there is no autocorrelation up to order k. It is computed as

$$Q = T(T+2)\sum_{j=1}^{k}\frac{r_j^2}{T-j}$$

Where r_j is the j-th autocorrelation and T is the number of observations. *k* is the number of lags being tested.

4. Analysis

In this section, we discuss our analysis of data. For the purpose of the study, data for 1st January 2008 to 31 December 2009 is used a total of 731 observations. The daily call volume depend on different variables such as Road Accident, Short of Breath, Fire Case, Heart Attack, fall from Height, Blast, Head Injury / Injured, Bullet Injury/Violence, Delivery Case / Gynae Problem and other Medical Problems. A scatter plot of our 731 daily calls volume observations is shown in figure 4.1.

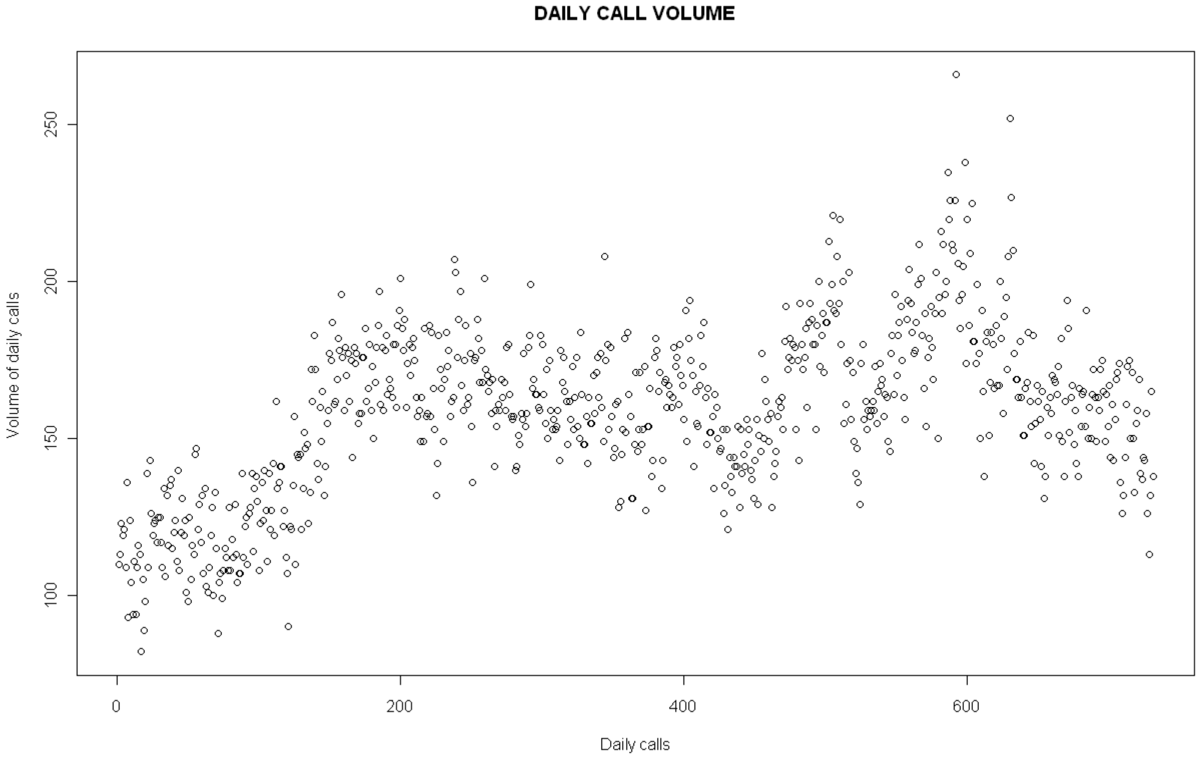

Figure 4. 1: The scatter plot of daily calls volume

and a time series plot in figure 4.2

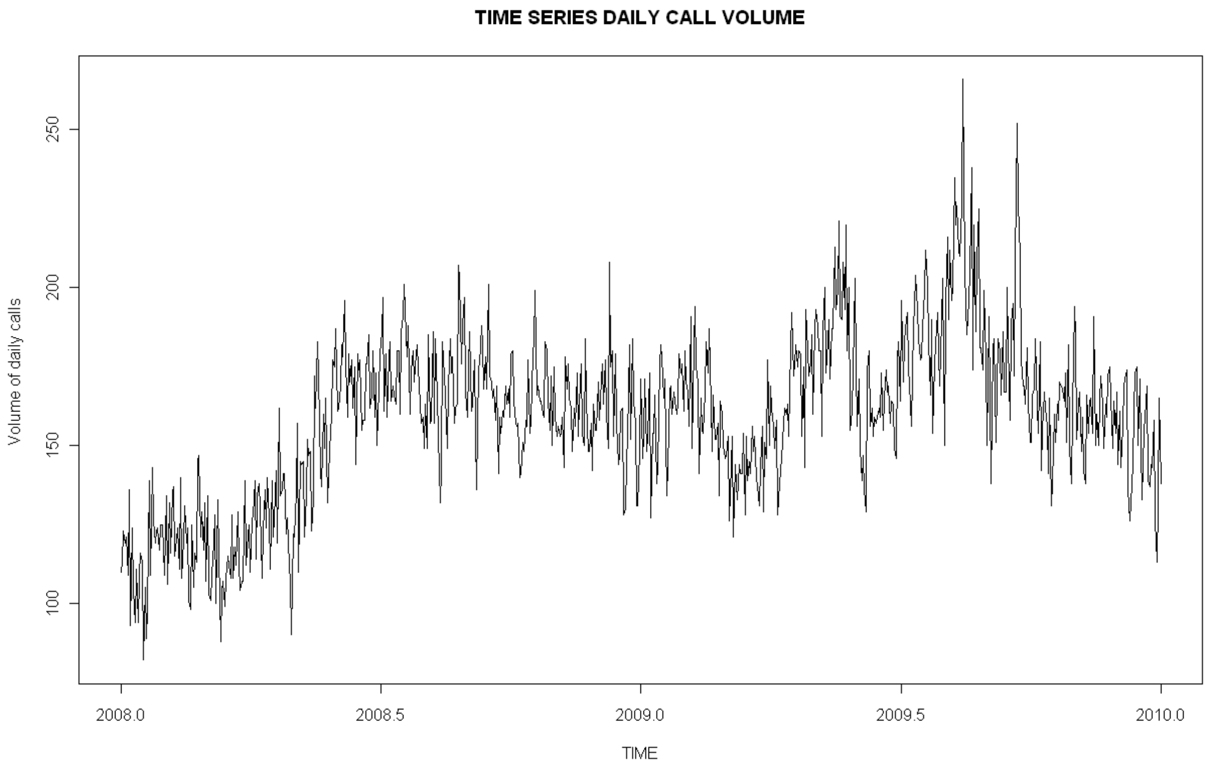

Figure 4. 2: The time series plot of daily calls volume

4.1 ARIMA(p, d, q):

In this portion, the ARIMA(p,d,q) technique is applied on the data of daily calls volume to find an appropriate ARIMA model.

