Load Adapted Solar Thermal Combisystems
– Optical Analysis and Systems Optimization

LICENTIATE THESIS

SVANTE NORDLANDER

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Abstract

In a northern European climate a typical solar combisystem for a single family house normally saves between 10 and 30% of the auxiliary energy needed for space heating and domestic water heating. It is considered uneconomical to dimension systems for higher energy savings. Overheating problems may also occur. One way of avoiding these problems is to use a collector that is designed so that it has a low optical efficiency in summer, when the solar elevation is high and the load is small, and a high optical efficiency in early spring and late fall when the solar elevation is low and the load is large. The study investigates the possibilities to design the system and, in particular, the collector optics, in order to match the system performance with the yearly variations of the heating load and the solar irradiation. It seems possible to design practically viable load adapted collectors, and to use them for whole roofs (≥ 40 m²) without causing more overheating stress on the system than with a standard 10 m² system. The load adapted collectors collect roughly as much energy per unit area as flat plate collectors, but they may be produced at a lower cost due to lower material costs. There is an additional potential for a cost reduction since it is possible to design the load adapted collector for low stagnation temperatures making it possible to use less expensive materials. One and the same collector design is suitable for a wide range of system sizes and roof inclinations. The report contains descriptions of optimized collector designs, properties of realistic collectors, and results of calculations of system output, stagnation performance and cost performance. Appropriate computer tools for optical analysis, optimization of collectors in systems and a very fast simulation model have been developed.

Keywords: solar collector, combisystem, concentrating, reflector, stagnation, optical, optimization, simulation, TRNSYS

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# Nomenclature

## Latin symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area</td>
<td>$[m^2]$</td>
</tr>
<tr>
<td>A&lt;sub&gt;abs&lt;/sub&gt;</td>
<td>Absorber area</td>
<td>$[m^2]$</td>
</tr>
<tr>
<td>A&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Collector aperture area</td>
<td>$[m^2]$</td>
</tr>
<tr>
<td>B</td>
<td>Radiation flux</td>
<td>[W]</td>
</tr>
<tr>
<td>C</td>
<td>Concentration ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>c&lt;sub&gt;1&lt;/sub&gt;</td>
<td>Linear heat loss factor</td>
<td>$[Wm^{-2}K^{-1}]$</td>
</tr>
<tr>
<td>c&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Second order heat loss factor</td>
<td>$[Wm^{-2}K^{-2}]$</td>
</tr>
<tr>
<td>CPI&lt;sub&gt;cm&lt;/sub&gt;</td>
<td>Cost performance index for collector materials</td>
<td>$[SEK kWh^{-1} a^{-1}]$</td>
</tr>
<tr>
<td>D</td>
<td>Thickness</td>
<td>[mm]</td>
</tr>
<tr>
<td>F'</td>
<td>Collector efficiency factor</td>
<td>[-]</td>
</tr>
<tr>
<td>f&lt;sub&gt;sav&lt;/sub&gt;</td>
<td>Fractional thermal energy savings</td>
<td>[-]</td>
</tr>
<tr>
<td>F&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Target function for optimization</td>
<td>[kWh]</td>
</tr>
<tr>
<td>G&lt;sub&gt;b&lt;/sub&gt;</td>
<td>Beam irradiance on the collector plane</td>
<td>[W m&lt;sup&gt;2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>G&lt;sub&gt;d&lt;/sub&gt;</td>
<td>Diffuse irradiance on the collector plane</td>
<td>[W m&lt;sup&gt;2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>G&lt;sub&gt;T&lt;/sub&gt;</td>
<td>Total irradiance on the collector plane</td>
<td>[W m&lt;sup&gt;2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>G&lt;sub&gt;TC&lt;/sub&gt;</td>
<td>Critical irradiance</td>
<td>[W m&lt;sup&gt;2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>I&lt;sub&gt;b&lt;/sub&gt;</td>
<td>Beam irradiation on the collector plane</td>
<td>[Wh m&lt;sup&gt;-2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>I&lt;sub&gt;d&lt;/sub&gt;</td>
<td>Diffuse irradiation on the collector plane</td>
<td>[Wh m&lt;sup&gt;-2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>I&lt;sub&gt;T&lt;/sub&gt;</td>
<td>Total irradiation on the collector plane</td>
<td>[Wh m&lt;sup&gt;-2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>k</td>
<td>Slope of optical efficiency curve segment</td>
<td>$[rd^{-1}]$</td>
</tr>
<tr>
<td>K&lt;sub&gt;L&lt;/sub&gt;</td>
<td>Correction factor for longitudinal component of beam radiation</td>
<td>[-]</td>
</tr>
<tr>
<td>K&lt;sub&gt;θb&lt;/sub&gt;</td>
<td>Incidence angle modifier for beam radiation</td>
<td>[-]</td>
</tr>
<tr>
<td>K&lt;sub&gt;θd&lt;/sub&gt;</td>
<td>Incidence angle modifier for diffuse radiation</td>
<td>[-]</td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
<td>[m]</td>
</tr>
<tr>
<td>L</td>
<td>Radiation intensity</td>
<td>$[W m^{-2} rd^{-1}]$</td>
</tr>
<tr>
<td>P&lt;sub&gt;cm&lt;/sub&gt;</td>
<td>Material cost</td>
<td>[SEK $m^{-2}$]</td>
</tr>
<tr>
<td>q&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Thermal output energy rate per unit aperture area</td>
<td>[W $m^{-2}$]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;aux&lt;/sub&gt;</td>
<td>Auxiliary energy</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Energy delivered by a collector</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;dbhw&lt;/sub&gt;</td>
<td>Domestic hot water energy</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;h&lt;/sub&gt;</td>
<td>Total heating load</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;lb&lt;/sub&gt;</td>
<td>Optical loss for beam radiation</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;ld&lt;/sub&gt;</td>
<td>Optical loss for diffuse radiation</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;Loff&lt;/sub&gt;</td>
<td>Collector losses when the system is off but not in stagnation</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;Loh&lt;/sub&gt;</td>
<td>Collector losses during stagnation</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;lop&lt;/sub&gt;</td>
<td>Collector losses during normal operation</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;LS&lt;/sub&gt;</td>
<td>Storage heat losses</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;Lth&lt;/sub&gt;</td>
<td>Thermal loss of collector</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;sh&lt;/sub&gt;</td>
<td>Space heating energy</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;T200&lt;/sub&gt;</td>
<td>Total irradiation for hours when $I_T &gt; 200$</td>
<td>[kWh]</td>
</tr>
<tr>
<td>Q&lt;sub&gt;u&lt;/sub&gt;</td>
<td>Useful energy delivered by a collector</td>
<td>[kWh]</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>[h], [a]</td>
</tr>
<tr>
<td>T&lt;sub&gt;a&lt;/sub&gt;</td>
<td>Ambient temperature</td>
<td>[$^o$C]</td>
</tr>
<tr>
<td>T&lt;sub&gt;a200&lt;/sub&gt;</td>
<td>Average ambient temperature when $I_T &gt; 200$</td>
<td>[$^o$C]</td>
</tr>
<tr>
<td>T&lt;sub&gt;i&lt;/sub&gt;</td>
<td>Inlet fluid temperature</td>
<td>[$^o$C]</td>
</tr>
</tbody>
</table>
T \text{m} \quad \text{Collector mean fluid temperature} \quad [\degree \text{C}]

T \text{stag} \quad \text{Stagnation temperature} \quad [\degree \text{C}]

T_o \quad \text{Outlet fluid temperature} \quad [\degree \text{C}]

t_{\text{ref}} \quad \text{Reference stagnation duration} \quad [\text{hour}]

T_{\text{ref}} \quad \text{Reference temperature} \quad [\degree \text{C}]

t_{\text{stag}} \quad \text{Stagnation duration} \quad [\text{hour}]

U_{\text{hx}} \quad \text{Heat exchanger heat transfer capacity} \quad [\text{W K}^{-1}]

U_L \quad \text{Collector heat loss factor} \quad [\text{W m}^{-2} \text{K}^{-1}]

V_{\text{st}} \quad \text{Storage volume} \quad [\text{m}^3]

W_{\text{abs}} \quad \text{Absorber width} \quad [\text{m}]

W_t \quad \text{Weighting factor for stagnation duration} \quad [\text{kW}]

W_T \quad \text{Weighting factor for stagnation temperature} \quad [\text{kWh K}^{-1}]

x_d \quad \text{Ratio of diffuse irradiation to total irradiation} \quad [-]

\textbf{Greek symbols}
\begin{align*}
\alpha & \quad \text{Absorptance} \quad [-] \\
\beta & \quad \text{Collector slope} \quad [\degree] \\
\Delta T & \quad \text{Temperature difference} \quad [\text{K}] \\
\eta_0 & \quad \text{Zero-loss collector efficiency} \quad [-] \\
\eta_{0b} & \quad \text{Zero-loss collector efficiency for beam radiation} \quad [-] \\
\eta_{0d} & \quad \text{Zero-loss collector efficiency for diffuse radiation} \quad [-] \\
\eta_{0b} & \quad \text{Optical efficiency for beam radiation} \quad [-] \\
\eta_{0d} & \quad \text{Optical efficiency for diffuse radiation} \quad [-] \\
\eta_{0d,01,02} & \quad \eta_{0d} \text{ within an angle interval } 0_1 < \theta < 0_2 \quad [-] \\
\theta_c & \quad \text{Acceptance half angle} \quad [-] \\
\theta_L & \quad \text{Longitudinal incidence angle} \quad [-] \\
\theta_{\text{noon}} & \quad \text{North-south projected solar elevation at noon} \quad [-] \\
\theta_{\text{pe}} & \quad \text{North-south projected solar elevation} \quad [-] \\
\theta_{\text{pew}} & \quad \text{Energy weighted north-south projected solar elevation} \quad [-] \\
\theta_T & \quad \text{Transversal incidence angle} \quad [-] \\
\rho & \quad \text{Reflectance} \quad [-] \\
\tau & \quad \text{Transmittance} \quad [-] \\
\tau_{\alpha\text{en}} & \quad \text{Optical efficiency for beam radiation at normal incidence} \quad [-]
\end{align*}

\textbf{Subscripts}
\begin{align*}
a, \text{abs} & \quad \text{absorber, absorbed} \\
b & \quad \text{beam} \\
c & \quad \text{cover, collector} \\
d & \quad \text{diffuse} \\
\text{FP, F} & \quad \text{Flat plate collector} \\
L & \quad \text{Longitudinal (parallel with optical axis)} \\
\text{LA} & \quad \text{Load adapted collector} \\
n & \quad \text{normal} \\
r & \quad \text{reflector} \\
T & \quad \text{Transversal (perpendicular to optical axis)} \\
u & \quad \text{useful}
\end{align*}
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1 Introduction

1.1 Objective, scope and limitations

The objective of this study is to analyze solar thermal combsystems for high solar fractions in a Northern European climate, and to arrive at design methods and parameters for the system, and in particular for the optical performance of the solar collector. The study is limited to a Stockholm climate, single-family houses with annual combined space heating and domestic hot water loads of approximately 11 MWh, and to systems with stationary collectors and short-term storage.

1.2 Solar thermal energy and materials physics

The essence of solar thermal technology is the conversion of radiative energy into heat. This occurs in the absorber surface of a solar thermal collector, which naturally has been a prime object of interest for physicists working with solar energy materials. The absorptance for solar radiation should of course should be as high as possible. The properties in the infrared part of the spectrum are also essential. At temperatures between 50 °C and 100 °C, where most collectors operate, the heat losses from the absorber by infrared radiation are of the same order of magnitude as the conductive and convective losses. Losses by thermal radiation have been reduced very succesfully by the development of selective surfaces, which are characterized by high absorptance for radiation in the solar spectrum and low emittance in the infrared part of the spectrum. Suitable surfaces have been developed by work in the field of materials physics, and several methods for manufacturing of selective surfaces are in industrial use, including electrolytical techniques and sputtering. Recently promising advances have been made with solution-chemistry techniques, which is of special interest because the coating can be applied with inexpensive methods at atmospheric pressure.

Materials physics has been important also in developing materials for covers and reflectors. The objective is usually to reduce the optical losses, by increasing the transmittance of covers and increasing the reflectance of reflectors. The application of multiple thin layers or materials with graded refractive index is often used to achieve these goals. High efficiency collectors usually have covers with durable anti-reflective coatings. The convective heat losses from the absorber may be reduced by anti-convection barriers, such as transparent polymer films. Transparent insulation materials like polymer or glass honeycomb structures and aero-gels reduce convective and conductive as well as radiative losses. The high efficiency of the modern flat plate collector has brought new problems, however. The stagnation temperature of the collector may be over 200 °C, forcing manufacturers to use more temperature-resistant materials, and the problem of degradation of the heat carrier fluid at high temperatures is not fully solved.

The present study is mainly concerned with the possibilities of increasing energy collection and avoiding overheating problems in solar thermal systems by suitable arrangements of absorbers and reflectors. The analysis is based on the optical and thermal properties of commonly available industrial materials. There are limitations to this approach, such that the resulting optimal collector designs presuppose a certain orientation of the collector and certain relations between the collector area and yearly variations in climate and in thermal load. There is also an inherent conflict between energy output and overheating protection. In the conclusion of this study is briefly discussed how future developments of collector materials might help to overcome these limitations.
1.3 Outline of the study

Chapters 2 and 3 provide a background and introduce the concepts and ideas behind the work. Chapter 2 also explains what is meant by a load adapted collector. In chapter 4 collector parameters for flat plate and concentrating collectors taken from the literature are presented.

Chapter 5 presents an estimation of consistent, representative collector parameters, optical and thermal, for concentrating collectors in a range of concentrations ratios deemed suitable for load adapted systems. This set of parameters will be used in subsequent chapters as input data for various calculations. Some newly developed theory on non-imaging concentrating collectors is presented in the chapter.

In chapter 6 several examples and detailed calculations are presented to provide a deeper understanding of systems for large solar fractions and the load adapted collector. The chapter also presents a method for calculating the desired optical properties of a load adapted collector and for estimating upper and lower limits for a suitable concentration ratio.

Chapter 7 is devoted to ideal collectors that are optimized for energy output with different restrictions. A semi-automatic method for optimization of the optical efficiency for concentrating collectors in systems is described. Ideal optical efficiency curves for many combinations of system sizes and collector concentration ratios are presented and discussed. Examples of how restraints on temperature and stagnation duration affect the useful energy output are given.

In chapter 8 a ray tracing technique is presented which is used for design and evaluation of realistical collectors. The optical performance of a number of designs are presented and compared with the performance of theoretically optimized collectors of chapter 7.

Chapter 9 is about the performance of realistic collector designs. System output with different collector designs are simulated, and estimations of stagnation temperatures and cost efficiency are presented.

Chapter 10 presents results from detailed system simulations with the TRNSYS programme. System parameters are varied, and heat exchangers and store sizes are optimized. Comparisons between systems with flat plate and load adapted collectors are made in order to determine whether a load adapted system will need to be dimensioned differently from a conventional system. A comparison between results from TRNSYS simulations and calculations with simplified, very fast Excel calculations is presented and discussed.

Chapter 11 contains a summary of the main results and a discussion.
2 Background

The global consumption of fossil fuels such as crude oil, natural gas and coal is growing by the rate of approximately 2% per year. During the last five years signs have emerged of difficulties to increase the extraction of these resources to meet the increasing demand fully, indicated by price increases and volatility. Also, due to the risk of global climate change caused by the emission of greenhouse gases it is strongly desirable to reduce the use of fossil fuels. Among the renewable sources, solar thermal energy is one means of avoiding the adverse environmental impacts arising from the use of conventional fuels. Solar thermal energy from a combisystem is most often used for space heating and domestic hot water preparation and represents a sensible way for house owners to reduce their dependence on non-renewable energy. The European solar thermal market is now firmly established with strong industrial actors delivering standard products of high quality and efficiency. The Swedish solar thermal market is smaller, per capita, than the most successful ones in Germany and Austria, but it has grown by about 25% annually since the late 1990's. There is continuing scientific and industrial work going on in order to increase the energy output, ease the installation work and lower the costs of solar thermal systems.

2.1 Solar thermal combisystems

Figure 1 shows the main components of a typical Swedish combisystem with external hot water preparation. The collected solar energy is delivered to the storage tank, and is used for the radiator system or domestic hot water as needed. When the temperature in the upper region of the store goes below a given limit, an auxiliary energy source charges the store. In this case it is an internal electric heater, but it could also be an external boiler. There is a separate control system for the solar subsystem. When the temperature in the upper part of the collector is above the temperature in the lower part of the store the control system starts the pumps connected to the solar loop heat exchanger. When the temperature in the lower part of the store exceeds the temperature in the upper part of the collector the pumps stop. If the temperature in the collector exceeds a given limiting temperature, for example 95 °C, the control system stops the pumps in order to protect the store and other components from harmful pressures and temperatures. This system state is called stagnation and will occur, for example, during summer if the system load is not large enough cool the tank sufficiently.
A typical Swedish solar combisystem may reduce the use of auxiliary energy (from e. g. oil or electricity) with approximately 20% (Lorenz et al., 2000; Weiss, 2003). Figure 2 shows results from a computer simulation of a year of operation of a typical combisystem with 10 m² flat plate collectors and a 750 liter storage tank in a Swedish single family house.