4.1.1 Stationarity test:

First step is to check if the series is stationarity. Dickey-Fuller test is applied first to the series of total daily calls volume.

The value of the statistic turned out to be -2.859 and p value 0.2146, which shows that our null hypothesis of daily call volume stationarity is rejected and we conclude that daily calls volume series is not stationary. This is also clear from figure 4.3 and figure 4.4 of the estimated autocorrelation and partial autocorrelation functions.

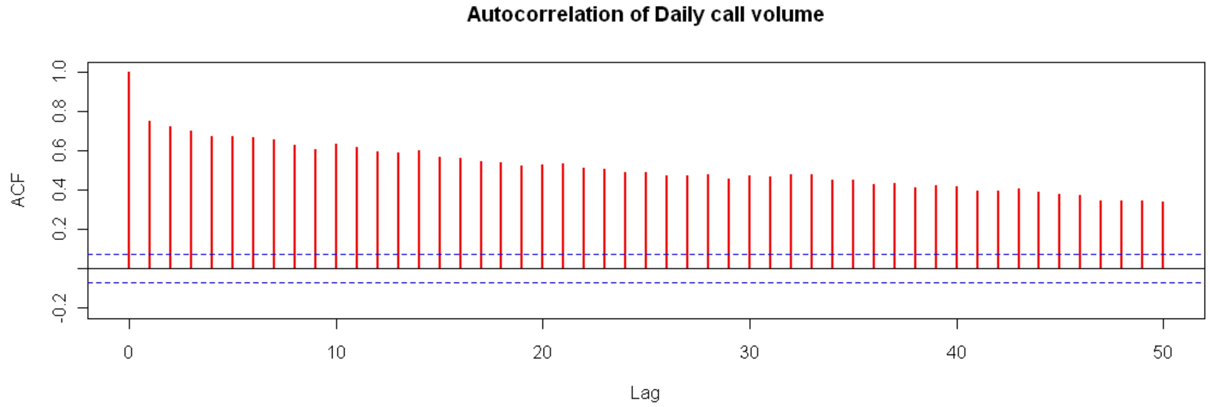

Figure 4. 3: ACF of daily calls volume

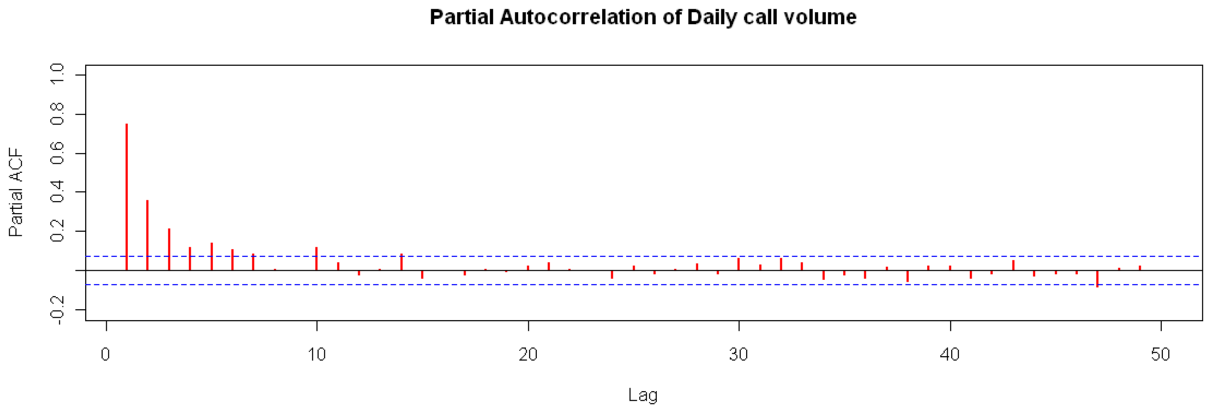

Figure 4. 4: PACF of daily calls volume

After taking the first difference the tests are applied again. This time the Dickey-Fuller test gives -13.4876 with p value 0.01. We conclude that our series after 1^{st} differencing is stationary and this is also clear from figure 4.5.

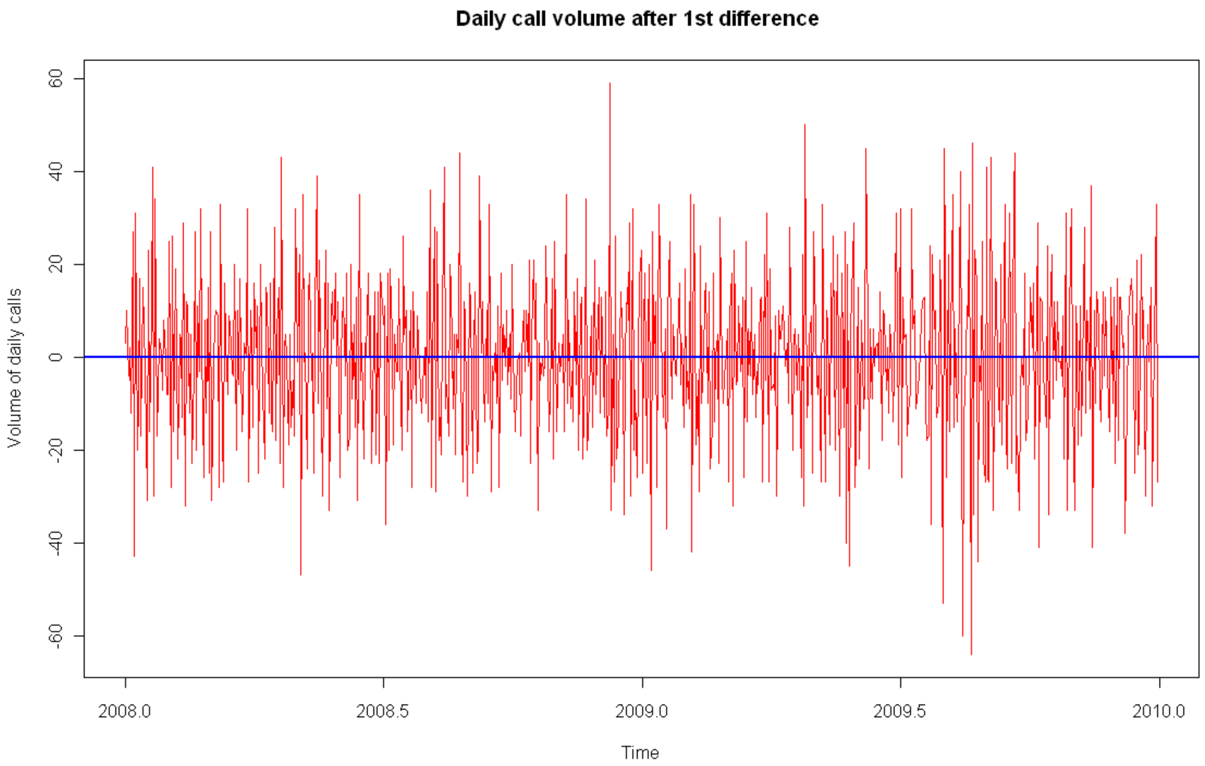

Figure 4. 5: The daily calls volume after 1st Difference

4.1.2 Selection of the parameters:

After deciding about the stationary of the series, the next step is to decide about the parameters “p” and ”q” of the ARIMA(p,d,q) model. Because the stationarity at first difference has ensured that the value of ‘d’ in the model is ‘1’, the values of ‘p’ and ‘q’ are left to be determined. For determining the value of ‘p’ and ‘q’, the graphs of the estimated autocorrelation and partial autocorrelation are obtained and plotted, in figure 4.6 and figure 4.7.

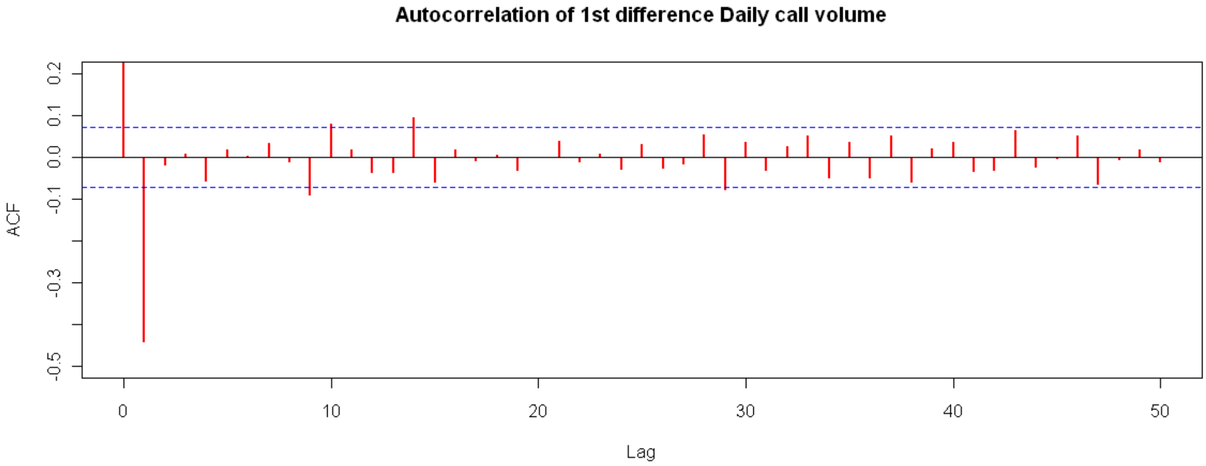

Figure 4. 6: ACF of daily calls volume after 1st Difference

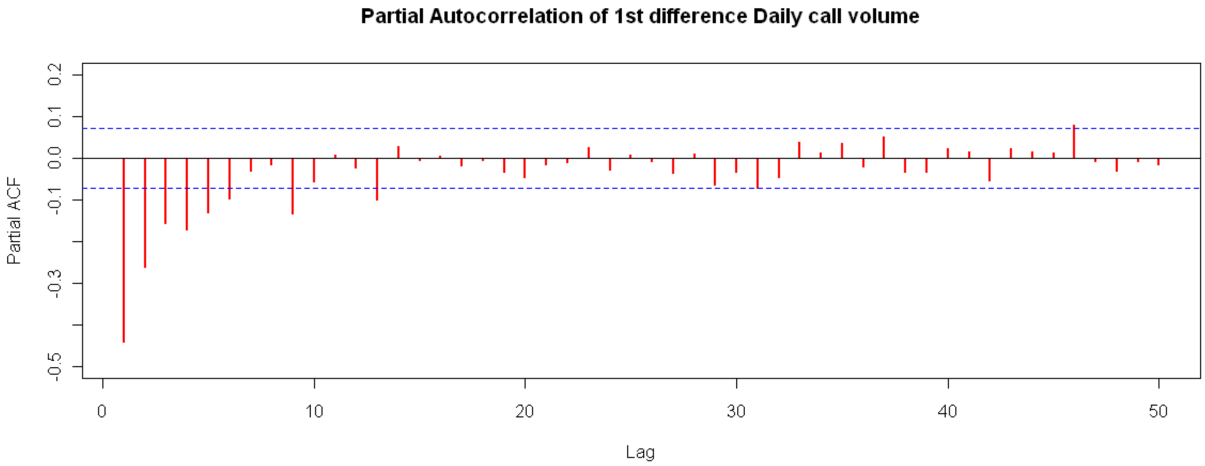

Figure 4. 7: PACF of daily call volumes after 1st Difference

Figure 4.6 and figure 4.7 clearly shows that the autocorrelation at lag 1 is outside the limits and we also have decreasing spikes in the partial autocorrelation. This indicates MA(1).

4.1.3 Model Selection:

After deciding parameter values, we used different ARIMA(p,d,q) models in order to find out, the best one for daily calls volume forecast. Different models with fitted criterion are given in the following table

Crit./Model	ARIMA(0,0,0)	ARIMA(0,1,1)	ARIMA(1,1,1)
AIC	6672.37	6089.4	6083.1
AICc	6672.4	6089.43	6083.02
BIC	6686.15	6097.18	6101.34
F. A. Error	28.83%	11.30%	11.67%

from above table it is clear that the best fitted model is ARIMA(0,1,1).

4.1.4 ARIMA(0,1,1)

The estimated parameters of ARIMA(0,1,1) are given in the following table with standard errors.

Variable	Coefficient	Std. Error
Const.	0.273	0.167
MA(1)	-0.7282	0.0312

Using the parameters given above, the following model has been obtained for estimation of daily calls volume.

$$\Delta y_t = u_t - \theta \varepsilon_{t-1}$$

$$\Delta y_t = 0.273 - 0.7282 \varepsilon_{t-1}$$

$$y_t - y_{t-1} = 0.273 - 0.7282 \varepsilon_{t-1}$$

$$y_t = 0.273 + y_{(t-1)} - 0.728\varepsilon_{t-1}$$

Where u_t is constant and θ is MA parameter.