In the middle of the summer there is enough irradiation to cover the whole load, which then consists mainly of domestic water heating. In spring, fall and winter the system provides both space heating and hot water. During the winter months the irradiation is so weak and the load is so large that only a small fraction of the heating load is covered by solar energy. The storage tank can at the most store heat for a week, so with this type of system it is impossible to save energy from summer to winter. According to common Swedish practice the collector area is dimensioned to cover the hot water load in summer, when there is practically no space.
heating demand. In case of prolonged periods of sunny summer days, or if there is no load, the store may reach a temperature of 95 ºC, causing the control unit to stop the pumps and allowing the collectors to reach a high stagnation temperature.

### 2.2 Possible strategies for increasing the solar fraction

The solar fraction of a typical combisystem with 10 m² collector area is around 20 %. This may be considered to be such a small part of the total energy consumption of the building that a prospective customer would not be interested to invest in the system, even if the cost of the solar energy were acceptable. There are several ways to increase the solar fraction:

- To increase the collector area
- To use more efficient collectors
- To use a larger store
- To improve the function of the store

If the solar fraction is increased by merely adding more collector area, some additional energy will certainly be collected before and after the period in summer when the load is fully covered, according to Figure 2. The system will probably get a shorter total yearly operation time. The system will operate on approximately the same days as with the smaller collector area, but it will sometimes start later or stop earlier during the day. It will start later if the tank temperature is significantly higher in the morning than in the case with the smaller collector area. This may happen when the solar energy collection on the previous day has resulted in a surplus and if the load has not removed the whole surplus from the store during the night and early morning. The solar energy collection will stop earlier in the day if there is enough solar radiation to take the system temperature up to the cut-off limit of 95 ºC. The stagnation periods will thus be longer, but the stagnation temperature, i. e. the collector temperature at stagnation conditions, will seldom be higher, since the stagnation temperature is dependent only on collector properties, solar radiation and ambient temperature, but not on collector area.

In order to increase the amount of collected energy it is also possible to use more efficient collectors that can operate at lower ambient temperatures and lower radiation levels than the standard flat plate (FP) collectors, for example vacuum collectors, compound parabolic (CPC) collectors and collectors with extra convection barriers or with transparent insulation (TI). This will be useful during winter, early spring and late fall, but it will also cause longer overheating periods, due to the lower heat losses of these collectors. Furthermore, since these collectors have higher stagnation temperatures than flat plate collectors they will also expose the system materials and the collector loop fluid to more severe temperature stress. High temperature collectors of vacuum, CPC or TI types are usually more expensive than FP collectors and they can rarely match flat plate collectors economically.

A normal storage tank has a volume of about 750 liters and can store enough heat for about five days of hot water consumption in summer, but less than a day's heating demand during winter. In summer, if there are several sunny days in a row, there is not enough load to cool the store, so it soon reaches its maximum temperature. With a larger storage volume some of the additional energy may be distributed to the load over a longer time span. This effect is limited to fall and spring, since in summer the store is always almost full even with a standard tank, and in winter the store will never be filled even with a small tank, since the load is so much larger than the irradiation. Also, the heat losses from the store increase with size, unless
extra insulation is added, so that increasing the volume above a few cubic meters volume will cause an increase of the store losses in the same order of magnitude as the additional energy gained from the collectors (Weiss, 2003).

A not so obvious, but surprisingly efficient way of increasing the solar fraction is to optimize the function of the heat store as demonstrated by Lorenz et al. (2000). These optimizations mainly serve to improve the stratification of the water in the storage tank and to make the heat removal to the load more efficient, in order to cool the tank as much as possible and to preserve the stratification. It is thus possible to increase the solar fraction of the "standard Swedish combisystem" from below 20 % to more than 25 % by a more efficient design of the heat store and optimal use of standard components, without increasing the collector area or the store volume.

2.3 Overheating problems

In the design handbook "Solar Heating Systems for Houses" of the IEA SHC task 26, a whole section is devoted to system stagnation behaviour, with the following introduction

"According to EN ISO 9488, stagnation is the status of a collector or system when no heat is being removed by a heat transfer fluid. Stagnation may occur both as the result of normal operation or following abnormal circumstances like a temporary interruption of the circulating-pump power supply." (Weiss, 2003)

As modern flat plate collectors can reach a temperature of 170 °C or more (SPF, 2002), the temperatures during periods of stagnation may cause several kinds of problems. Short term exposure at sufficiently high temperatures can cause sudden damage or other types of failures, from the high temperature, from a high pressure (temperature induced) of the collector liquid, or from a combination of these two factors. Since the fluid in the collector may become partially vaporized during stagnation and since energy is efficiently transported by evaporation and condensation of the fluid, the possible damage is not limited to the collector, but may occur in parts of the system at a distance from the collector array. Known examples of failures are:

- Deformation or rupture of plastic collector components such as the absorber or the insulation.
- Melting of soldered joints between absorber strips and piping, causing leakage.
- Degradation of components of the collector fluid, which is usually a mixture of water and glycol and certain additives. It is the additives that decompose at high temperatures and turn into substances that foul the liquid and can cause congestion or complete clogging of the system. New glycol mixtures with better properties are being developed but the problem is not completely solved.
- Destruction of rubber or plastic foam pipe insulation.
- Blow-out of collector liquid through the safety valve. If the liquid is collected properly it may be refilled into the system and the system can be started again, but this work normally requires the attention of a skilled person, and can not usually be performed until the system has cooled down and the solar radiation has decreased sufficiently.

Exposure to high temperatures during longer periods of time may cause gradual degradation of materials and slowly developing defects, such as:
- Outgassing from mineral wool insulation, causing deposits on the absorber and the inside of the cover that lower the optical performance of the collector.
- Brittleness or deformation of polymeric sealing components, permitting rain to leak into the collector. The rate of degradation of polymers approximately doubles for every temperature increase of 10 °C (Perers, 2004).

An unwanted temperature related effect from the use of large collector areas or highly efficient collectors is that the store will be very hot during most of the summer. In Sweden many houses are uncomfortably warm indoors during summer, even without a solar system. If the solar heat store is located in the house, the temperature rise due to the heat losses from the store of a system in stagnation may decrease the comfort level of the living zone significantly.

Manufacturers are continuously working in order to increase the efficiency of commercial collectors. The improvements include better absorptance and lower emittance of the absorbers, anti-reflection treated covers, anti-convection barriers and improved insulation. As the efficiency, and hence the stagnation temperatures, of the collectors increase, the materials must be increasingly more resistant to temperature stress, which normally means more expensive materials. Thus, the increased efficiency of the collectors also implies higher costs, both for the collectors and for parts of the systems that may be affected by the stagnation effects of boiling and condensation.

In the Task 26 handbook a number of protection schemes used by system suppliers are described:

A  Cooling of the store by automatic running of domestic hot water to waste
B  Cooling of the store by operation of the collector loop after sunset if the store temperature is above a given limit
C  Allowing high pressures, over 6 bars, in the collector loop, which prevents boiling
D  Drainback of the collector fluid by gravity when the system is not in operation
E  System and collectors are designed so that the hot vapor in the collector can quickly force the liquid down into an expansion vessel; only a small amount of vapor will be left in the collector and exposed to the high temperatures.

The last method, E, is considered to be the most modern one. Perers et al. (2004) have recently shown experimentally that this method works well with commonly available Swedish components, provided that the system is properly dimensioned and installed. Method E requires that the liquid is able to exit the collector easily without being trapped in, for example, vertical U-bends. One efficient way of accomplishing this is to have the absorber channels arranged horizontally. The gravity drainback principle is also recommended, but it requires that all piping, including the absorber tubing, has a downward slope. These two methods, D and E, will protect the collector liquid and the system from high temperatures. Methods A and B, if working as intended, will protect all parts of the system from stagnation, but they are not foolproof since they require that the control system, the electricity supply and the water mains are functioning. Method C, which hitherto is the most common one in Sweden, may protect the non-collector parts of the system from high temperatures, but the collector and in particular the fluid will suffer the full extent of the stagnation temperature stress. It requires a setting of the safety valve of around nine bars. More efficient collectors would require even higher settings. The pressure with method E will be below 2 bars at the same temperatures since the vapor is superheated, not saturated as with method C.
All the protection schemes discussed above provide stagnation overheating protection for the system, but not for the collector itself. Methods A and B may protect the collector, but not in all cases.

2.4 The load adapted collector

In this work, the main focus is to explore the possibilities of adapting the collector optical properties to the load and the climate in such a way that:

- The thermal losses are as small as possible at all times in order to enable a high output.
- The optical losses are higher at higher solar elevations, in order to decrease the overall collector efficiency and prevent overheating during summer, late spring and early fall.
- The optical efficiency is as high as possible at times when there is solar radiation available and at the same time there is enough load to use it. For a Swedish climate this is during winter, early spring and late fall.

With such a collector, it ought to be possible to install large collector areas (>10 m²) on single family houses without unnecessary heat being produced in summer and without overheating problems. In this report a collector with these properties is called a load adapted solar collector, or shorter, an LA collector.

Using the system in Figure 2 as an example, it means that the efficiency of a load adapted collector should be such that it is capable of producing as much energy as indicated by the load curve, but not more than that. The difference between the total radiation and the load suggests that there is quite a potential for a lower optical efficiency during summer, without losing any useful energy. The total efficiency of the collector during the period when there is an abundance of solar radiation in relation to the load will be the same for the load adapted collector as for a conventional one, but the loss mechanism will be different. The load adapted collector will have high optical losses during summer, and the conventional system will have high thermal losses from the collector, the piping and the storage. Thus the mean temperature of the load adapted system, including collector, piping and storage, will be lower during the periods when the collector has reduced optical efficiency.

The study is limited to glazed, non-imaging concentrating collectors with internal reflectors. The glazing, with the reflectors and the absorber beneath it, is there in order to reduce heat losses from the absorber and to give the collector a long life. Since the collector is a concentrating one, the absorber area will be smaller than the aperture area. This has two favourable implications:

- The heat losses from the collector will be lower, because the heat losses from the collector are proportional to the absorber area.
- The cost of the collector will be reduced because the cost per unit area of a good reflector is in the order of a third of the cost of a good absorber.

The use of reflectors allows several degrees of freedom that are useful in the design of the optical properties of the collector:

- It is possible to use an absorber that is illuminated on one side, as in a flat plate collector, or on both sides.
- The reflectors make it possible to tailor the incidence angle dependence of the optical efficiency for the beam radiation within broad limits.

![Diagram of Collector type A](image1)

**Figure 3.** Load adapted collector type A with single sided absorber, designed for a southward tilt of 30° at a latitude of 60° N.

![Diagram of Collector type B](image2)

**Figure 4.** Load adapted collector type B with double sided absorber, designed for a southward tilt of 30° at a latitude of 60° N.

Figure 3 and Figure 4 show two examples of load adapted collectors. Collector A has an absorber that is illuminated on one side and collector B has an absorber that is illuminated on both sides. The optical function during summer (dotted arrows) and winter (solid arrows) is indicated in the figures. At summer solar elevations most radiation will be reflected out of the collector, while at winter solar elevations all beam radiation will be directed towards the absorber. In the spring there will be a gradual decrease of the optical efficiency, with increasing solar elevation, and in the fall the reverse change takes place. This dependence of the optical efficiency on solar elevation is evident for noon solar elevations, but as will be discussed in more detail later, it is also sufficiently valid at other times of the day. In this manner it is possible to achieve a high optical efficiency in winter and early spring and a lower optical efficiency in summer.

Type A will have slightly higher optical efficiency at lower solar elevations than type B, since a larger fraction of the radiation hits the absorber directly, without reflections. On the other hand type B might be more cost effective since the absorber strip is utilized on both sides for energy collection. But again, type A uses less reflector material and may be the lightest, simplest and most material efficient design. These issues will be discussed more in depth in the chapters on collector design and performance.
A number of questions arise from this choice of collector concept, concerning optimal optical properties, the choice of absorber type (single or double sided), energy production, system dimensioning, the possible impact on overheating problems and cost efficiency. In short, the study gives the following results:

A collector with a concentration ratio of around 2 suits a large span of applications. The incidence angle dependence generally should be such that the beam efficiency is as good as possible from winter until some time in spring, depending on system size, and then decrease gradually to a much lower value in summer. The absorber may be single or double sided, the final choice depending on practical circumstances. A load adapted system will produce less energy per collector unit area than an FP collector in systems with small collector areas, but about as much energy in larger systems. The LA collector has a good potential for reduction of stagnation problems, although it is necessary to compromise between energy collection and overheating protection. The optimal storage and heat exchanger capacities are similar for LA systems and FP systems. In applications with large collector areas the LA collector will probably be more cost effective than an FP collector.

2.5 Related work

The idea of adapting the collector optics to the load and the climate is not new. Rabl (1976), Mills and Giutronich (1978) and Mills et al. (1994) have proposed asymmetric collector designs for matching of the collector output to the load and the climate. A variety of reflector and absorber (planar, tubular and evacuated tubular) designs have been suggested in these papers.

Muschaweck et al. (2000) report on design and testing of an asymmetric concentrating collector with tubular absorber that is optimized for maximum collection of the solar irradiation of a German climate, but the seasonal variations of the load is not taken into account. Tripanagnostopoulos et al. (2000) have investigated different versions of asymmetrical collectors with flat absorbers, but the main argument for these collectors was the lower heat losses from the absorber, not that the collectors could match the yearly variations in climate or load.

The Swedish MaReCo collector (Karlsson and Wilson, 2000) in its standard configuration is optimized for Swedish solar irradiation, but the seasonal variation of the load is not considered. Another MaReCo version that is designed to match the seasonal variations of a domestic heating load, the Spring/Fall MaReCo, is further described and analyzed by Helgesson et al. (2002) and Adsten (2002).

Mills and Morrison (2003) have simulated different designs of asymmetric concentrating collectors for domestic hot water preparation in an Australian climate and concluded that they are able to replace considerably more auxiliary energy than symmetric and flat plate designs. The analysis takes the yearly variations of the climate and the load into consideration.
3 Concepts and definitions

3.1 Projected radiation

For an east-west-oriented trough-like concentrator with its absorber parallel to the trough it is convenient to work with the concept of projected radiation. The direct irradiance on a surface can be treated as a three-dimensional vector which can be separated into two components, one component parallel to the collector axis and one component perpendicular to the axis, of which the latter will be called the projected component or the transversal component. The parallel component, also called the longitudinal component, will not contribute to the irradiation on the absorber, and thus it is sufficient to consider only the projected component when calculating the irradiation on the absorber, exclusive of optical losses. However, in order to calculate the optical losses it will be necessary to consider all three dimensions, since the absorptance and reflectance of surfaces usually are incidence angle dependent. In this study the transversal incidence angle $\theta_T$ is defined as the angle between the transversal beam component and the aperture normal. The value of $\theta_T$ is defined to be zero when the transversal beam component is perpendicular to the collector aperture, negative when the beam component is south of the surface normal and positive when it is north of the normal. The longitudinal incidence angle $\theta_L$ is defined as the angle between the aperture normal and the projection of the beam radiation vector onto a plane that is parallel to the collector axis and perpendicular to the aperture. It is taken to be zero for normal incidence, negative for directions east of the normal and positive for directions west of the normal. The incidence angles $\theta_T$ and $\theta_L$ are exemplified in Figure 5. Projected radiation is treated in detail by Rönnebäck (1998).

![Figure 5. Illustration of the transversal incidence angle $\theta_T$ and the longitudinal incidence angle $\theta_L$ for a surface with inclination $\beta$ to the south and surface normal $n$. $\theta_T$ is taken as negative when the solar position is south of the surface normal, as is the case in the figure.](image)

For solar beam radiation, the angle $\theta_{pe}$ is defined as the angle between the sun position vector and the horizontal plane, projected onto a north-south vertical plane. $\theta_{pe}$ will be called the
north-south projected solar elevation, or shorter, the projected elevation. The transversal incidence angle $\theta_T$ for a south-facing collector with tilt $\beta$ is related to $\theta_{pe}$ by the equation

$$\theta_T = -90^\circ + \beta + \theta_{pe}. \quad (1)$$

![Figure 6. Illustration of the transversal incidence angle $\theta_T$ and projected solar elevation $\theta_{pe}$ for a surface with inclination $\beta$ to the south and surface normal $\mathbf{n}$. $\theta_T$ is taken as positive when the solar position is north of the surface normal, as is the case in the figure.](image)

### 3.2 The solar collector

The collectors in this study are assumed to be either of a standard flat plate type or versions of a non-imaging concentrating collector with internal reflectors. Both types will be described by the following collector energy balance equation, which conforms with the quasi-dynamic collector model specified in the EN 12975-2 standard (CEN 2001), with the exception of a collector heat capacity term:

$$q_c = F'(\tau a)_{en} K_{\theta b}(\theta_T, \theta_L) G_b + F'(\tau a)_{en} K_{\theta d} G_d - c_1 (T_m - T_a) - c_2 (T_m - T_a)^2 \quad (2)$$

where

- $q_c$: Thermal output energy rate per unit aperture area [W m$^{-2}$]
- $F'$: Collector efficiency factor [-]
- $(\tau a)_{en}$: Optical efficiency for beam radiation at normal incidence [-]
- $K_{\theta b}(\theta_T, \theta_L)$: Modifier for non-normal incidence of beam radiation [-]
- $\theta_T$: Transversal incidence angle [$^\circ$C]
- $\theta_L$: Longitudinal incidence angle [$^\circ$C]
- $G_b$: Beam irradiance on the aperture plane [W m$^{-2}$]
- $K_{\theta d}$: Incidence angle modifier for diffuse radiation [-]
- $G_d$: Diffuse irradiance on the aperture plane [W m$^{-2}$]
- $c_1$: Linear heat loss factor [W m$^{-2}$ K$^{-1}$]
- $c_2$: Second order heat loss factor [W m$^{-2}$ K$^{-2}$]
- $T_m$: Collector mean fluid temperature [$^\circ$C]
- $T_a$: Ambient temperature [$^\circ$C]

The collector fluid mean temperature $T_m$ is defined as

$$T_m = (T_i + T_o) / 2 \quad (3)$$
where $T_i$ is the inlet fluid temperature and $T_o$ is the outlet fluid temperature. At stagnation conditions when the fluid mass flow is zero $T_m$ may also be interpreted as an absorber mean temperature.

In equation (2) the products $(\tau\alpha)_{en}$ $K_{0b}$ for the beam radiation and $(\tau\alpha)_{en}$ $K_{0d}$ for the diffuse radiation express the fraction of the radiation energy incident on the collector aperture that will be converted to heat in the absorber. The factor $F'$ takes into account the fact that the equation relates to the fluid average temperature, not the absorber surface temperature. There is a heat resistance between the absorber surface and the fluid, consisting of the conduction resistance in the absorber material and the convection resistance between the fluid and the wall of the pipe, and a bond resistance between the pipe and the absorber plate. Because of these resistances the absorber will have a higher average temperature than the fluid when heat is delivered to the fluid. At a fluid temperature of $T_m$ equal to the ambient temperature $T_a$ a fraction of the radiation energy absorbed in the absorber will be lost to the environment because the absorber is warmer than the fluid and the environment. With a typical absorber fin and at normal flow rates it is the resistance in the fin that constitutes the main heat resistance, and normally the value of $F'$ is close to the fin efficiency of the absorber. In this study it will always be assumed that $F'$ is a constant with the value 0.95. These matters are treated in considerable detail in Duffie and Beckman (1991).

In order to make the nomenclature more readable and the analysis easier to follow, several of the factors in equation (2) have been combined into single factors by use of the following relations:

\begin{align}
\eta_{0b}(\theta_r, \theta_L) &= F' (\tau\alpha)_{en} K_{0b}\theta_r, \theta_L) \\
\eta_{0d} &= F' (\tau\alpha)_{en} K_{0d}
\end{align}

(4) 

(5)

where

\begin{align*}
\eta_{0b} & \quad \text{Zero-loss collector efficiency for beam radiation ("beam efficiency") [-]} \\
\eta_{0d} & \quad \text{Zero-loss collector efficiency for diffuse radiation ("diffuse efficiency") [-]}
\end{align*}

Equations 2, 4 and 5 give the more readable expression for the collector output

\[ q_c = \eta_{0b} G_b + \eta_{0d} G_d - c_1 (T_m - T_a) - c_2 (T_m - T_a)^2 \] 

(6)

$\eta_{0b}$ is dependent on the beam radiation incidence angle. In subsequent treatment of the incidence angle dependence of the beam efficiency, $\theta_L$ will be assumed to have a value of zero, i.e. only meridional rays are considered, unless otherwise stated. However, in all full year simulations the collector gains will be calculated with both $\theta_r$ and $\theta_L$ taken into account. $\eta_{0d}$ will be treated as a constant for a particular collector and collector slope. If the diffuse radiation is assumed to be isotropic, $\eta_{0d}$ for a particular collector depends only on the slope of the collector. In the present study all collectors are assumed to be facing south with the surface sloping 30º from the horizontal. This angle was chosen because it is near the roof slope of typical modern Swedish single-family houses. For simplicity $\eta_{0b}$ and $\eta_{0d}$ will often be referred to as "beam efficiency" and "diffuse efficiency".

When only the optical properties of a collector are treated it is convenient to work with the concept of optical efficiency. It is defined as the fraction of the irradiation incident on the
collector aperture that will be absorbed and converted to heat in the absorber. Since beam radiation and diffuse radiation are treated separately in this study two such efficiencies will be used: the beam optical efficiency $\eta_{ob}$ and the diffuse optical efficiency $\eta_{od}$. The following two equations express the relationship between optical efficiency and zero-loss collector efficiency

$$\eta_{ob} = \frac{\eta_{0b}}{F'} \quad (7)$$
$$\eta_{od} = \frac{\eta_{0d}}{F'} \quad (8)$$

In some cases a version of equation (6) will be used that refers to integrated or summed energies and average temperatures, calculated for suitable periods of time when the collector is in operation. It then takes this form:

$$Q_c = A_c (\eta_{0b} I_b + \eta_{0d} I_d - t (c_1 (T_m - T_a) - c_2 (T_m - T_a)^2)) \quad (9)$$

where

- $Q_c$: Useful energy produced by the collector [kWh]
- $A_c$: Collector aperture area [m$^2$]
- $t$: Time over which the energy rate is integrated or summed [h]
- $I_b$: Direct solar irradiation on surface [Wh m$^{-2}$]
- $I_d$: Diffuse solar irradiation on surface [Wh m$^{-2}$]

$\eta_{0b}$, $T_m$, and $T_a$ are average values over the period $t$ when used with equation (9). $I_b$ and $I_d$ are the sums over $t$ of the beam and diffuse irradiation, respectively.

### 3.3 Concentrating collectors

A concentrating solar collector is a collector designed so that the irradiance on the absorber can be greater than the irradiance on the aperture. This is only possible if the aperture area $A_c$ is larger than the absorber area $A_{abs}$, and a concentrating collector is commonly characterized by its geometrical concentration ratio $C$:

$$C = \frac{A_c}{A_{abs}} \quad (10)$$

The symbol $X$ is commonly used for denoting the concentration ratio, for example when a collector with concentration ratio 2.5 is referred to as a 2.5X collector. In this study a notation of the type "a C 2.5 collector" refers to a collector of concentration ratio $C=2.5$.

In this study only collectors with flat absorbers are treated. If a flat absorber with length $L$ and width $W_{abs}$ is exposed for radiation on both sides, directly or indirectly, the absorber area is

$$A_{abs} = 2L W_{abs} \quad (11)$$

If it is exposed only on one side the absorber area is

$$A_{abs} = L W_{abs} \quad (12)$$

An imaging concentrator has an optical function such that an image of the radiation source is formed on the absorber. For the production of thermal energy this is unnecessary and can also
be harmful, since it may produce hot spots which can damage the absorber. A non-imaging concentrator does not produce an image, but it may still, depending on the optical arrangement and the incidence angle of the beam radiation, cause an uneven illumination of the absorber surface.

The best-known type of non-imaging concentrating solar collector is the compound parabolic concentrating collector, commonly referred to as the CPC collector. The absorber can have any shape and the concentrating function is often accomplished by reflectors. Some examples are shown in Figure 11. CPC collectors are produced commercially by a number of companies.

### 3.4 Thermal storage

In a typical Swedish combisystem, which is examplified by Figure 1, the heat store is a water tank with a volume of 0.75 m$^3$. It includes an internal or external heat exchanger for domestic hot water preparation and it has some means of auxiliary heating, usually an external boiler or an internal electric heater. The store is directly connected to the space heating system with a shunt valve. Only the upper part of the tank is heated by the auxiliary heater, leaving as large a volume as possible for the solar energy to operate on. Usually the return water from the heating system enters near the bottom of the tank. This means that during the heating season the lower part of the tank will seldom have a temperature lower than the return water temperature, which often is around 30 °C. During collector operation the fluid leaving the tank has the same temperature as the fluid entering the collector, and the fluid entering the tank has the same temperature as when exiting the collector, disregarding any temperature drop in the heat exchanger and heat losses in the piping system. This fact allows the rough estimation that the mean operating temperature of the collector is near the mean temperature of the store during a day when at least a volume corresponding to the store volume is circulated through the collector.

### 3.5 Heating load

The heating load consists of two parts, space heating and domestic hot water heating:

$$Q_h = Q_{sh} + Q_{dhw}$$

where

- $Q_h$ The total heating load [kWh]
- $Q_{sh}$ Energy used for space heating [kWh]
- $Q_{dhw}$ Energy used for producing domestic hot water [kWh]

The load is assumed to be typical for a well insulated Swedish single family house in a Stockholm climate, with yearly values of $Q_{sh}=7950$ kWh and $Q_{dhw}=3200$ kWh and $Q_h=11150$ kWh, unless otherwise stated. Figure 7 shows how $Q_h$ is distributed over the year. The space heating load was produced by a detailed computer simulation of a modern, well insulated Swedish house in Stockholm (Lorenz et al. 2000). The space heating load is defined as hourly values of the radiator system mass flow and forward and return temperatures. The hot water load represents the consumption of a typical family with two adults and two children and consists of three daily uses of water at 45 °C: 72 liters at 6 a.m., 36 liters at noon and 104 liters at 6 p.m. When reference is made to "load" or "heating load" it is always $Q_h$ that is implied, unless otherwise stated.
3.6 Climate

The climate data used in this study is for a Swedish test reference year (TRY) for Stockholm. The climate data is in the form of hourly average values for a synthetic year, statistically composed from measured data for ten years (Skartveit et al. 1994). Values for $T_a$, $I_b$, $I_d$ and the total irradiation on a surface $I_T$ were derived from the reference year. $I_b$ and $I_d$ were calculated for a south facing surface with a slope of 30°. $I_T$ is the sum of $I_b$ and $I_d$.

Corresponding biweekly and yearly summed values $Q_b$, $Q_d$ and $Q_T$ of irradiation for a collector area $A_c$ and different time spans will be used according to

$$Q_x = A_c \sum I_x$$

where $x$ denotes index $b$, $d$ or $T$. (13)

The average ambient temperature for hours when $I_T$ is greater than 200 Wh/m² is called $T_{a200}$ and the $Q_T$ for the same hours is called $Q_{T200}$. The limiting value of 200 W/m² is chosen because radiation below 200 W/m² contributes very little to the yearly collected energy. Thus the values for $Q_{T200}$ and $T_{a200}$ offer an indication of what climatic operating conditions a solar collector is subjected to over a year. Figure 8 shows the yearly variations of the solar elevation at noon $\theta_{noon}$, $T_{a200}$ and $Q_{T200}$.

Figure 8. Solar irradiation $Q_{T200}$ and average ambient temperature $T_{a200}$ for hours with $I_T$ above 200 Wh/m² and solar elevation at noon $\theta_{noon}$, on a south facing surface sloping 30° for Stockholm.
The amount of energy irradiated below 200 Wm$^{-2}$ is quite small; the yearly sums for $Q_T$ and $Q_{T200}$ are 1085 kWh m$^{-2}$ and 920 kWh m$^{-2}$ respectively. As expected the solar irradiation has a maximum around midsummer and a minimum near the winter solstice. The ambient temperature follows the irradiation but with a displacement of some weeks, due to the thermal capacity of the land and the waters of the earth.

Wind cooling and radiation losses to the sky may affect the output of a solar collector significantly, especially unglazed collectors with high heat loss coefficients. Since this study deals with collectors with low heat losses, effects of varying wind speed and sky radiation are not taken into account.

### 3.7 Critical radiation

For assessments of collector performance the concept of critical radiation, $G_{TC}$, is useful. It will be used in chapter 6 for a first approximation of the desirable beam efficiency of a load adapted collector. $G_{TC}$ is the irradiance on the collector when the optical gain of the collector is equal to the thermal losses. At radiation levels above $G_{TC}$ the collector can deliver useful energy to the system. An expression for the critical radiation is

$$G_{TC} = \frac{(c_1 (T_m - T_a) + c_2 (T_m-T_a)^2)}{\eta_0}$$  \hspace{1cm} (14)

where $\eta_0$ is a weighted average of $\eta_{0b}$ and $\eta_{0d}$. Knowledge of the proportions of diffuse and beam radiation is required for proper weighting. When analyzing concentrating collectors it is preferable to treat beam and diffuse radiation separately. Then the following equation applies

$$G_{TC} = \frac{(c_1 (T_m - T_a) + c_2 (T_m-T_a)^2)}{(\eta_{0b} (1-x_d) + \eta_{0d} x_d)}$$  \hspace{1cm} (15)

where $x_d$ is the ratio of diffuse irradiation to total irradiation. Since different collectors have different optical and thermal properties they will also have different corresponding critical radiation levels for a particular collector fluid temperature and ambient temperature. According to equation (15) collectors with low heat losses and high optical efficiencies will have lower values for the critical radiation. Table 1 was calculated from the Stockholm climate data, for a south facing surface sloping 30º to the horizontal, and it shows yearly sums of beam, diffuse and total irradiation for hours when the average irradiance exceeds different $G_T$ levels.

<table>
<thead>
<tr>
<th>Irradiance $G_T$ [Wm$^{-2}$]</th>
<th>Total irradiation $Q_T$ [kWh/y]</th>
<th>Beam irradiation $Q_b$ [kWh/y]</th>
<th>Diffuse irradiation $Q_d$ [kWh/y]</th>
<th>Diffuse fraction $x_d$ [-]</th>
<th>Ambient temperature $T_a$ [ºC]</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;100 Wm$^{-2}$</td>
<td>1011</td>
<td>592</td>
<td>419</td>
<td>0.41</td>
<td>8</td>
</tr>
<tr>
<td>&gt;200 Wm$^{-2}$</td>
<td>920</td>
<td>573</td>
<td>347</td>
<td>0.38</td>
<td>8</td>
</tr>
<tr>
<td>&gt;300 Wm$^{-2}$</td>
<td>813</td>
<td>537</td>
<td>276</td>
<td>0.34</td>
<td>8</td>
</tr>
<tr>
<td>&gt;400 Wm$^{-2}$</td>
<td>686</td>
<td>480</td>
<td>206</td>
<td>0.30</td>
<td>9</td>
</tr>
</tbody>
</table>
In sections 4.1 and 4.2 typical collector parameter values for flat plate and concentrating collectors are estimated. Using these values with equation (15) will yield typical $G_{TC}$ values of between 200 and 300 W/m². According to Table 1 the $x_d$ ratios are then 0.38 and 0.34 respectively, indicating that when working with concentrating collectors in a Swedish climate it is necessary to consider the influence of the diffuse radiation.
4 Performance of real collectors

In this study comparisons are made between hypothetical, practically possible and real, tested concentrating collectors and a generic flat plate collector. In order for the comparisons to be realistic, a literature survey was made over test results for various collectors. There are many brands of commercial collectors, but almost all collectors are of the flat plate or evacuated tube type. Of the commercial collectors only data for flat plate collectors were compiled since the evacuated tubes are far from being economically competitive with flat plate collectors at the relatively low operating temperatures that are common for collectors in combisystems.