4.1.5 Diagnostic of model:

In order to check the diagnosis of the model, there are some tests and procedures like Anderson Darling normality test, Box pierce test, histogram, QQ plot and correlograms of ACF and PACF that are applied to the residuals of the model. We apply them to the ARIMA(0,1,1)

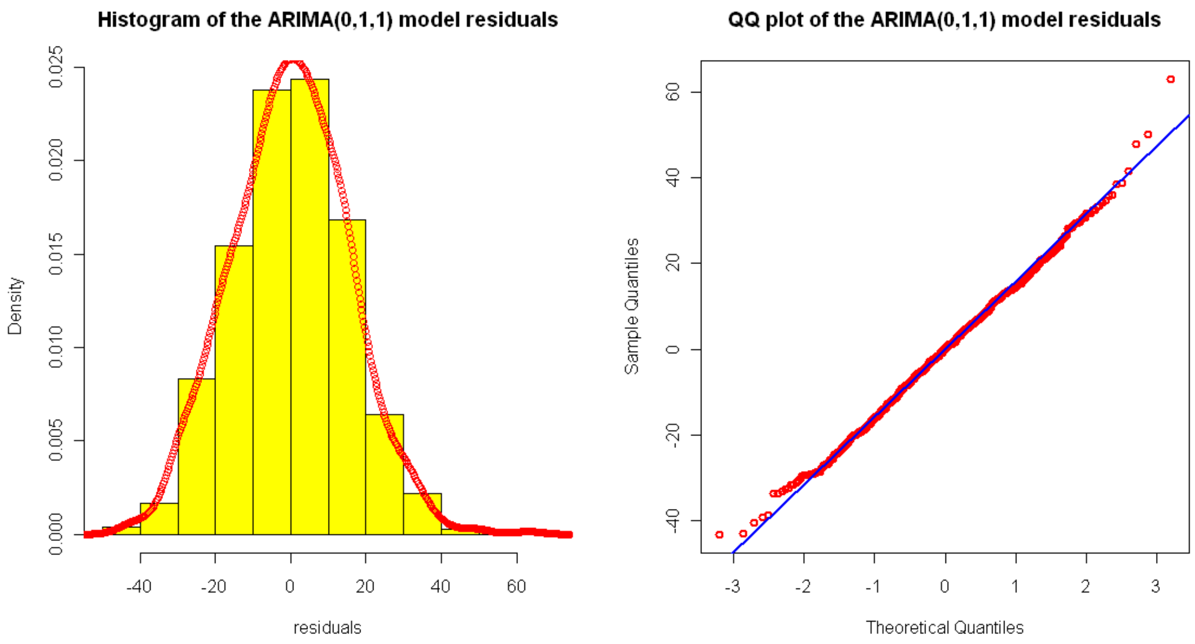

Figure 4. 8: Histogram and QQ Plots ARIMA (0, 1, 1) Residuals

The values of statistic and p-value of the Anderson Darling normality test are 0.2603 and 0.7095. The histogram is close to the bell shape. and The QQ plot is close to a straight line indicating normality. So all in all Figure 4.8 and the tests indicates that the residuals of ARIMA(0,1,1) are normally distributed.