4.1 Performance of flat plate collectors

For flat plate collectors, data published by the testing institute SPF (2002) was used. SPF has tested more than 90% of the commercially available collectors in Europe. The SPF database contained price and performance data for 185 tested collectors, of which 162 were glazed flat plate collectors, 21 were evacuated tube collectors, one was unglazed, and one was concentrating with internal reflectors. 18 of the flat plate collectors were stated as suitable for roof integration, and 17 of these were of typical flat plate design. Price and performance data for these collectors was compiled and processed and the result is presented in Figure 9 and Figure 10. The zero-loss efficiency value $\eta_0$ given by SPF is a single value which does not distinguish between efficiency for diffuse and beam radiation. The collector equation used is

$q_c = \eta_0 G_T - c_1 (T_m - T_a) - c_2 (T_m - T_a)^2$

where $G_T$ is the total irradiance on the collector surface. Since the measurements were made in conditions with very little diffuse radiation the $\eta_0$ value for the tested collectors should be comparable with the $\eta_{0b}$ values used in this study. The heat loss factors $c_1$ and $c_2$ are directly comparable to the ones used in this study. Some of the collectors were tested with wind effects and some were not, giving incompatible heat loss factors. The values were recalculated so that all heat loss factors $c_1$ and $c_2$ in Figure 9 and Figure 10 represent heat losses with a wind speed of 3 ms$^{-1}$ (SPF uses the notations $a_1$ and $a_2$ for heat loss factors with wind effects, however).

![Figure 9. a) Zero-loss efficiency $\eta_0$ versus first order heat loss factor $c_1$ for 17 tested collectors b) Temperature dependent heat loss factor $c_2$ versus first order heat loss factor $c_1.$](image-url)
Figure 10. a) Heat loss factor \(c_1\) versus collector module area \(A_c\) for 17 tested flat plate collectors. b) Collector cost in SEK per m\(^2\) divided by calculated yearly output in kWh/m\(^2\) versus heat loss factor \(c_1\).

Figure 9 indicates that there is no relation between the optical efficiency and the heat losses of a collector. The mean value for \(\eta_0\) of the 17 collectors is 0.81. Figure 9 b) shows that there might be a negative correlation between the first and the second order heat loss factors \(c_1\) and \(c_2\), but since the spread is so large no attempt was made to calculate any correlation. In Figure 10 a) no significant correspondence between collector module size and the heat loss factor \(c_1\) can be traced. Finally, a calculation of the yearly collector output in Stockholm at a fixed collector temperature \(T_m\) of 60 °C was performed, according to a straightforward utilisability method proposed by Perers and Karlsson (1990), using \(\eta_0\), \(c_1\) and \(c_2\) from the SPF data. The collector cost, in SEK/m\(^2\), excluding VAT, was divided by the yearly output, and the result is presented in Figure 10 b), which does not suggest any evident correlation between heat loss factor and economic efficiency.

### 4.2 Performance of concentrating collectors

Up to 1990 there was a great interest for CPC collectors and a number of studies were published, both theoretical ones, mainly on optical properties, and reports of measurements on prototypes. These collectors most often had tubular absorbers. But apparently the designs were not able to compete with the flat plate collectors on the market for low and medium temperature applications. Today only a few brands of collectors with internal reflectors are commercially available in Europe. There is a Portuguese company that produces a CPC collector of type D in Figure 11, and in Sweden the company Boröpannan AB produces and markets a collector, also of type D. In the SPF database there is only one collector with internal reflectors, of an Austrian brand. It has a measured \(\eta_0\) of 0.76 and \(c_1\) and \(c_2\) values of 3.15 and 0.015 respectively. Since 1999 the Swedish companies Finsun AB and Vattenfall Utveckling AB (VUAB) have designed a number of interesting variants of asymmetric CPC collectors with bifacial absorber and internal reflectors, called Mareco (Maximum Reflection Collector). They are presented in Karlsson and Wilson (1999) and Adsten (2002).

Approximately a thousand m\(^2\) of demonstration installations have been made with different versions of these collectors. Two Mareco types are designed for installation on roofs with a slope of approximately 30°, and one of these is designed to be load adapted, since it has a much reduced optical efficiency for summer solar beam radiation.

Measurement data for 17 different collectors reported between 1979 and 2002 were compiled and the result is presented in Table 2 and Figure 12. In Table 2 the letter in the "Type" column indicates the reflector and absorber arrangement which is examplified in Figure 11:
Figure 11. Examples of configurations of concentrating collectors

A Flat reflector, single-sided flat absorber
B Asymmetrically truncated CPC, flat double-sided absorber
C Symmetrical CPC, tubular absorber
D Symmetrical CPC, flat double-sided absorber
E Symmetrical CPC, flat single-sided absorber
F Semicircular reflector, flat double-sided absorber

Table 2. Collector parameters based on measurements. Collectors 14, 15 and 16 have a teflon film as an extra convection barrier.

<table>
<thead>
<tr>
<th>N:o</th>
<th>Type</th>
<th>Concentration ratio</th>
<th>Optical efficiency $\eta_0$ [-]</th>
<th>Heat loss factor $F'U_L$ [Wm$^{-2}$ K$^{-1}$]</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>3.00</td>
<td>0.70</td>
<td>2.70</td>
<td>Collares-Pereira et al. (1979)</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>5.20</td>
<td>0.68</td>
<td>1.85</td>
<td>Rabl et al. (1980)</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>1.60</td>
<td>0.68</td>
<td>2.62</td>
<td>Collares-Pereira et al. (1979)</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>1.50</td>
<td>0.67</td>
<td>2.64</td>
<td>Collares-Pereira (1985 a)</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
<td>1.20</td>
<td>0.68</td>
<td>2.84</td>
<td>Collares-Pereira (1985 b)</td>
</tr>
<tr>
<td>6</td>
<td>D</td>
<td>1.12</td>
<td>0.74</td>
<td>4.00</td>
<td>Carvalho et al. (1995)</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>2.17</td>
<td>0.59</td>
<td>2.40</td>
<td>Karlsson et al. (1999)</td>
</tr>
<tr>
<td>8</td>
<td>B</td>
<td>1.50</td>
<td>0.69</td>
<td>2.40</td>
<td>Karlsson et al. (1999)</td>
</tr>
<tr>
<td>9</td>
<td>B</td>
<td>2.02</td>
<td>0.59</td>
<td>2.00</td>
<td>Adsten (2002)</td>
</tr>
<tr>
<td>10</td>
<td>B</td>
<td>1.85</td>
<td>0.56</td>
<td>2.30</td>
<td>Adsten (2002)</td>
</tr>
<tr>
<td>11</td>
<td>B</td>
<td>1.50</td>
<td>0.61</td>
<td>2.00</td>
<td>Adsten (2002)</td>
</tr>
<tr>
<td>12</td>
<td>B</td>
<td>1.50</td>
<td>0.63</td>
<td>2.38</td>
<td>Fiedler (2002)</td>
</tr>
<tr>
<td>13</td>
<td>A</td>
<td>3.00</td>
<td>0.59</td>
<td>2.35</td>
<td>Fiedler (2002)</td>
</tr>
<tr>
<td>14</td>
<td>E</td>
<td>1.53</td>
<td>0.75</td>
<td>2.31</td>
<td>Rönnelid et al. (1996)</td>
</tr>
<tr>
<td>15</td>
<td>D</td>
<td>1.12</td>
<td>0.69</td>
<td>3.00</td>
<td>Carvalho et al. (1995)</td>
</tr>
<tr>
<td>16</td>
<td>B</td>
<td>2.17</td>
<td>0.64</td>
<td>2.20</td>
<td>Adsten (2002)</td>
</tr>
<tr>
<td>17</td>
<td>F</td>
<td>1.13</td>
<td>0.756</td>
<td>3.60</td>
<td>SPF (2002)</td>
</tr>
</tbody>
</table>

Most of the results were reported according to a collector equation of the type

$$q_0 = \eta_0 G_T - F'U_L (T_m - T_a)$$

where $\eta_0$ is the zero-loss efficiency with radiation from a clear sky and $F'U_L$ is the heat loss factor. In the reports it is not usually stated what difference between collector and ambient
temperature the heat loss factor refers to. The parameters for collectors 12, 13 and 14 were reported according to equation (6), and they have been recalculated for a temperature difference of 40 °C in order to be comparable with the other values.

![Graph showing collector parameters versus concentration ratio for 17 concentrating collectors.](image)

**Figure 12.** Collector parameters versus concentration ratio for 17 concentrating collectors

a) The diagram on the left shows zero-loss efficiency $\eta_0$ versus concentration ratio $C$, and a solid line that represents the resulting $\eta_0$ from applying theoretically estimated optical efficiencies.

b) The diagram on the right shows the heat loss factor $F'U_L$ versus concentration ratio $C$. The solid line represents an empirical relation between concentration ratio and heat loss factor.

Figure 12 is based on data from Table 2. There is a correspondence between optical efficiency and concentration ratio, such that a higher concentration ratio implies lower optical efficiency. This is a known effect, readily explained by the average number of reflections of the radiation before it hits the absorber. The line in Figure 12 a) shows a semi-empirical correlation between optical efficiency and concentration ratio that will be explained in section 5.2. The correlation seems to overestimate the optical efficiency, but it is a compromise that is expected to overestimate the optical efficiency for double sided absorbers and tubular absorbers and underestimate it for single sided absorbers. Since only two of the 17 collectors have single sided absorbers it is reasonable that the correlation line should be above most of the points in the diagram. The line in Figure 12 b) represents an empirical correlation between heat loss factor and concentration proposed by Carvalho (1985) that will be used for estimation of the heat loss factor for hypothetical collectors further on in this study. The correlation seems very reasonable for concentration ratios less than 3. For a concentration ratio of 1.0 it gives a value of 3.5 which is a normal value for a good flat plate collector.

Explicit values for the temperature dependent heat loss factor $c_2$ were reported for only six collectors. Due to the low number and a large spread between the values, it was not considered meaningful to try to find any correlation between $c_2$ and $C$. The average value of $c_2$ for the six collectors was 0.011.
5 Estimations of properties of concentrating collectors

In chapters 6 and 7 the properties of ideal and practical load adapted collectors are investigated. At the outset nothing is presupposed about the configuration of the collectors, but in order to do the investigations in a structured manner some limitations and basic relations for the collector optical and thermal performance must be established. It is practical to classify the collectors by their concentration ratios C. Since the heat losses occur from the absorber it is reasonable that there should be some relationship between C and the heat loss factors. In order to perform and compare energy output calculations it is necessary to assume values for the beam and diffuse optical efficiencies for collectors with different concentration ratios. In this chapter approximate relations between the geometric concentration ratio C and the upper limit for the beam efficiency $\eta_{0b}$, the diffuse efficiency $\eta_{0d}$ and the heat loss factors $c_1$ and $c_2$ will be established. These relations will be used in later chapters. The chapter ends with a presentation of a new equation for the maximum concentration of asymmetrical ideal concentrators.

5.1 Selection of concentration ratios

An upper limit for the concentration ratio may be derived from the theory for non-imaging optics. It is shown by Welford and Winston (1989) that the upper limit for the concentration ratio of a perfect trough concentrator is $C = 1 / \sin \theta_c$, where $\theta_c$ is the acceptance half angle. This means that all beams with a transversal incidence angle $\theta_T$ within an incidence angle interval of $2 \theta_c$, centered about the aperture normal, will reach the absorber, though the intensity of the radiation will be reduced by optical losses. In order to collect a large fraction of the total solar radiation it is necessary to have a large acceptance angle. This is desirable, for example, when the collector area is small in relation to the load, so that there is a high probability that the load can remove the energy collected. Rönnelid (1998, p 40) has shown that the acceptance half-angle for an east-west aligned concentrator in Stockholm should be at least 35° - 40 ° in order to receive 90 % of the radiation that an optimally tilted flat plate collector receives over a year. The half-angle of 40° corresponds to a concentration ratio of 1.6, which will be taken as a representative for the lower end of the range of concentration ratios of interest. The other extreme may be found by examination of the diagram in Figure 17, which shows the useful energy collected by a system which has a large collector area compared to the load. Before week 7 there is very little energy collected, and after week 17 the irradiation is so much greater than the load that a chance might be taken that the diffuse radiation alone could cover the small load, making it unnecessary to have any beam efficiency at all at higher solar elevations than that in week 17. The corresponding noon solar elevations, 18° and 50° are taken from Figure 27. This angle interval yields a maximum concentration ratio of 3.6. In order to cover this range of possible concentration ratios, collectors with concentration ratios of 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0 were selected for investigation in subsequent chapters.

5.2 Relation between diffuse optical efficiency and concentration ratio

According to Rabl (1985) the fraction of isotropic radiation accepted by a solar collector of concentration C is 1/C. The actual amount of radiation absorbed by the absorber must be less
than this due to various losses. Thus the relation between concentration ratio $C$ and optical efficiency $\eta_{od}$ for diffuse isotropic radiation for a concentrating collector can be written

$$\eta_{od} < 1/C$$  \hspace{1cm} (16)$$

The practical efficiency is lower than $1/C$ due to optical losses, mainly consisting of reflection losses in the collector cover, absorption in the reflector and reflection losses from the absorber. If the collector is equipped with reflectors, one fraction of the radiation will reach the absorber without reflections and another fraction will reach the absorber after one or more reflections. The fraction that reaches the absorber directly may be assumed to suffer approximately the same optical losses as in a flat plate collector, since the glass cover is the same in both cases, and since the incidence angle distribution on the absorber also can be considered to be sufficiently similar. Thus an upper limit for the diffuse optical efficiency of a practical concentrating collector would be approximately

$$\eta_{od} = \frac{\eta_{od,FP}}{C}$$  \hspace{1cm} (17)$$

where $\eta_{od,FP}$ is the diffuse optical efficiency of a flat plate collector with the same cover and absorber material properties.

The intensity of the reflected fraction of the radiation reaching the absorber will be reduced by a factor $(1-\rho_r)$ for each average reflection, where $\rho_r$ is the reflectance of the reflector. Generally there is no simple relationship between the concentration ratio and the fraction of the radiation that undergoes reflection. By studying figure 13 it is possible to establish an approximate relationship. Consider a small area near the middle of the absorber strip of collector A. It is evident, for reasons of symmetry, that more than half of the diffuse radiation reaching this area will come directly from the aperture, and that less than half will be reflected once in the reflector. At a point closer to the reflector a smaller fraction will come directly without reflection, but as long as the angle between absorber and reflector is greater than 90°, less than half of the diffuse radiation reaching the absorber comes via the reflector. This relation is, maybe surprisingly, independent of the concentration ratio. With a small angle between absorber and cover the non-reflected fraction will always be close to 1, and with a very large angle the non-reflected fraction will always be small. By using standard equations for view factors for an enclosure with three sides the fractions of reflected and non-reflected radiation reaching the absorber could be calculated exactly, but since this would involve additional assumptions about the length of the reflector, it would hardly add any precision to the estimation.

![Figure 13](image.png)

Figure 13. Two types of concentrating collectors with internal reflectors. Type A has an absorber that is illuminated on one side and type B has an an absorber that is illuminated on both sides.

The collector type B in Figure 13 has its absorber parallel to the cover. The absorber is illuminated on both sides, and it is obvious that no radiation can reach the side of the absorber
facing the inside of the collector without being reflected at least once. The fraction of reflected diffuse radiation reaching the upper side of the absorber is small but not negligible. Thus we can conclude that more than half of the diffuse radiation with a possibility of reaching the absorber can do so only after one or several reflections. Also this relationship is independent of the concentration ratio.

A simple relation for the reflected fraction is chosen, such that half of the diffuse radiation with a possibility of reaching the absorber will be reflected once. This will underestimate the diffuse efficiency for all collectors of type A with an angle between reflector and absorber greater than 90º and overestimate the diffuse efficiency for all collectors of type B. Combining this relation with equation (17) gives

$$\eta_{od} = \eta_{od,FP} (0.5 + 0.5 \rho_r) / C$$

(18)

The results of the ray tracing calculations in chapter 8 show that indeed equation (18) is quite a good approximation, especially for collectors of type B.

5.3 Relation between beam optical efficiency and concentration ratio

Normally, a concentrating collector is designed to have its maximum attainable optical efficiency within an interval of incidence angles where there is much available beam solar radiation and where the system can put the collected energy to good use. At these incidence angles all beams incident on the cover have such a direction that they will reach the absorber, directly or after reflections, but the intensity of the radiation will be reduced due to optical losses. If the absorber is single-sided and parallel to the cover, the fraction of beam radiation that can reach the absorber directly is 1/C, at any incidence angle. The remainder will be reflected at least once before reaching the absorber. For such a collector, using similar arguments as in the previous section on diffuse optical efficiency, a reasonable estimation for the maximum beam optical efficiency would be

$$\eta_{ob,max} = \eta_{ob,max,FP} (1/C + (1-1/C) \rho_r)$$

(19)

This relation will be used for the upper limit for the beam efficiency of hypothetical concentrating collectors in the following chapters. For a collector of type B with a double-sided absorber, a larger fraction than (1-1/C) will suffer reflection losses and equation (19) will yield an overestimation. For a collector of type A, in a case when the beam radiation is perpendicular to the absorber, the fraction of radiation reaching the absorber directly will be greater than 1/C. For beam radiation parallel to the reflector the fraction is 1. In such cases equation (19) will result in an underestimation. Furthermore, at larger incidence angles, for example at an incidence angle of 45º, the absorptance of the absorber in a flat plate collector will be significantly lower than at normal incidence. Thus, in certain angle intervals a concentrating collector of type A may actually have a higher beam optical efficiency than a flat plate collector.

5.4 Relation between heat loss factors and concentration ratio

Since the heat losses occur from the absorber it is reasonable that there should be some relationship between C and the heat loss factors. Carvalho et al. (1985) have suggested the following empirical relationship between the concentration factor C and the heat loss factor $F'U_L$ for low-concentrating CPC collectors:
In this study equation (20) will be used for calculating the temperature-independent heat loss factor $c_1$. It is assumed that Carvalho’s relation is given for a collector temperature that is 40 °C above the ambient temperature. The correlation of equation (20) gives a value of 3.5 for $F'U_L$ of a flat plate collector and implies that its first-order heat loss coefficient $c_1$ is 3.1 W/m²K, if a value of 0.01 for the second order heat loss factor $c_2$ is assumed. This is quite a low value and similar to test values for the very best available flat plate collectors without extra convection barriers, as can be seen in in Figure 9 a).

The second order heat loss factor $c_2$ was chosen to have the constant value of 0.01 for all collectors, a typical value for flat plate collectors, as shown in Figure 9 b), and also close to the average value of 0.011 for concentrating collectors discussed in section 4.2.

5.5 Summary of estimated collector parameters

With typical values for $\eta_{\text{nob,max},\text{FP}} = 0.88$, $\eta_{\text{od,FP}} = 0.77$, $F'=0.95$ and $\rho_r = 0.85$ (for aluminium reflectors) and the use of the relations described in the previous sections the values in table 3 were calculated.

Table 3. Estimated collector parameters for collectors with different concentration ratios. The values for the flat plate (FP) collector are representative for a high performance flat plate collector.

<table>
<thead>
<tr>
<th>$C$</th>
<th>$\eta_{\text{nob,max}}$</th>
<th>$\eta_{\text{od}}$</th>
<th>$\eta_{\text{od,FP}}$</th>
<th>$\eta_{\text{od}}$</th>
<th>$c_1$ [Wm⁻²K⁻¹]</th>
<th>$c_2$ [Wm⁻²K⁻²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>0.88</td>
<td>0.77</td>
<td>0.83</td>
<td>0.73</td>
<td>3.50</td>
<td>0.010</td>
</tr>
<tr>
<td>1.5</td>
<td>0.83</td>
<td>0.47</td>
<td>0.79</td>
<td>0.45</td>
<td>2.37</td>
<td>0.010</td>
</tr>
<tr>
<td>2.0</td>
<td>0.81</td>
<td>0.35</td>
<td>0.77</td>
<td>0.34</td>
<td>1.95</td>
<td>0.010</td>
</tr>
<tr>
<td>2.5</td>
<td>0.80</td>
<td>0.28</td>
<td>0.76</td>
<td>0.27</td>
<td>1.70</td>
<td>0.010</td>
</tr>
<tr>
<td>3.0</td>
<td>0.79</td>
<td>0.24</td>
<td>0.75</td>
<td>0.22</td>
<td>1.53</td>
<td>0.010</td>
</tr>
<tr>
<td>3.5</td>
<td>0.78</td>
<td>0.20</td>
<td>0.74</td>
<td>0.19</td>
<td>1.41</td>
<td>0.010</td>
</tr>
<tr>
<td>4.0</td>
<td>0.78</td>
<td>0.18</td>
<td>0.74</td>
<td>0.17</td>
<td>1.33</td>
<td>0.010</td>
</tr>
</tbody>
</table>

5.6 Methods for calculating the diffuse optical efficiency

When calculating the energy collected by a collector in detail, it is necessary to take into account both beam and diffuse radiation. The absorbed beam radiation is readily calculated by direct use of measured or calculated values for $\eta_{\text{ob}}$ at different incidence angles, if necessary after interpolation. Figure 14 shows an example of a typical curve of $\eta_{\text{ob}}$ versus incidence angle for an assymetrical concentrating collector.
Figure 14. An example of a beam optical efficiency versus incidence angle for an asymmetrical concentrating collector of concentration factor $C=1.5$. Curve A shows calculated points and curve B is an approximated curve consisting of linear segments between a fewer number of points.

The optical efficiency for hemispherical diffuse radiation may be roughly estimated by equation (18). When the incidence angle dependent beam efficiency is known it may be calculated more exactly. Assume hemispherical diffuse radiation with intensity $L$ falling on a concentrator aperture with area $A$. The incremental flux $dB$ through $A$ from a small angle interval $d\theta$ at an incidence angle $\theta$ is

$$dB = A L \cos \theta d\theta$$

where

- $B$: flux [W]
- $A$: aperture area [$m^2$]
- $L$: intensity [$W m^{-2} rd^{-1}$]
- $\theta$: incidence angle $-\pi/2 < \theta < \pi/2$ [rd]

The total flux incident on the aperture from the angle interval $-\pi/2 < \theta < \pi/2$ is

$$B_{ap} = \int_{-\pi/2}^{\pi/2} A L \cos \theta d\theta = 2 A L$$  \hspace{1cm} (21)

and the flux absorbed by the absorber is

$$B_{ab} = \int_{-\pi/2}^{\pi/2} A L \eta_{ob}(\theta) \cos \theta d\theta .$$  \hspace{1cm} (22)

The diffuse efficiency is the ratio of $B_{ab}$ to $B_{ap}$, and combining equations (21) and (22) gives

$$\eta_{od} = 0.5 \int_{-\pi/2}^{\pi/2} \eta_{ob}(\theta) \cos \theta d\theta .$$  \hspace{1cm} (23)

The hemispherical diffuse optical efficiency may be regarded as a sum of contributions of efficiencies from a number of angle intervals. For an angle interval $\theta_1 < \theta < \theta_2$ the contribution is
\[ \eta_{od,01,02} = 0.5 \int_{\theta_1}^{\theta_2} \eta_{ob}(\theta) \cos \theta \, d\theta \]  

(24)

and the total hemispherical diffuse optical efficiency for a curve with \( m \) points may be approximated by

\[ \eta_{od} = 0.5 \sum_{n=1}^{m} \int_{\theta_n}^{\theta_{n+1}} \eta_{ob}(\theta) \cos \theta \, d\theta \]  

(25)

In some applications, e.g. when modeling a collector in a simulation program, or when only few measured or calculated points are available, it may be more efficient or convenient to represent the efficiency curve as connected straight line segments between a fewer number of points, as curve B in Figure 14. The efficiency at a certain angle could then be derived by interpolation and the hemispherical efficiency for diffuse radiation as the sum of a suitable number of intervals according to equation (25). It is also possible to obtain a more exact result with less computing effort by integration of the beam efficiency along each line segment.

Consider a line element connecting points with incidence angles \( \theta_1 \) and \( \theta_2 \) and optical beam efficiencies of \( \eta_1 \) and \( \eta_2 \). Let the slope of this line segment be

\[ k = (\eta_2 - \eta_1) / (\theta_2 - \theta_1) ; \]  

(26)

then the optical beam efficiency \( \eta_{ob} \) at an angle \( \theta, \theta_1 < \theta < \theta_2 \), is

\[ \eta_{ob}(\theta) = \eta_1 + k (\theta - \theta_1) \]  

(27)

Combining equations (24) and (27) gives

\[ \eta_{od} = 0.5 \int_{\theta_1}^{\theta_2} (\eta_1 + k(\theta - \theta_1)) \cos \theta \, d\theta \]  

(28)

Integration gives the following analytical expression for the optical efficiency for isotropic diffuse radiation over an incidence angle interval of arbitrary width with linearly changing beam optical efficiency:

\[ \eta_{od,01,02} = 0.5 \left( (\eta_1 - k*\theta_1) (\sin\theta_2 - \sin\theta_1) + k (\cos\theta_2 - \cos\theta_1 + \theta_2 \sin\theta_2 - \theta_1 \sin\theta_1) \right) \]  

(29)

where \( \theta_1 \) and \( \theta_2 \) are expressed in radians and

\[ k = (\eta_2 - \eta_1) / (\theta_2 - \theta_1). \]  

(30)

For a collector with a tilt angle of \( \beta \) from the horizontal, the total optical efficiency for diffuse sky radiation will be the sum of the efficiencies for all the line segments at incidence angles greater than the limit \( \theta_0 = -90^\circ + \beta \), where the sun is on the horizon:

\[ \eta_{od} = \sum \eta_{od,0i,0(i+1)} \quad \text{(for } -90^\circ + \beta < \theta < 180^\circ \).}  

(31)
5.7 Relation between concentration ratio and optical efficiency

Combining equations (16) and (23) gives a relation between the incidence angle dependent beam optical efficiency and the concentration ratio for 2D concentrators:

\[ \frac{1}{2} \int_{-\pi/2}^{\pi/2} \eta_{ob}(\theta) \cos \theta \, d\theta \leq \frac{1}{C} \]  

(32)

This relation is proved for the general 3D case by Jones (1980). Furthermore, Jones shows that if no optical losses are incurred, and if there is an optical path to the aperture from every point and every angle of the absorber, the relation is exact. The 2D version of this relation can be written

\[ \frac{1}{2} \int_{-\pi/2}^{\pi/2} P(\theta) \cos \theta \, d\theta = \frac{1}{C} \]  

(33)

where the acceptance function \( P(\theta) \) is defined as the fraction of energy incident on the aperture at incidence angle \( \theta \) that reaches the absorber, provided no optical losses occur (the fraction that does not reach the absorber, \( 1-P(\theta) \), must thus be rejected from the concentrator). The beam optical efficiency at a certain incidence angle is the acceptance reduced by the optical losses. If we now neglect the effects of the variation of the optical losses with incidence angle, we may write

\[ \frac{1}{2} \int_{-\pi/2}^{\pi/2} \eta_{ob}(\theta) \cos \theta \, d\theta = \text{constant} \leq \frac{1}{C} \]  

(34)

for a given concentration ratio \( C \). The physical interpretation of equation (34) is that, for a given \( C \), it is possible to increase the optical efficiency within one angle interval only at the expense of a corresponding decrease of the optical efficiency within another angle interval. Due to the effect of the cosine factor a certain change in optical efficiency at larger (absolute) incidence angles corresponds to a smaller change in optical efficiency at smaller (absolute) angles, for the same size of angular interval.

For cases where the acceptance function for a collector is unity within an angular interval \(-\theta_c < \theta < \theta_c\) and zero outside of this interval, integration of equation (33) results in

\[ \sin \theta_c = \frac{1}{C} \]  

(35)

the well-known equation for the angular acceptance of an ideal 2D concentrator with acceptance half-angle \( \theta_c \), according to Winston (1970). The corresponding equation for an ideal asymmetrical concentrator is arrived at by integration of equation (33) over an acceptance interval \( \theta_1 < \theta_2 \), with \( P(\theta)=1 \), which results in

\[ \sin \theta_2 - \sin \theta_1 = \frac{2}{C} \]  

(36)

This relation may be useful for working with asymmetrical ideal concentrators and has not been presented before. Since this study deals with "non-ideal" concentrators with varying acceptance functions, equation (36) is not used in the analysis in subsequent chapters, but it is
included here for the sake of completeness and general interest. A separate, more coherent, proof of equation (36), based on the conservation of phase space volume, is given in the next section.

### 5.8 Maximum concentration for ideal asymmetrical concentrators

Equation (35) is valid for symmetrical ideal concentrators, which have no optical losses and which accept all radiation within an angular interval \(-\theta_c < \theta < \theta_c\) and accept no radiation outside of this interval.

For work with asymmetrical concentrators it would be practical to use a corresponding relation, but with two different cut-off angles, both related to the normal of the aperture, as to be directly comparable with the incidence angles for incoming beam radiation. Hitherto no such relation has been presented. Rabl (1976) presents a relation with two cut-off angles, but the angles are related to the normal of the absorber, not the entrance aperture, of an asymmetrical CPC collector and are thus not readily comparable to the incidence angle for incoming radiation. Mills (1978) presents equations for the angular acceptance width for more and less asymmetric concentrators, but neither do these relations contain the corresponding angles with the aperture normal.

Here is proposed a new relation between the maximum concentration factor \(C\) and the upper and lower acceptance angles, measured relative to the entrance aperture normal, that is valid for asymmetrical as well as for symmetrical ideal non-imaging concentrators: The proof supplied below is a generalization of the proof of equation (35) for symmetrical concentrators presented by Welford and Winston (1989).

Using the concepts of phase space optics in the 2D version, for an incremental beam element passing through a loss-free concentrator with no radiation turning back through the entrance aperture the following expression holds:

\[
dU = dy \, dM = dy' \, dM'
\]  

where

- \(dU\) is an invariant for the beam and equation (37) is valid for coordinate systems of any orientation. Now assume coordinate systems with the \(y\) and \(y'\) axes parallel to the entrance and exit apertures respectively and beam incidence angles \(\theta\) and \(\theta'\) to the surface normals of the entrance and the exit apertures respectively. Figure 15 shows the original and the displaced rays at the aperture. Then \(dM = d(sin \, \theta)\) and \(dM' = d(sin \, \theta')\). Integration over the apertures and angle intervals gives

\[
\int_y \int_0 dy \, d(sin \, \theta) = \int_y' \int_0 dy' \, d(sin \, \theta')
\]

and
y (sin \( \theta_2 \) -sin \( \theta_1 \)) = y' (sin \( \theta'_2 \) -sin \( \theta'_1 \)) \hspace{1cm} (39)

Figure 15. Displacement in position \( dy \) and in direction cosine of a ray at the aperture of a concentrator, defining an incremental volume in 2-dimensional phase space.

The maximum possible concentration is for \( \theta'_1 = -\pi/2 \) and \( \theta'_2 = \pi/2 \), resulting in

\[
C = \frac{y}{y'} = \frac{2}{(\sin \theta_2 - \sin \theta_1)}
\]  \hspace{1cm} (40)

and the equivalent equation

\[ \sin \theta_2 - \sin \theta_1 = \frac{2}{C} \]  \hspace{1cm} \text{(identical to equation (36))}

This proof is similar to the one supplied by Welford and Winston (1989), the main difference being that in the present proof the integration at the aperture is carried out between two arbitrary angles rather than between two angles symmetrically positioned about the normal. For a symmetrical concentrator \( \theta_2 = -\theta_1 \) and in that particular case equation (36) is equivalent with the well-known equation (35).
6 Desirable beam efficiency for load adapted collectors

6.1 Introduction

The object of this chapter is to demonstrate a method for determination of the desired optical properties of a load adapted collector in a system with a large collector area. The chapter is intended to create an understanding of how and why variations of the collector properties affect the system performance, and what considerations should be made during the design process. The examples and relations explored in this chapter will be used in later chapters to explain some of the results.

The calculations in the present chapter are extensive. It is also possible to arrive at the desired incidence angle dependence for a load adapted collector by performing a great number of simulations with successive improvements, a method that will be described in chapter 7. That method offers greater precision than the one described in this chapter. However, such a "black box" method is not very transparent and does not provide much insight into how certain results are arrived at.

First two example simulations with flat plate collectors are presented, one with a typically dimensioned solar collector and one with a much larger area. Then an example for one single day of the year is presented, where the performance of a flat plate system and a load adapted system are compared in detail. Thereafter a method using the critical radiation concept is applied in order to arrive at a desired incidence angle dependence of the beam efficiency for a load adapted collector. Finally upper and lower limits for the concentration ratio of load adapted collectors for a 40 m² system are suggested.

6.2 A 10 m² system: radiation, load and useful output

A simulation with the TRNSYS programme (Klein et al. 2002) was made with a flat plate collector area of 10 m² and a 0.75 m³ store. TRNSYS was chosen because it allows detailed observation of how the collector, the store and the control system interact for both longer and shorter periods of time. The system size is typical for a Swedish combisystem and similar systems have been analyzed and optimized in great detail by Lorenz (2000, 2001). The simulation was made with the following collector and system parameters:

Table 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta_{0b,\text{max}})</td>
<td>0.83</td>
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<tr>
<td>(\eta_{0d})</td>
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<tr>
<td>(c_1)</td>
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<tr>
<td>(c_2)</td>
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</tr>
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<td>(A_c)</td>
<td>10</td>
</tr>
<tr>
<td>(V_{st})</td>
<td>0.75</td>
</tr>
</tbody>
</table>

In this example the yearly sum of the solar energy delivered to the store, \(Q_{0u}\), amounts to 3213 kWh. This is in good accordance with detailed simulation studies made by Lorentz (2001) and in the IEA task 26 work (Weiss 2003) on similar systems.
Figure 16. Simulation results for a 10 m² flat plate combisystem: Biweekly sums of simulated losses when the system is off but not overheated \( (Q_{l,\text{off}}) \), losses during stagnation \( (Q_{l,\text{stag}}) \), losses during normal operation \( (Q_{l,\text{op}}) \), useful solar energy \( (Q_u) \) and total heating load including tank losses \( (Q_h + Q_{l,t}) \).