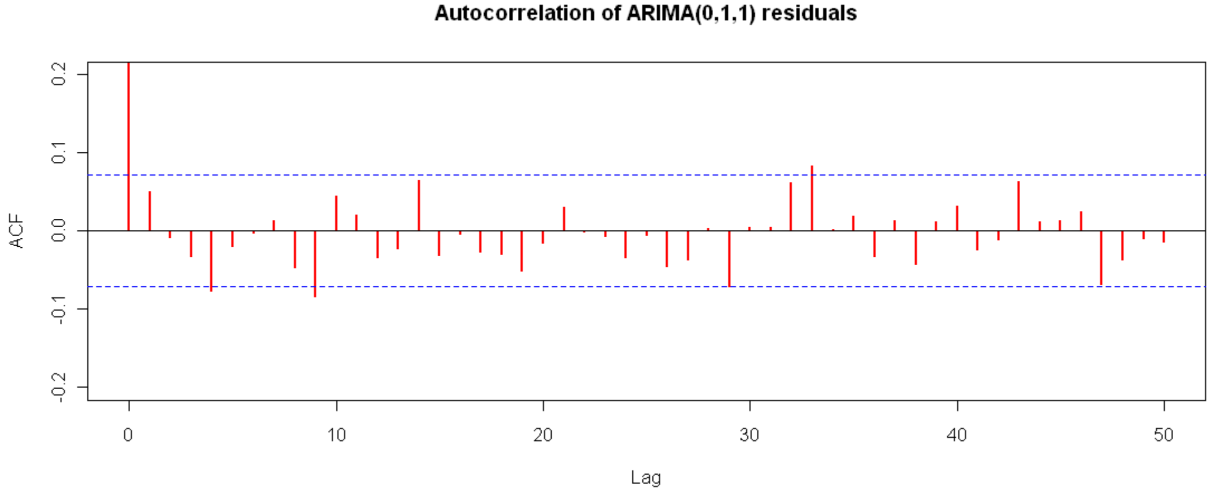

Figure 4. 9: ACF of ARIMA (0, 1, 1) residuals

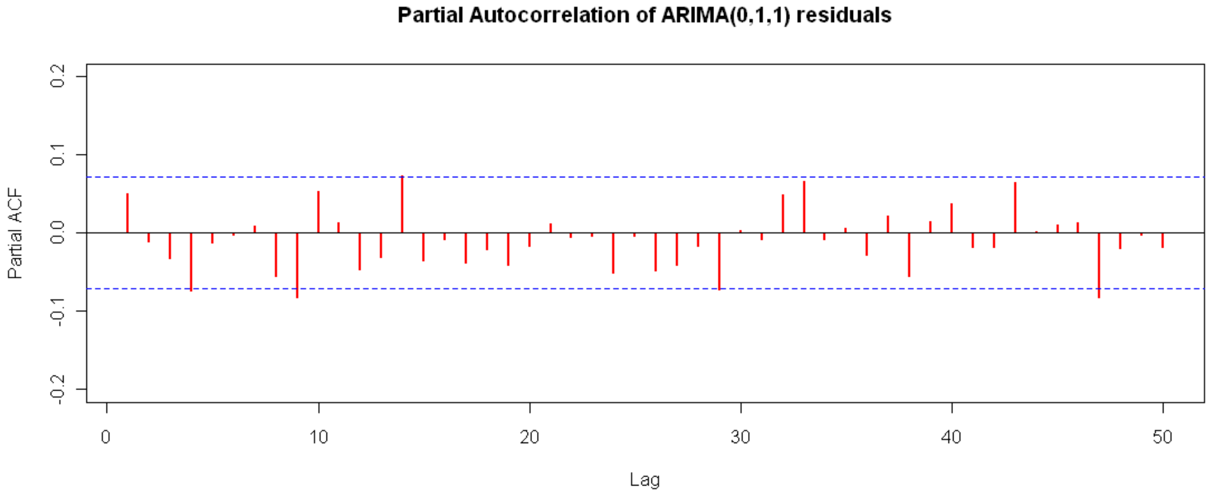

Figure 4. 10: PACF of ARIMA (0, 1, 1) residuals

In the above figure 4.7 and figure 4.8 most of the spikes of ACF and PACF lies inside the confidence band. The Box-Pierce statistic and its p values are 2.421 and 0.1197 respectively, so the Box-Pierce test shows that the residuals of ARIMA(0, 1, 1) model are not significant for different lags.

Hence the diagnosis of the model shows that the ARIMA(0, 1, 1) model is adequate.

4.1.6 Fitted Model:

The values of the fitted model are plotted over the daily calls volume data time series plot in figure 4.8.

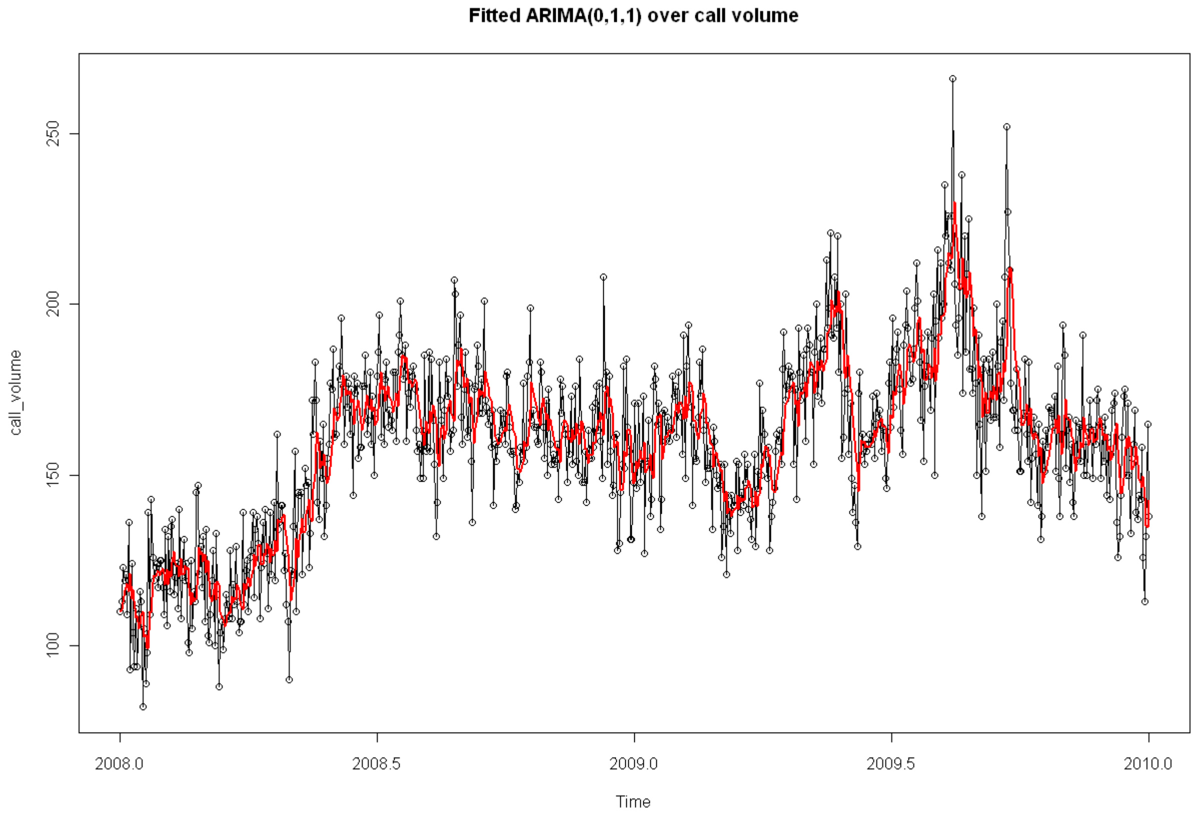

Figure 4. 11: Fitted ARIMA(0, 1, 1) over calls volume

4.1.7 Forecasting graph of ARIMA (0, 1, 1):

In the following figure 4.12, the last red curve shows one week forecasted daily calls volume of the fitted ARIMA (0, 1, 1) model.

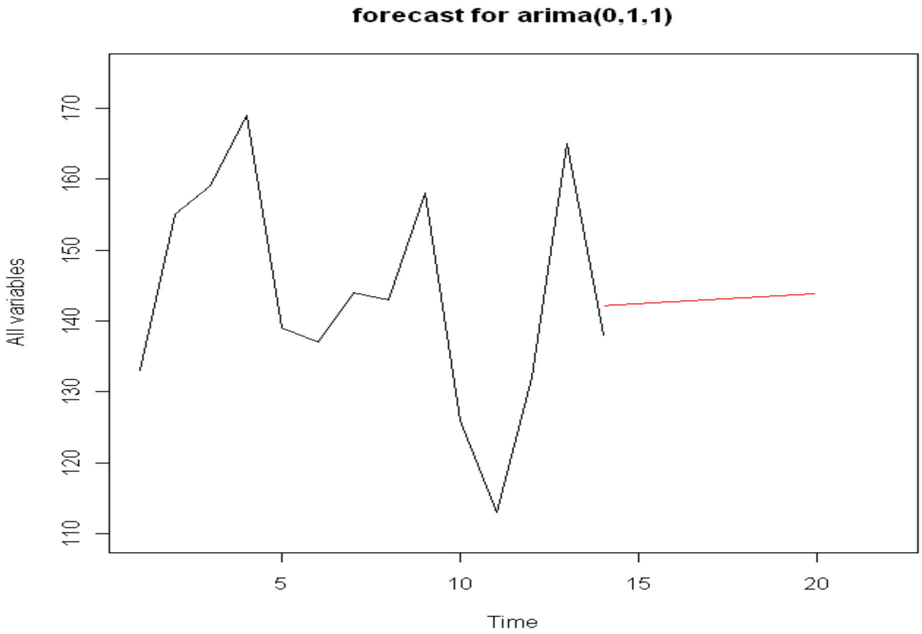

Figure 4. 12ARIMA(0, 1, 1) forecast week for daily calls volume

4.1.8 Forecasting Accuracy Measurement:

Different measurements of ARIMA(0,1,1) which are most suitable than all other ARIMA(p, d, q) models (which detail is given in Appendix)

F.A.M.	ARIMA (0, 1, 1)
ME	-0.85
MSE	241.4
RMSE	15.6
MAE	12.4
MPE	-1.4
MAPE	7.987
MASE	0.82
FORECAST ACCURACY ERROR	11.3 %

4.2.1 Exponential-Smoothing

In this section we apply the single exponential and the Holt-Winter non-seasonal method to the daily call volume. As expected we have no trend in data and hence we end up with simple exponential smoothing.