Figure 16 shows biweekly sums (sums over 14 days) recorded from the simulation. The curves represent:

- \( Q_{l,\text{off}} \): Collector losses, optical and thermal, when the system is off but not in stagnation
- \( Q_{l,\text{stag}} \): Collector losses, optical and thermal, during stagnation
- \( Q_{l,\text{op}} \): Collector losses, optical and thermal, during normal operation
- \( Q_u \): Useful solar energy delivered to the storage tank
- \( Q_h + Q_{l,t} \): The sum of the heating load and the tank losses

The solar energy contribution covers the load between weeks 22 and 33. Between weeks 13 and 29 significant amounts of energy are lost as thermal losses from the collector during stagnation. The diagram illustrates that in order to deliver larger amounts of energy to the heating load during spring and fall there must be a certain overcapacity during summer, when using a flat plate collector. All radiation energy that is absorbed by the absorber must go somewhere. When the store is fully charged the energy must be dissipated as thermal losses, mainly from the collector itself, unless some additional system for active cooling is added to the system.

### 6.3 A 40 m² system: radiation, load and useful output

Exactly the same kind of simulation as in the 10 m² example was performed for a 40 m² flat plate collector area. This is about three times as much as is recommended by installers for a typical combisystem for a single family house in Sweden. The store volume, and the collector loop pump and heat exchanger were upsized to fit the larger collector area. The store volume was 2.0 m³. This kind of system is not recommended for a normal Swedish single-family house, since it is not considered cost effective and since it may cause serious overheating problems.

In this 40 m² example the yearly sum of the useful solar contribution \( Q_u \), amounts to 5700 kWh, to be compared with 3213 kWh for the 10 m² system. The yearly useful output per collector unit area has decreased from 321 to 142 kWh/m², less than half. The losses during stagnation have increased from 74 to 296 kWh/m²; they are four times as large.
Figure 17. Simulation results for a 40 m² flat plate combisystem: Biweekly sums of simulated losses when the system is off but not in stagnation (Q_{l,off}), losses during stagnation (Q_{l,stag}), losses during normal operation (Q_{l,op}), useful solar energy (Q_u) and total heating load including tank losses (Q_h + Q_{l,t}).

Figure 17 shows biweekly sums recorded during the simulation. The system covers the load between weeks 14 and 39. The excess solar radiation during summer is very large. From the figure it is obvious that large solar collector areas for single family combisystems may cause serious overheating problems of the kind discussed in section 2.3. This is a case where it would be useful to have a collector with reduced optical efficiency at higher solar elevations.

### 6.4 Study of April 13 with a 40 m² system

The general idea of a load adapted solar collector is that it should have a reduced optical efficiency when there is excessive solar radiation in relation to the load. In order to look closer at this possibility a particular day of the 40 m² simulation was studied in detail. With data recorded at 12 minute intervals from the simulation of the flat plate system, Figure 18

Figure 18. Collector thermal losses Q_{l,th,F}, beam optical losses Q_{l,b,F}, diffuse optical losses Q_{l,d,F}, collector output Q_{c,F}, and collector mean temperature T_{m,F} for a 40 m² flat plate system on April 13. The shaded areas represent energies and the bold line represents the collector outlet temperature. Subscript F denotes flat plate collector.
was produced for April 13, a day with a normal spring thermal load and a full day of sunshine, when the system was subject to overheating. Figure 18 shows useful collector output, beam optical losses, diffuse optical losses and thermal losses from 5 am until 6 pm, summed over 12 minute periods. These energies add up to the total irradiation $Q_T$ on the collector, as expressed by the equation

$$Q_T = Q_c + Q_{l,b} + Q_{l,d} + Q_{l,th}, \quad (41)$$

where

- $Q_T = Q_d + Q_b$ Total incident radiation
- $Q_c$ Useful output from the collector
- $Q_{l,b} = (1-\eta_{0b}) Q_b$ Beam optical loss
- $Q_{l,d} = (1-\eta_{0d}) Q_d$ Diffuse optical loss
- $Q_{l,th} = t \left( c_1 (T_m-T_a) + c_2 (T_m-T_a)^2 \right)$ Thermal loss during a time step $t$
- $T_m$ Average collector mean temperature during $t$
- $T_a$ Average ambient temperature during $t$

At seven in the morning the system starts working, and at 10 the store has reached 95 °C and the system stops in order to avoid overheating of the store. The solar loop pump stops and the collector reaches its stagnation temperature. Now the useful output is zero, and the collector must dispose of all absorbed energy as thermal losses. At 11:20 there is a small load because domestic hot water is used and the system operates again for a short while until the store is fully charged again at 11:40. Most of the excess energy is lost as thermal losses, causing the collector to endure more than 4 hours of temperatures above 95 °C.

How should an LA collector work during this example day? It cannot collect more useful energy than the FP collector, since the store cannot receive any more. In order to prevent overheating it should have a somewhat lower total efficiency during its operation time than the FP collector. Then the operation time during the day must be longer than the operation time for the FP collector. An operation time slightly shorter than the sum of the operation time and the duration of the overheating state for the FP collector may be expected, since the LA collector probably will start a little later, but will operate through most of the the overheating period since there is enough radiation to keep the FP collector above 95 °C. In order to calculate the gain of the LA collector the operating temperature of the collector must be estimated. Since the store will be charged with the same amount of energy as in the FP case, and thus have the same temperature at the beginning and end of operation in both cases, it is reasonable to assume that the LA collector experiences the same rise of the operating temperature $T_m$ as the flat plate collector. The ambient temperature $T_a$ used should be the actual one during the LA system operation. The collector gain over a suitable time step can then be calculated as

$$Q_{c,L} = \eta_{0b} Q_b + \eta_{0d} Q_d - t \left( c_1 (T_m-T_a) + c_2 (T_m-T_a)^2 \right), \quad (42)$$

where $T_m$ and $T_a$ are chosen according to the principles just described and $t$ is the time step. With a specific concentration ratio, and thus $\eta_{0d}$ and heat loss factor according to table 2, all input data to the equation are known, except for $\eta_{0b}$. By choosing $\eta_{0b}$ so that the calculated $Q_{c,L}$ equals the useful gain from the simulation of the flat plate system the suitable $\eta_{0b}$ will be established.
Following this method and using temperature and energy data from the simulation of April 13, the performance of an LA collector over the day was calculated in 12 minute time steps. An LA collector with $C=1.5$ was chosen, and with parameters chosen according to Table 3, the system performed as shown in Figure 19. The optical losses are now dominating, and the thermal losses are quite small. The operation time is 6.5 hours, to be compared with the combined operation and overheating period of 7 hours for the FP system. Because the losses during operation are larger than for the flat plate system the store and collector temperatures rise more slowly, and actually never exceed the overheating level. The useful collector output for the whole day is exactly the same, 18.3 kWh, for both cases. An advantageous side effect could be that the maximum power for the collector loop heat exchanger to transfer is smaller with the LA collectors; thus it might be possible to use a somewhat smaller and less expensive heat exchanger.

Figure 19. Collector thermal losses $Q_{l,th,L}$, beam optical losses $Q_{l,b,L}$, diffuse optical losses $Q_{l,d,L}$, output $Q_{c,L}$, and collector mean temperature $T_{m,L}$ for 40 m$^2$ load adapted collectors on April 13. The shaded areas represent energies and the bold line represents the collector outlet temperature. Subscript $L$ denotes load adapted collector.

The example of April 13 illustrates the concept of load adaption of a solar collector. But using this method for a whole year would be very time consuming since it would demand calculations with manual trial and error adjustments of $\eta_0$, operation times and temperature values for every day when the system is in operation. In the following section a faster method based on the critical radiation concept is presented.

### 6.5 Using the critical radiation level for estimating a desirable optical efficiency

The example of April 13 in the previous section demonstrated a calculation of the desirable beam efficiency of a load adapted collector for one single day. Implicit in that calculation was that the beam optical efficiency, and thus also the projected incidence angle and the projected solar elevation, were fairly constant over the operation time. Over a whole year they obviously are not. In order to design a load adapted collector the desired beam optical efficiency must be estimated for the whole range of possible incidence angles. A suitable time span for such calculations is two weeks, since it is long enough to smooth out stochastic variations in the climate, but at the same time provides a good angular resolution for the incidence angle dependence of the collector beam efficiency. Assuming that the projected solar elevation is fairly constant and near the elevation at noon during the part of a day when
the main share of the energy is collected, such calculations, performed over a whole year, would establish a relation between the desirable $\eta_{0b}$ and the projected incidence angle.

This section shows how the desired beam efficiencies for three load adapted collectors with different concentration ratios were estimated using the critical radiation concept. The calculation was done for biweekly periods over a year and was carried out in the following steps, which will be described in detail in the subsequent part of the section:

1. An average collector operating temperature for each biweekly period was estimated.
2. Aggregated biweekly values of beam radiation $I_b$, diffuse radiation $I_d$, ambient temperature $T_a$ and operating time $t$ for three different values of the critical irradiance $G_{TC}$ were created from climate data. Three sets of data were needed in order to interpolate between them.
3. The critical radiation $G_{TC}$ for each collector and biweekly period was calculated.
4. $I_b$, $I_d$, $T_a$ and $t$ values for each collector and period were estimated by interpolation.
5. The maximum possible collector output for each period and collector was calculated, with use of $I_b$, $I_d$, $T_a$ and $t$ values.
6. The smallest value of the maximum possible collector output and the load was selected as the useful collector output for each period.
7. For the periods when the collector output exceeded the load a desired, reduced $\eta_{0b}$ of each collector was calculated.

The procedure may seem complicated, but it is implemented in one single Excel spreadsheet and can be repeated for new sets of collector parameters in a matter of a minute or two.

The collector parameters shown in table 5 for the three concentration ratios and a reference flat plate collector were obtained from table 3.

Table 5. Parameters for sample collectors.

<table>
<thead>
<tr>
<th>C</th>
<th>$\eta_{0b,max}$</th>
<th>$\eta_{0d}$</th>
<th>$c_1$ Wm$^{-2}K^{-1}$</th>
<th>$c_2$ Wm$^{-2}K^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat plate</td>
<td>0.83</td>
<td>0.73</td>
<td>3.50</td>
<td>0.01</td>
</tr>
<tr>
<td>1.5</td>
<td>0.79</td>
<td>0.45</td>
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<tr>
<td>2.0</td>
<td>0.77</td>
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<td>1.95</td>
<td>0.01</td>
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<tr>
<td>4.0</td>
<td>0.74</td>
<td>0.17</td>
<td>1.33</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Step 1. Estimation of the collector operating temperature

$\Delta T$, the difference between $T_m$ and $T_a$, affects the calculated collector output strongly, and should do so, since it is the only link between the collector output and the system load. In an overdimensioned flat plate system $\Delta T$ will be very high during summer, causing the collectors to work with a poor efficiency. In a load adapted system $\Delta T$ should be lower, so low that the system can barely supply energy to cover the load. In winter the load adapted system may be expected to perform approximately as a system of the same size with a good flat plate collector and consequently have approximately the same $\Delta T$. Approximate operating temperatures can be arrived at by system simulations. This was accomplished by running detailed TRNSYS simulations of the 40 m$^2$ system, once with the FP collector and once with the same collector with a much reduced beam efficiency, in order to model a system that barely produces enough energy to meet the summer load. Figure 20 shows the resulting $\Delta T$ values and the auxiliary energy $Q_{aux}$.

In summer the reduced system covers the load between week 21 and 29, so the small system $\Delta T$ was chosen for this period. Before week 9 and after week 39 there is no overheating, so the $\Delta T$ for the full system was chosen for this period.
the periods between, a weighted average between the $\Delta T$ of the full system and the reduced system was calculated. The $\Delta T_{LA}$ curve in the diagram shows the values chosen for the estimation of the critical radiation. This $\Delta T$ was used for all collectors in the following steps.

Figure 20. $\Delta T$, Difference between collector mean operating temperature and ambient temperature for two 40 m² systems, and curves for auxiliary energy $Q_{aux}$ and overheating energy $Q_{oh}$. Index 'std' refers to results with a standard flat plate collector, and index 'red' refers to results with the same collector, but with the beam efficiency reduced to 30% of the original value.

Step 2. Biweekly values of $I_b$, $I_d$, $T_a$ and operating time $t$
Three sets of biweekly sums of beam irradiation, diffuse irradiation and total duration for hours with average irradiance $G_T$ greater than 200, 300 and 400 Wm⁻², for each set respectively, were calculated for a 30° sloping, south facing surface in Stockholm. The beam and diffuse irradiation values were kept separate, because the collectors have different efficiencies for beam and diffuse radiation. The result is shown in table 6.
Table 6. Biweekly sums of beam irradiation $I_b$, diffuse irradiation $I_d$ and total duration $t$ for hours with average irradiance $G_T$ greater than 200, 300 and 400 Wm$^{-2}$, on a south facing 30º slope in Stockholm.

<table>
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<tr>
<th>Week</th>
<th>$I_{b200}$</th>
<th>$I_{d200}$</th>
<th>$t_{200}$</th>
<th>$I_{b300}$</th>
<th>$I_{d300}$</th>
<th>$t_{300}$</th>
<th>$I_{b400}$</th>
<th>$I_{d400}$</th>
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</table>

Sum 573 537 480 347 276 206 1850 1418 1058

Step 3. Calculation of the critical radiation level
The concept of critical radiation $G_{TC}$, as demonstrated in section 3.7, was used for creating aggregated data for $Q_b$, $Q_d$, $T_a$ and the operation time $t$. A rewriting of equation (15) for the critical irradiance, using $\Delta T$ instead of $(T_m-T_a)$, gives

$$G_{TC} = \frac{(c_1 \Delta T + c_2 \Delta T^2)}{(\eta_{0b}(1-x_d) + \eta_{0d} x_d)}$$

(43)

The critical radiation level for each collector and biweekly period was calculated using equation (43), collector parameters from Table 5 and the $\Delta T$ determined in step 1. In order to do this a value for the diffuse fraction of the irradiation $x_d$ is needed, which is not known exactly at this stage of the calculation. It was approximated by the the $x_d$ value for hours with $G_T$ greater than 300 Wm$^{-2}$. 

39
Step 4. Determination of $I_{b}$, $I_{d}$ and operation time
Values for the biweekly collectable beam and diffuse irradiation and the operation times corresponding to the $G_{TC}$ for each collector were calculated by linear interpolation between the values in table 5.

Step 5. Calculation of possible collector output
The biweekly collector output $Q_c$ at a $\Delta T$ according to the $\Delta T_{LA}$ curve in figure 20 was calculated with equation (9). The results are shown in figure 21. The difference between $Q_c$ for the different collectors and the load $Q_h$ represents the amount of possible collector output that the system can not utilise because of the load being too small. The difference must be discarded by the system in some way. In the case of the flat plate collector this is accomplished in two ways: For small differences between possible collector output and load, the store temperature and thus the $\Delta T$ will be somewhat higher than necessary, causing the collector to work with a lower efficiency, and for large differences the store will be fully charged and the system will stop and the surplus irradiation will be lost during stagnation. In the case of a load adapted collector the difference between the load and the possible collector output corresponds to the thermal losses plus a possible and desirable reduction in beam efficiency $\eta_{0b}$. If $\eta_{0b}$ has an ideal value the collector will discard a suitable amount of irradiation as optical losses and operate at the $\Delta T$ needed to cover the load, but not more.

![Figure 21. Biweekly sums of maximum possible collector output $Q_c$ for a flat plate (FP) collector and three concentrating collectors, and the heating load $Q_h$, for a 40 m$^2$ system.](image)

The $Q_c$ curves for the concentrating collectors in figure 21 are remarkably similar. This illustrates the fact that the lower heat loss due to higher concentration is balanced by a lower optical efficiency, particularly for diffuse radiation. As can be seen in table 6, the diffuse irradiation accounts for between 20 % and 40 % of the total irradiation for hours with irradiance above the critical irradiance, which means that the diffuse efficiency is important for a concentrating collector in a Swedish climate.

Step 6. Calculation of the useful collector output for each period
Between week 14 and week 39 the load $Q_h$ is smaller than the possible collector output $Q_c$. During this period the useful collector output is equal to the load. The useful collector output $Q_u$ was calculated for each period as

$$Q_u = \min(Q_c, Q_h) \quad (44)$$
Step 7. Determination of the desired beam efficiency for each biweekly period

It is now possible to estimate the $\eta_{0b}$ necessary to barely cover the heating load during the periods when there is an excess of useful energy. An expression for this is obtained by rearranging equation (43):

$$\eta_{0b} = \frac{(Q_u - \eta_{0d} I_d + t (c_1 \Delta T + c_2 \Delta T^2))}{I_b}$$  \hspace{1cm} (45)

In equation (45) the nominator on the right hand side is simply an expression for the amount of beam radiation that is absorbed and converted into heat. The application of equation (45) on the biweekly values for $Q_u$ and corresponding values for $I_b$, $I_d$ and $\Delta T$ results in the curves in Figure 22.

![Figure 22. Calculated desirable beam efficiency $\eta_{0b,i}$ for a flat plate and three load adapted collectors with different concentration ratios.](image)

Before week 14 and after week 39 the calculation will naturally only return the same $\eta_{0b}$ as was used for calculating the useful output, since $Q_u$ and $Q_c$ are equal for these periods. Between weeks 14 and 39 the result is dependent on the ratio of the load to the possible collector output. Equation (45) takes this into account, as well as differences in diffuse efficiency and heat loss factors, and thus yields different beam efficiencies $\eta_{0b}$ for different collectors. Since the differences in diffuse efficiency $\eta_{0d}$ and $c_1$ tend to cancel each other, the curves in Figure 22 are also quite similar. The collector with $C=4$ needs a slightly higher $\eta_{0b}$ in summer in order to compensate for the low $\eta_{0d}$ value of 0.17, as compared with the $\eta_{0d}$ of 0.45 for the $C=1.5$ collector.

### 6.6 Choice of a relevant design incidence angle

As shown in the previous section it is possible to dispose of unwanted irradiation during summer, spring and fall by optical losses instead of by thermal losses, by choosing a suitable incidence angle dependence of the beam optical efficiency for the collector. The $\eta_{0b}$ at a certain incidence angle is fixed and cannot be varied if a stationary collector is used, unless materials with adjustable optical properties are used. But such materials tend to be expensive and decrease the overall optical efficiency. In this and the following sections it will be investigated whether it is possible to design a stationary concentrating collector that has desirable $\eta_{0b}$ values at all times of the year.
Figure 23 shows the yearly variation of the calculated beam efficiency $\eta_{0b}$ of the load adapted collector with $C=2$, and the projected solar elevation at noon, $\theta_{\text{noon}}$. In order to design the optics of a suitable collector it is necessary to decide which projected transversal incidence angle $\theta_T$ and thus which projected solar elevation angle $\theta_{pe}$ is the proper one to associate with the implied beam efficiency. For example, the calculation for a load adapted collector on April 13 represented in Figure 19 is made with an assumed constant $\eta_{0b}$ throughout the day. If $\theta_{pe}$ during the operation time varies significantly this assumption may be less valid. Also, if a collector were designed for a $\eta_{0b}$ of 0.45 in summer, at a $\theta_{pe}$ of 53º, as suggested by Figure 23, but with a lower $\eta_{0b}$ at higher angles, it may deliver too little energy because $\theta_{pe}$ will be 53º only at noon, but higher at other times.

Figure 23. Calculated beam efficiency $\eta_{0b}$ and solar elevation at noon $\theta_{\text{noon}}$ for a 40 m² load adapted collector with $C=2$.

Figure 24 illustrates how $\theta_{pe}$ varies from morning to afternoon for some selected days between January and June. Only near the equinox in March is the angle constant over the day. On winter days $\theta_{pe}$ is always less than or equal to the noon solar elevation $\theta_{\text{noon}}$, and on summer days $\theta_{pe}$ is always greater than or equal to $\theta_{\text{noon}}$.

Figure 24. Projected solar elevation $\theta_{pe}$ for selected days between January and June.

Assuming that all energy collection occurs at solar elevations near the elevation at noon, the desired optical efficiency as a function of solar elevation could be established from Figure 23. This may be quite true for winter days, when the radiation is potentially large only during a
few hours around noon. Figure 25 illustrates this for February 26, when over 90% of the daily beam irradiation on a 30° south sloping surface occurs within a $\theta_{pe}$ angle interval of 3°.

![Figure 25](image)

**Figure 25.** Projected solar elevation $\theta_{pe}$ and hourly beam irradiation $Q_b$ on a 30° south sloping surface on February 26.

But in summer it is different, as shown in Figure 26, where an angle interval as large as 18° is needed to collect 90% of the beam radiation.

![Figure 26](image)

**Figure 26.** Projected solar elevation $\theta_{pe}$ and hourly beam irradiation $Q_b$ on a 30° south sloping surface on June 18.

It is reasonable that the projected solar elevation angle to be associated with the desired beam efficiency $\eta_{0b}$ should be representative for some average operation condition for the collector, which in this case is chiefly the amount of incident radiation. Thus the projected elevation weighted with the total irradiation above 200 Wm$^{-2}$, $I_{T200}$, will be used instead of the elevation at noon when establishing the desirable relation between $\eta_{0b}$ and the incidence angle for a collector. The energy weighted projected elevation $\theta_{pew}$ is calculated over each biweekly period as

$$\theta_{pew} = \frac{\int \left( \theta_{pe} Q_{T200} \right)}{\int Q_{T200}}$$

(46)

The value 200 Wm$^{-2}$ is chosen because it is close to the average critical radiation value found for the LA collectors in the previous calculations. Figure 27 shows biweekly averages of the projected elevation at noon and $\theta_{pew}$. At times before the spring equinox and after the fall
equinox the weighted angle differs less than 2º from the angle at noon, as should be expected from Figure 25, but in summer the difference is larger, about 10º, as should be expected from Figure 26.

![Figure 27](image-url)

Figure 27. Biweekly averages of the noon solar elevation θ_{noon}, and the projected elevation weighted with total irradiation above 200 W/m² θ_{pew}.

Plotting η_{0b} versus the projected incidence angle is a practical way of illustrating the acceptance function for 2-D non-imaging concentrators, which makes it easy to analyze, design and compare collectors without having to use expressions for the collector slope and the solar elevations in the calculations. The transversal incidence angle θ_T is directly related to the projected solar elevation by equation (1). In this study it is always assumed that the collector is located in Stockholm, at 60º latitude, facing due south and having a slope β to the horizontal of 30º. Thus

θ_T = -90º + β + θ_{pew} = -60º + θ_{pew}. \hspace{1cm} (47)

Figure 28 shows the η_{0b} values from Figure 23 plotted against two sets of incidence angles, the noon incidence angle θ_{noon} and the energy weighted incidence angle θ_{pew}. At incidence angles below -30º, corresponding to the darker half of the year, the curves are almost identical, but in summer, at angles above -20º, there is a significant difference. Using the angle at noon would imply that the collector does not need to collect any beam energy at projected incidence angles larger than -5º, whereas using the energy weighted angle implies that the collector needs a beam efficiency of approximately 0.35 up to a projected incidence angle of 3º. Also, using the noon angle indicates that beam radiation at a θ_T of 0º causes no overheating, whereas use of the weighted angle indicates danger of overheating.
Figure 28. Calculated beam efficiency for a load adapted collector, versus incidence angle of beam radiation at noon $\theta_{\text{noon}}$ (solid line) and versus the energy weighted mean incidence angle $\theta_{\text{pew}}$ (circles). Each of the two sets of values represents 26 biweekly periods over a year.

### 6.7 Suitable beam optical efficiency for a 40 m$^2$ collector

The diagram in Figure 29 is made with the same calculated values of the beam efficiency $\eta_{0b}$ as in Figure 23 and Figure 28, but here $\eta_{0b}$ is plotted versus the transversal incidence angles $\theta_T$ corresponding to the irradiation weighted projected incidence angles $\theta_{\text{pcw}}$, calculated with equation (47). The beam efficiencies for the first and second halves of the year are distinguished by different line styles.

![Graph showing calculated beam efficiency $\eta_{0b}$ versus the projected incidence angle $\theta_T$ (corresponding to an energy weighted projected solar elevation $\theta_{\text{pew}}$) for a load adapted collector with concentration factor 2 in a 40 m$^2$ system. The curve with circles represents $\eta_{0b}$ values for the first half of the year and the bold line represents $\eta_{0b}$ values for the second half of the year. The dotted vertical lines represent:
- $-60^\circ \theta_T$ for the south horizon which is the lower limit for beam and diffuse radiation from the sky.
- $-52^\circ \theta_T$ for the winter solstice.
- $-30^\circ \theta_T$ for the fall and spring equinoxes.
- $3^\circ \theta_T$ for the summer solstice.

Figure 29 reveals an asymmetry in the desirable $\eta_{0b}$, such that the collector needs more optical efficiency in the spring than in the fall, at equal incidence angles. This should not be a surprise, since the heating load, the ambient temperature and the irradiation are not symmetrical about the summer solstice, as can be observed in Figure 8. In the spring the ambient temperature is lower than in the fall, causing a higher heating demand. According to
figure 29 an ideal collector should have a maximum $\eta_{0b}$ up to a transversal incidence angle $\theta_T$ of -25° in the spring, in order to collect the full amount of energy possible. On the other hand, in the fall it ought to have a reduced $\eta_{0b}$ at $\theta_T$ above -42° in order not to absorb unnecessary energy and risk overheating of the system. It can be concluded that an ideal collector that both collects the maximum possible amount of energy and avoids all overheating is impossible. There must be a compromise between these two criteria. It can also be noted that there is a very large angle interval to the right of the curve where the optical efficiency can be high in order to collect more diffuse radiation, with less risk of overheating. At these angles only diffuse radiation and beam radiation in summer mornings and afternoons will reach the absorber. From Figure 29 it is possible to define three characteristic incidence angle regions:

R1. The region of high $\eta_{0b}$, from the horizon direction at -60° up to -25°, if maximum energy collection is desired, or up to -42° if maximum overheating protection is desired. In region R1 the $\eta_{0b}$ should be as high as possible.

R2. The region of change, where the $\eta_{0b}$ should decrease. This decrease is necessary if overheating is to be reduced. Keeping the $\eta_{0b}$ low in region R2 also makes it possible to have higher optical efficiency at other angles, since for a given concentration ratio the diffuse optical efficiency has an upper limit, as discussed in section 5.7.

R3. The region of different possible $\eta_{0b}$ values, above $\theta_T$ of 3°. It depends on the concentration ratio of the collector if it is useful to have any beam efficiency in this region. Only collectors with a low C and thus a high $\eta_{0d}$ can be optically efficient for incidence angles in both region R1 and R3.

For other ratios between load and collector size and for other concentration ratios these regions may be limited by other angles.

### 6.8 Collector efficiency for diffuse radiation

In the calculation of the desired $\eta_{0b}$ it was assumed that the collector had a concentration ratio of 2.0 and a diffuse collector efficiency $\eta_{0d}$ of 0.34. Now it is interesting to determine if the curve for the implied beam efficiency in Figure 29 is consistent with a diffuse efficiency in the neighbourhood of 0.34. Since $\eta_{0d}$ and $\eta_{0b}$ are directly proportional to $\eta_{0d}$ and $\eta_{0b}$, respectively, with the factor $F'$, we can use new versions of equations (29) and (31) for calculation of the possible diffuse efficiency for the collector (all collectors in the study are assumed to have an $F'$ of 0.95, for simplicity and comparability). Equations (7) and (8) combined with equations (29) and (31) give:

$$\eta_{0d,01,02} = 0.5 \left( (\eta_{01}-k*\theta_1) (\sin \theta_2 - \sin \theta_1) + k (\cos \theta_2 - \cos \theta_1 + \theta_2 \sin \theta_2 - \theta_1 \sin \theta_1) \right)$$  \hspace{1cm} (48)

$$\eta_{0d} = \sum \eta_{0d,\theta_i,\theta_{i+1}} \quad \text{(for } -90° < \theta < 90° \text{)}$$  \hspace{1cm} (49)

These equations can be applied to curves of $\eta_{0b}$ vs incidence angle, composed by linear segments, for calculating the collector efficiency for diffuse radiation, $\eta_{0d}$. The curve A in Figure 30, arranged as interconnected linear segments, represents the beam efficiency of a hypothetical collector designed with the following criteria:
A simple shape
\( \theta_T \) between -70º and -60º : a gradual increase
\( \theta_T \) between -30º and 5º : follows approximately the calculated \( \eta_{0b} \) curve
\( \theta_T \) between 5º and 90º : chosen by trial and error

Figure 30. Curve A represents the beam efficiency \( \eta_{0b} \) versus incidence angle for a hypothetical collector with a diffuse efficiency \( \eta_{0d} \) of 0.32. The curve with circles represents the calculated \( \eta_{0b} \) values for a load adapted C 2 collector.

Using equation (48) and (49) and the coordinates for the line segments in curve A, the diffuse efficiency for this collector was calculated to be 0.32, and thus in good accordance with the assumption of \( \eta_{0d} \) of 0.34 for the calculation of the desired beam efficiency. The shape of curve A is simple and it should be possible to manufacture a collector with similar optical properties.

### 6.9 Upper and lower limits for concentration ratio

A high concentration ratio \( C \) for the load adapted concentrator is desirable in order to:

- save money, because the absorber is more expensive than the reflectors
- gain energy, because a high \( C \) implies lower thermal losses

A high \( C \) does not, however, improve the optical efficiency. Only in specific cases, and within rather small intervals of the incidence angle range, as was discussed in section 5.3, will a non-imaging concentrating collector have higher optical efficiency than a flat plate collector. Thus the load adapted collector must gain from a lower unit area cost and lower heat losses what it loses in optical efficiency.

In order to estimate approximate upper and lower limits for the concentration ratio, two hypothetical \( \eta_{0b} \) curves were designed. Figure 31 shows the curve for a collector with \( C=3 \) and Figure 32 shows the curve for a collector with \( C=1.2 \), as initial guesses for the upper and lower concentration limits.
The high C collector in Figure 31 is designed so that it only accepts the beam radiation necessary to follow the $\eta_{0b}$ curve. This is in order to achieve a maximum concentration ratio. Using equations (48) and (49) the diffuse efficiency for the high C collector was calculated to be 0.25, which corresponds to a concentration ratio of $C = 2.7$, according to Table 3. With a higher concentration ratio, optical efficiency would have to be sacrificed somewhere in the useful angle region, so $C=2.7$ can be said to be an approximate upper limit for the concentration ratio for this application.

The curve for a low C collector is depicted in Figure 32. The curve for the low C collector gives a calculated $\eta_{0b}$ of 0.56. This corresponds to a concentration ratio of 1.3 according to Table 3. It is thus necessary to have a concentration ratio of at least 1.3 to have both good energy collection and avoid overheating. If the concentration ratio is lower it will probably be difficult to achieve the reduced optical efficiency between -30° and 5° needed to avoid overheating problems. Thus, for a 40 m² system the suitable concentration ratio of a load adapted collector is between a low value of C=1.3 and a high value of C=2.7.
7 Optimization of theoretical load adapted collectors

7.1 Introduction

The analysis of load adapted collectors in the previous chapter resulted in approximate upper and lower limits for the concentration ratio. It also gave an estimation of a suitable incidence angle dependence of the beam efficiency $\eta_{0b}$ for transversal incidence angles between -60° and 3°. But the analysis did not provide information about the $\eta_{0b}$ curve for projected angles outside of this range. Also, if the stagnation temperature of the collector is important, it would be complicated to introduce such considerations into the calculations. Furthermore, the heat store was not modelled at all, but the influence of the load on the collector output was represented only as an operating temperature derived from a simulation of one system and thus valid only for systems with an output profile for that particular system.

This chapter presents a numerical semi-automatic method and results for optimization of the $\eta_{0b}$ curve for stationary concentrating collectors in systems with different ratios between collector area system heating load load. The optimization process has two distinct subprocesses which work closely together, the simulation subprocess and the subprocess that performs the numerical optimization of the $\eta_{0b}$ curve. The yearly collector output is simulated with a timestep of one hour. The $\eta_{0b}$ curves can be optimized for yearly energy output, stagnation temperature, stagnation duration or any combination of the three. The result of the optimization is an ideal curve of the beam efficiency $\eta_{0b}$ versus the transversal incidence angle $\theta_T$. In order to determine how close a real collector can come to the ideal, one can compare simulation results for ideal collectors with results for collectors modelled on basis of ray traced and measured curves for practical collectors. This will be done in chapter 8.