We tried different constants but it turned out that 0.266 was the best for simple exponential smoothing. A graph of the original series with the smoothed series superimposed may be seen in figure 4.13.

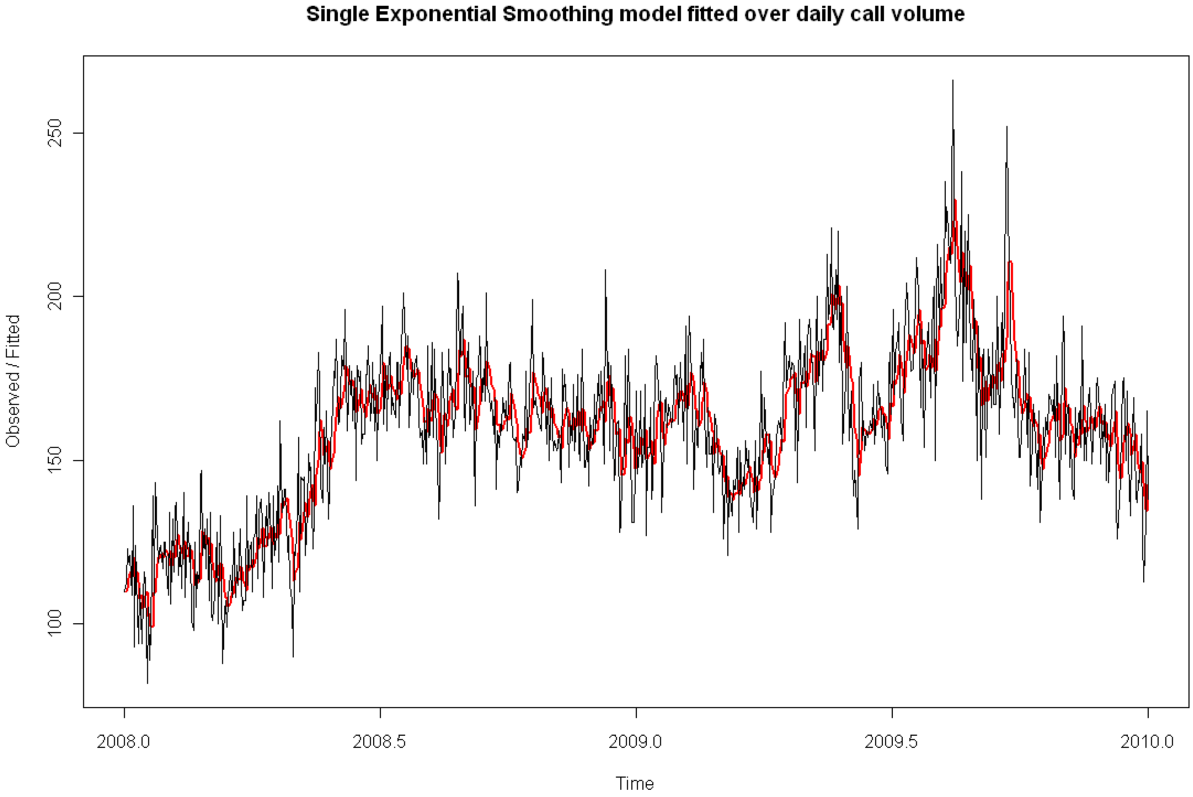

Figure 4. 13: Original Series with Smoothed Series

4.2.2 Diagnostic of model:

In order to check the diagnosis of the single exponential smoothing model, there are some tests and procedures like Anderson Darling normality test, Box pierce test, histogram, QQ plot and correlograms of ACF and PACF are applied on the residuals of the model single exponential smoothing are given below.

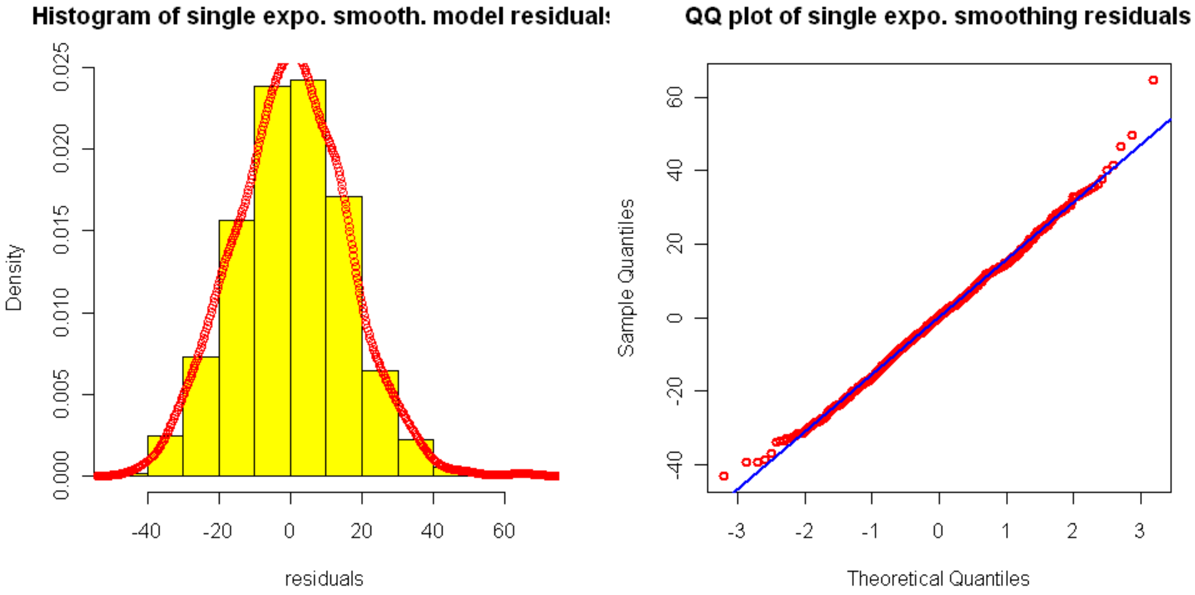

Figure 4. 14: Histogram and QQ Plots single exponential smoothing Residuals

The values of statistic and p-value of the Anderson Darling normality test are 0.2604 and 0.7093. The histogram is close to the bell shape. and The QQ plot is close to a straight line indicating normality. So all in all Figure 4.14 and the tests indicates that the residuals of single exponential smoothing are normally distributed.

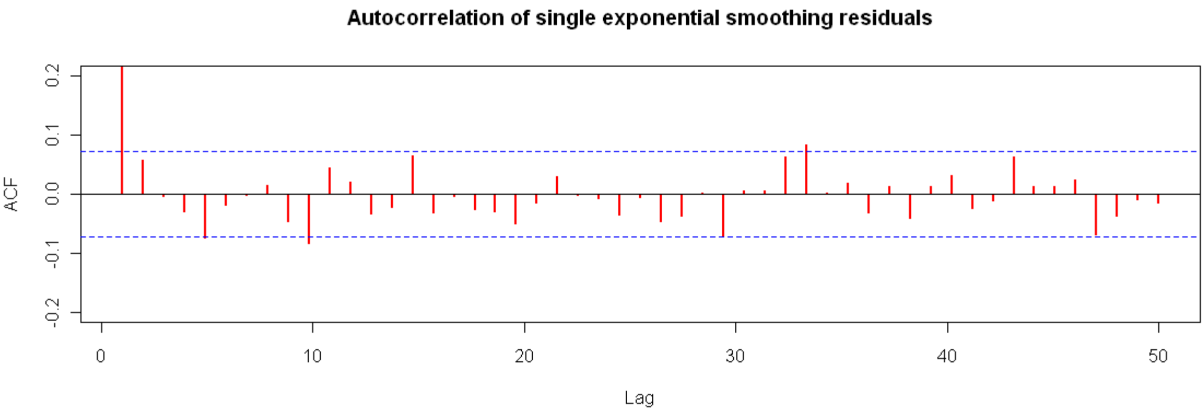

Figure 4. 15: ACF of Residuals for daily calls volume

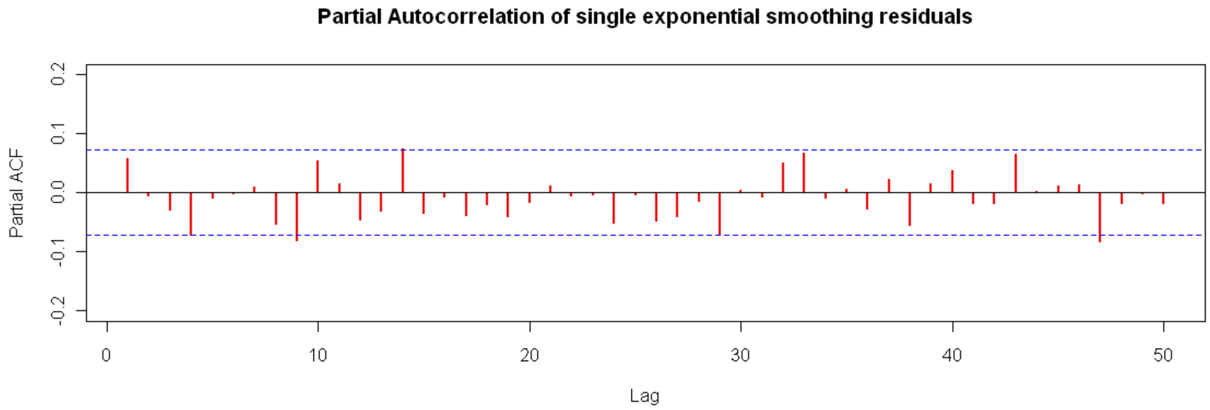

Figure 4. 16: PACF of Residuals for daily calls volume

In the above figure 4.15 and figure 4.16 most of the spikes of ACF and PACF lies inside the confidence band. The Box-Pierce statistic and its p values are 2.9288 and 0.0870 respectively, so the Box-Pierce test shows that the residuals of single exponential smoothing are not significant for different lags.

Hence the diagnosis of the model shows that the single exponential smoothing model is adequate.

4.2.3 Forecasting graph of single exponential smoothing:

In the following figure 4.17, the last red curve shows one week forecasted daily calls volume of the fitted single exponential smoothing model.

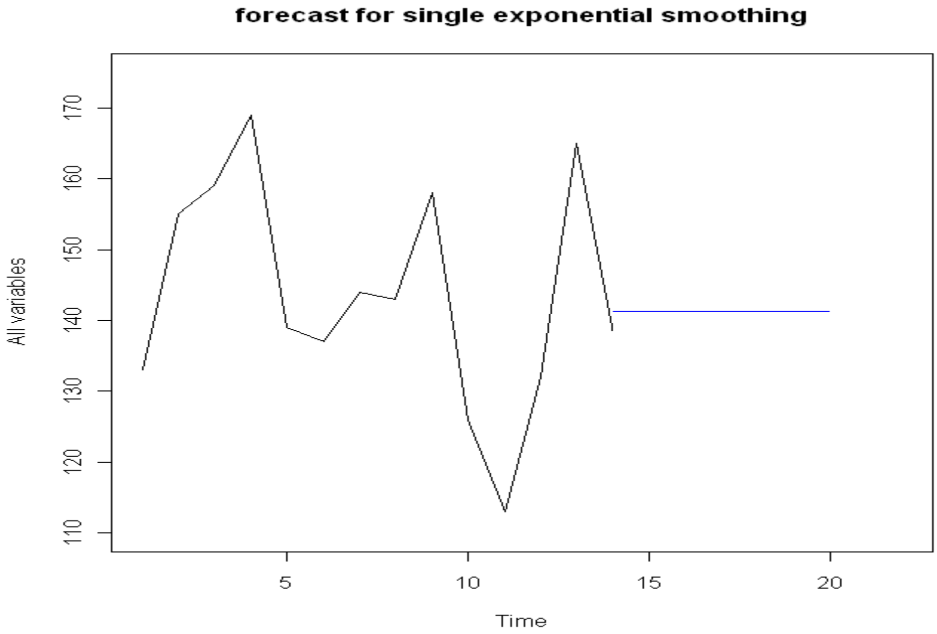

Figure 4. 17: single exponential forecast week for daily calls volume

4.2.4 Forecasting Accuracy Measurement:

Different measurements of single exponential smoothing and Holt's Linear smoothing models

Forecast Accuracy measurement	Single exp. Smoothing	Holt's Linear smoothing
ME	0.16	52.34
RMSE	15.58	93.66
MAE	12.36	61.2
MPE	-0.73	32.33
MAPE	7.948	38.7
MASE	0.82	4.07
FORECAST ACCURACY ERROR	10.8 %	10.1 %

Hence, from the above table it is very to find out that single exponential is much bitter fitted model for daily calls volume.

4.3. Naive Simple Average Procedure:

In this procedure, we forecasted for coming week of daily calls volume by taking averages of past two years corresponding days of daily calls volume data. We found 14.8 % forecasting accuracy error for this procedure as followed by the same procedure that is used for ARIMA and single exponential smoothing models, which show that both statistical models we used give more better results than simple naive average.

4.3.1 Forecasting graph of Naive simple average:

In the following figure 4.18, the last red curve shows one week forecasted daily calls volume of the Naive Simple Average.

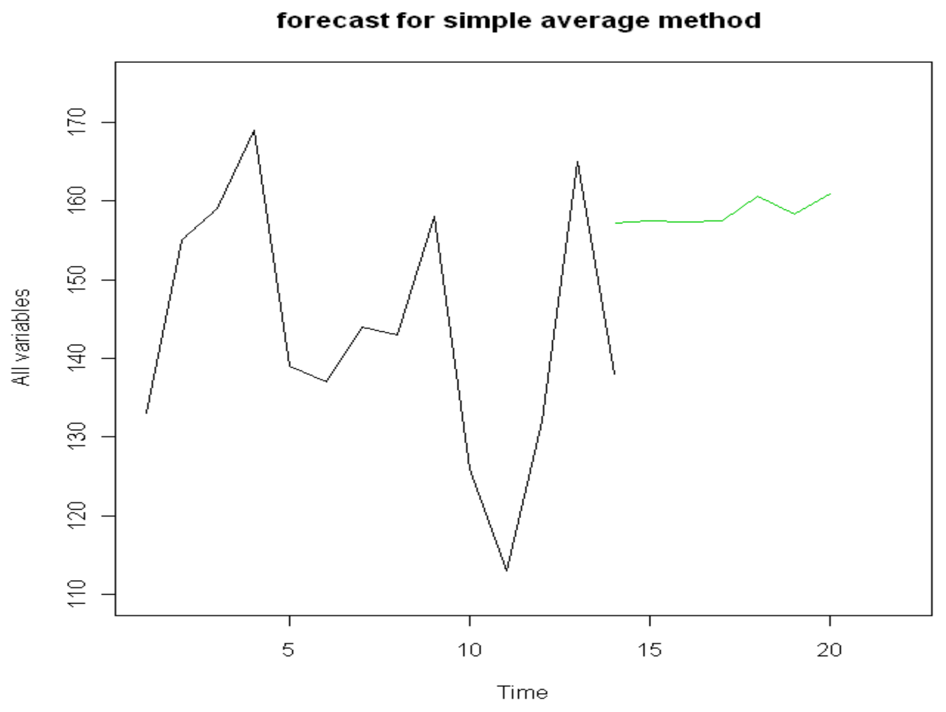

Figure 4. 18:Naive Simple Average forecast week for daily calls volume

5. Summary and Conclusion

In this study different time series techniques and models for estimating and forecasting daily call volume for the Emergency service Rescue 1122 was considered. In the first part of the analysis, the model is found by ARIMA methodology. Among the different combinations of the order of ‘p’, ‘d’ and ‘q’, the most suitable model found was ARIMA (0,1,1) and the diagnostic checks satisfied this model.

Then single exponential smoothing method was used, since there is no trend in the data nor seasonality, and the diagnostic checks confirmed a good fit. The comparison through the forecasting accuracy measurements for both models is given below:

Model	ME	RMSE	MAE	MPE	MAPE	MASE	Forecast Error	And test p value(residual)	Box test p value
ARIMA(0,1,1)	-0.85	15.6	12.4	-1.4	7.987	0.82	11.3 %	0.7095	0.1197
Single exp.	0.16	15.58	12.36	-0.73	7.948	0.82	10.8 %	0.7093	0.0870

The above comparison indicate that overall both models give equivalent results, Since the algorithm of single exponential smoothing is easier than ARIMA(0,1,1) model, Hence the single exponential smoothing model is preferred for daily calls volume forecasting. Although it. Although it is expected that such models should be applicable in other cities it is important to investigate whether this is the case.

APPENDIX

Function Reference

Function	Description
abline( )	Graphics command
acf( )	Estimation of the autocorrelation function
and.test()	Anderson normality test
arima( )	Fitting ARIMA–models
Box.test( )	Box–Pierce and Ljung–Box test
c( )	Vector command
density( )	Density estimation
diff( )	Takes differences
dnorm( )	Normal distribution
hist( )	Draws a histogram
HoltWinters( )	Holt–Winters procedure
ks.test( )	Kolmogorov–Smirnov test
length( )	Vector command
lines( )	Graphics command
mean( )	Calculates means
pacf( )	Estimation of the partial autocorrelation function
plot( )	Graphics command
predict( )	Generic function for prediction
qqnorm()	QQ plot
qqline()	Graphics command
read.csv( )	Data import from CSV–files
summary( )	Generic function for summaries
ts( )	Creating time–series objects
tsdiag( )	Time series diagnostic

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References:

[1] Lisa Bianchi, Jeffrey Jarrett and R. Choudary Hanumara (1998). Improving forecasting for telemarketing centers by ARIMA modeling with intervention: University of Rhode Island, Kingston, RI, USA.

[2] Yantai Shu, Minfang Yu, and Jiakun Liu (2003).Wireless Traffic Modeling and Prediction Using Seasonal ARIMA Models: Tianjin University, China.

[3] Channouf, Nabil (2006). The application of forecasting techniques to modeling emergency medical system calls in Calgary, Alberta, Health Care Management Science, Vol. 10, No. 1. (February 2007), pp. 25-45.

[4] P. Holcomb, John and Radke Sharpe, Norean (2007). Forecasting Police Calls during Peak Times for the City of Cleveland: CS-BIGS 1(1): 47-53.

[5] Bergh, Koen (2006). Predicting call arrivals in call center: Vrije University, Netherlands.

[6] Peijie Wang, Financial econometrics: methods and models, pp. 15-16, (2003).

[7] Bernhard Pfaff, Analysis of Integrated and Cointegrated Time Series with R, 2nd Edition, pp.14-20, 2008.