7.2 Iteration process

By using an iterative process for successive changes of an $\eta_{0b}$ curve for a collector in combination with system simulations it is possible to optimize the useful collector output $Q_u$, or the value of some other target function $F_t()$, with a given system size and collectors of an arbitrary concentration ratio $C$. The concentration ratio is related to the diffuse efficiency as shown in Table 3, and the diffuse efficiency is related to the $\eta_{0b}$ curve by equations (29) and (31). The main idea is that for a certain concentration ratio a collector can only accept a certain amount of diffuse radiation. An increase of the optical efficiency in some incidence angle interval is possible only at the expense of a decrease of the optical efficiency in some other angle interval. The optimization involves successive variations of the angular distribution of the optical efficiency in order to find an optimum. An example of an optimized curve is shown in Figure 33. The following steps are taken in the process:

1. Choose a concentration ratio
2. Select a $\eta_{0b}$ curve with arbitrary shape that gives a $\eta_{od0}$ consistent with $C$
3. Denote the original $\eta_{od}$ value by $\eta_{od0}$
4. Make a $\eta_{0b}$ curve $B_0$, shaped as the original curve, combined from 18 linear segments
5. Perform a system simulation with curve $B_0$ and record the target function value $F_t(B_0)$
6. Add a small increment to the $\eta_{0b}$ value of the first vertex $v_1$ of curve $B_0$. The resulting new curve is called $B_{11}$. 
7 Compute the resulting diffuse efficiency for $B_{1t}$, call it $\eta_{od1}$. $\eta_{od1}$ will be larger than $\eta_{od0}$. Since equations (48) and (49) are linear with respect to $\eta_{ob}$, a multiplication of the $\eta_{ob}$ values for all the 17 vertices in curve $B_{1t}$ with a factor ($\eta_{od0} / \eta_{od1}$) will give the curve $B_1$ with $\eta_{od} = \eta_{od0}$ which is thus consistent with the original concentration factor $C$.

8 Do a simulation with curve $B_1$ and record the resulting $F_t(B_1)$.

9 Reset $\eta_{ob1}$ for vertex $v_1$ to its original value.

10 Carry out steps 6 through 9 for all vertices.

11 Evaluate what effect the increase in $\eta_{ob}$ has on $F_t(B_i)$ for each vertex $v_i$.

12 Adjust the $\eta_{ob}$ values for all vertices in proportion to their effect on $F_t(B_i)$.

13 Carry out step 7 to make a new, improved curve $B_0$ with $\eta_{od} = \eta_{od0}$.

14 Repeat steps 5 through 13 until the change in $F_t(B)$ is within a given convergence limit.

At convergence some vertices are at certain predesignated upper or lower limits, and the remaining vertices all have identical incremental ratios of $dF_t(B_i) / d\eta_{ob,i}$ and no further improvement is possible. This simple optimization method requires that the function $F_t(\eta_{ob})$ does not have local maxima, which is quite plausible if only $Q_a$ is considered, since an increased $\eta_{ob}$ at any angle always increases $Q_a$ or leaves it unaffected. There is a possible problem in that an increase in $\eta_{ob}$ may cause overheating effects that influence $F_t$ in the opposite direction. The practical experience from working with the method is that optimizations where only $Q_a$ is included in the target function $F_t$ require approximately half the number of steps to converge, compared to cases when stagnation duration and temperature are included in $F_t$.

The iterative process is time-consuming if performed manually, and it might be automatized using an optimization program, e.g., GenOpt® from the Simulation Research Group at Lawrence Berkeley National Laboratory or dFitTRN (Spirkl 1999) together with a simulation program, for example TRNSYS (Klein et al. 2002). For this study a dedicated semi-automatic procedure consisting of a combination of a Microsoft Excel simulation model and a Visual Basic optimization procedure was developed, for reasons of speed. With this procedure convergence is normally attained after between 30 and 50 trial curves, each new curve requiring 17 calculations of the yearly $Q_a$, one for each vertex. Each yearly calculation takes 0.35 seconds and a few minutes of data organizing is needed for each new system and collector combination. Altogether it takes between 5 and 10 minutes to make an optimization for the combination of one system and one concentration factor. The TRNSYS simulation model used in this study uses 5 minutes for one yearly run, so doing the same kind of optimization with the TRNSYS model instead of the Excel system model would require 55 hours of simulations instead of 5 or 10 minutes. In order to do a combination of 6 collectors and 5 systems the Excel method required about 5 hours, but would require over 2000 hours with TRNSYS. It would have been practically impossible to do the optimization with TRNSYS and one of the available optimization programs.

### 7.3 System simulation model

The Excel simulation model performs hourly energy balances for the collector according to equation (6), including incidence angle effects from both $\theta_I$ and $\theta_L$. The heat store is simply modelled as one single thermal mass, with maximum and minimum temperature limits within which the store average temperature is allowed to fluctuate. When performing simulations for the purpose of comparing different collector designs it is not necessary to do any detailed modelling of the store and the heating load. However it is important to model the store temperature realistically, since it influences the collector mean temperature which in turn
affects the collector heat losses strongly. The store temperature was calculated in the following manner:

A simple solar heating system was considered, consisting of

- A heat store with a single thermal mass, with store heat losses to the surrounding room
- A solar collector that charges the store, with collector heat losses to the ambient air
- A heating load that is supplied with energy from the store

The following nomenclature was used

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<td>mC_p</td>
<td>[J K⁻¹]</td>
<td>Heat capacity of the store</td>
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<td>T(t)</td>
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<td>[°C]</td>
<td>Initial store temperature</td>
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<tr>
<td>T_a</td>
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<td>Ambient temperature to which collector and pipe losses occur</td>
</tr>
<tr>
<td>T_r</td>
<td>[°C]</td>
<td>Room temperature to which store losses occur</td>
</tr>
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<td>P_s</td>
<td>[W]</td>
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</tr>
<tr>
<td>P_h</td>
<td>[W]</td>
<td>Energy rate for the heating load</td>
</tr>
<tr>
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</tr>
<tr>
<td>k_st</td>
<td>[W K⁻¹]</td>
<td>Heat loss coefficient for the store</td>
</tr>
</tbody>
</table>

It is assumed that the collector mean temperature during operation is equal to the tank mean temperature. This is reasonable because the temperature at the inlet to the collector is equal to the outlet temperature of the store, and the outlet temperature of the collector is equal to the inlet temperature of the store. During collector operation the energy balance for the store is

$$P_s - k_c (T(t) - T_a) - k_st (T(t) - T_r) - P_h - mC_p \frac{dT}{dt} = 0 \quad (50)$$

Now consider a time period when all factors except $T$ and $t$ are stationary, with initial condition $T=T_0$ at $t=0$. Then equation (50) is a differential equation with a solution

$$T(t)=T_0 \exp(- t \frac{k_c+k_st}{mC_p}) + \left(1-\exp(- t \frac{(k_c+k_st)}{mC_p})\right)(P_s + k_c T_a + k_st T_r - P_h)/(k_c+k_st) \quad (51)$$

Equation (51) gives the temperature $T$ of the store at time $t$. For periods when the collector is not in operation the solution reduces to

$$T(t)=T_0 \exp(- t \frac{k_st}{mC_p}) + \left(1-\exp(- t \frac{k_st}{mC_p})\right)(k_st T_r - P_h)/k_st \quad (52)$$

Equations (51) and (52) were used with hourly time steps in the simulations. Since the collectors were modelled with a temperature dependent heat loss factor the temperature from the previous time step was used to calculate $k_c$. This introduces a slight error, but if the store is of normal size and thus the temperature rise is moderate, the error will be quite small. For example, if the store temperature rises 5 °C from 50 °C and the ambient temperature is 20 °C and the first order heat loss coefficient is $3.5 \text{ W K}^{-1} \text{ m}^{-2}$ and the second order coefficient is $0.1 \text{ W K}^{-2} \text{ m}^{-2}$, the total heat loss coefficient will be $3.80 \text{ W K}^{-1} \text{ m}^{-2}$ at the beginning of the hour and $3.85 \text{ W K}^{-1} \text{ m}^{-2}$ at the end of the hour. The difference is only 1.3 %. Furthermore, if the store temperature decreases towards the end of the day part of the error will be cancelled out.
The space heating load was created in an earlier project by a detailed building simulation with TRNSYS (Lorentz 2001), and consists of hourly values for the heating system forward and return temperatures and the distribution system mass flow. The domestic hot water load is modelled as a constant withdrawal of energy. The loads are exactly the same ones as used in the TRNSYS simulation study in chapter 10, except for that the hot water load is continuous with this model but intermittent in the TRNSYS simulations.

Six collectors with concentration ratios C between 1.5 and 4 were optimized for five system sizes. A flat plate collector was simulated with each system size for reference. The parameters used in the simulations are shown in Table 7 and Table 8. The incidence angle modifier (IAM) constant $b_0$ is used for calculation of the correction factor $K_L$ for the incidence angle dependence of the longitudinal component $\theta_L$ of the beam radiation, according to

$$\eta_{\text{ob}}(\theta_T, \theta_L) = \eta_{\text{ob}}(\theta_T, 0) \cdot K_L(\theta_L)$$

and

$$K_L = 1 - b_0 (\frac{1}{\cos(\theta_L)} - 1)$$

where $\eta_{\text{ob}}(\theta_T, 0)$ is the beam optical efficiency for the transversal component of the incident radiation when $\theta_L = 0$. Unless otherwise stated, the notation $\eta_{\text{ob}}$ in text and diagrams refers to $\eta_{\text{ob}}(\theta_T, 0)$. The expression for $K_L$ is similar to the common expression for the incidence angle dependence of beam optical efficiency for flat plate collectors (Duffie and Beckman 1991), but here it is used only for modification due to the longitudinal component. The $b_0$ value of 0.13 used for all the collectors in the optimization was derived by a numerical correlation with calculated data from a ray trace described in chapter 8, using data for one flat plate collector and one collector with internal reflectors and a concentration factor of 3.0. In both cases the value for $b_0$ of 0.13 gave a best fit with the ray tracing data. In the optimization process the dependence of the beam optical efficiency on the transversal angle $\theta_T$ is optimized by the iterative process involving successive changes and improvements in $\eta_{\text{ob}}$. Figure 33 shows a typical curve of the beam efficiency during optimization, as well as an upper limiting curve for $\eta_{\text{ob}}$. The limiting curve has a maximum value according to section 5.3 and a shape that is proportional to a $\eta_{\text{ob}}$ curve for a flat plate collector, determined by ray tracing.

![Figure 33. Curves for the optimized beam optical efficiency $\eta_{\text{ob}}$ and for maximum allowed values of $\eta_{\text{ob}}$ for a C 2.5 collector in a 40 m² system after the optimization process.](image-url)
Table 7. Collector parameters used for the optimization.

<table>
<thead>
<tr>
<th>Type</th>
<th>1st order heat loss coefficient $c_1$</th>
<th>2nd order heat loss coefficient $c_2$</th>
<th>IAM constant $b_0$</th>
<th>Maximum beam efficiency $\eta_{\text{ib},\text{max}}$</th>
<th>Diffuse efficiency $\eta_{\text{id}}$</th>
<th>Concentration factor $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>3.50</td>
<td>0.010</td>
<td>0.13</td>
<td>0.832</td>
<td>0.727</td>
<td>1.0</td>
</tr>
<tr>
<td>C 1.5</td>
<td>2.37</td>
<td>0.010</td>
<td>0.13</td>
<td>0.790</td>
<td>0.449</td>
<td>1.5</td>
</tr>
<tr>
<td>C 2.0</td>
<td>1.95</td>
<td>0.010</td>
<td>0.13</td>
<td>0.769</td>
<td>0.336</td>
<td>2.0</td>
</tr>
<tr>
<td>C 2.5</td>
<td>1.70</td>
<td>0.010</td>
<td>0.13</td>
<td>0.757</td>
<td>0.269</td>
<td>2.5</td>
</tr>
<tr>
<td>C 3.0</td>
<td>1.53</td>
<td>0.010</td>
<td>0.13</td>
<td>0.749</td>
<td>0.224</td>
<td>3.0</td>
</tr>
<tr>
<td>C 3.5</td>
<td>1.41</td>
<td>0.010</td>
<td>0.13</td>
<td>0.743</td>
<td>0.192</td>
<td>3.5</td>
</tr>
<tr>
<td>C 4.0</td>
<td>1.33</td>
<td>0.010</td>
<td>0.13</td>
<td>0.738</td>
<td>0.168</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 8. System parameters used for the optimization.

<table>
<thead>
<tr>
<th>Collector area [m²]</th>
<th>Storage volume [m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td>20</td>
<td>1.1</td>
</tr>
<tr>
<td>40</td>
<td>1.7</td>
</tr>
<tr>
<td>60</td>
<td>2.4</td>
</tr>
<tr>
<td>80</td>
<td>2.5</td>
</tr>
</tbody>
</table>

The storage volumes in Table 8 were chosen according to the results of an optimization described in section 10.2.

The target function for the optimization, $F_t$, is a weighted combination of useful collector output and the maximum collector temperature at any time during the year:

$$F_t = Q_u - W_T \max(0, T_{\text{stag}} - T_{\text{ref}}) - W_t \max(0, t_{\text{stag}} - t_{\text{ref}}) \quad [\text{kWh}] \quad (55)$$

where

$W_T$ Weighting factor for stagnation temperature
$T_{\text{stag}}$ Maximum possible stagnation temperature during the simulation
$T_{\text{ref}}$ Reference temperature
$W_t$ Weighting factor for stagnation duration
$t_{\text{stag}}$ Total stagnation duration during the simulation
$t_{\text{ref}}$ Reference stagnation duration

The useful collector output $Q_u$ is defined as the difference between the auxiliary energy $Q_{\text{aux,0}}$ needed in a reference system with no solar collectors and the auxiliary energy $Q_{\text{aux}}$ needed for the simulated system:

$$Q_u = Q_{\text{aux,0}} - Q_{\text{aux}} \quad (56)$$

For the smaller size systems the simulated maximum temperature over the year may be lower than the maximum possible stagnation temperature for a collector, in cases when the load is able to remove the energy collected so that the store is not fully charged at the critical periods.
The temperature $T_{stag}$ is calculated from the solar radiation, the ambient temperature and the collector properties only, so that it represents the worst possible case during the year, regardless of the state of the store.

### 7.4 Optimization for energy output

First an optimization for maximum useful energy was performed by setting the weighting factors $W_T$ and $W_t$ in equation (55) to zero. Figure 34 through Figure 38 show the resulting optimized $\eta_{ob}$ curves for the system sizes 10, 20, 40, 60 and 80 m$^2$. Figure 39 shows the yearly collector output $Q_u$ for all the combinations of system sizes and collector types.

![Graph showing beam optical efficiency of collectors optimized for energy output in a 10 m$^2$ combisystem with a 0.75 m$^3$ store.](image)

Figure 34. Beam optical efficiency of collectors optimized for energy output in a 10 m$^2$ combisystem with a 0.75 m$^3$ store. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

With the 10 m$^2$ system the FP collector is the best, according to Figure 39. The collector area is so small that a large fraction of the irradiation must be collected in order to meet the summer load, and the high optical efficiency over the whole angle range of the FP collector, and thus also the high efficiency for diffuse radiation, makes it superior in this case. The curves for the collectors with concentration ratios of 3.5 and 4 are located approximately in the angle interval limited by the incidence angles corresponding to the equinox and the midsummer solar elevations.
Figure 35. Beam optical efficiency of collectors optimized for energy output in a 20 m² combisystem with an 1.1 m³ store. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

The curves for 20 m² collector are in Figure 35 are slightly different from the ones for 10 m². The curves for C 3, C 3.5 and C 4 are shifted towards lower angles, indicating that with the larger area they collect enough energy in summer and thereby have some spare optical capacity for use at lower solar elevations.

Figure 36. Beam optical efficiency of collectors optimized for energy output in a 40 m² combisystem with a 1.7 m³ store. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

With the 40 m² system, Figure 36, even less optical efficiency is needed in summer, so that the curves for C 3, C 3.5 and C 4 have moved even further towards lower angles, when compared with the curves for the 20 m² system. The curves for C 1.5 and C 2 have not moved at all and the curve for C 2.5 has moved only slightly.
Figure 37. Beam optical efficiency of collectors optimized for energy output in a 60 m² combisystem with a 2.4 m³ store. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

The systems for 60 m² and 80 m² in Figure 37 and Figure 38 show the same tendency as for the smaller collector areas, that the curves are shifted towards lower angles as more collector area is added. The difference between 60 m² and 80 m² is small, since optical efficiency at lower angles than -60° is useless, due to the sun being below the horizon.

Figure 38. Beam optical efficiency of collectors optimized for energy output in a 80 m² combisystem with a 2.5 m³ store. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

The yearly useful collector gains for all combinations of system sizes and optimized collectors are shown in Figure 39. For the smallest system size, 10 m², none of the concentrating collectors can match the flat plate system, due to their limited optical efficiency. The lower heat loss factors of the LA collectors do not compensate for this fully, since with the small collector area the collector operating temperatures are still moderate. With systems of 40 m² and larger, the collectors with a C of 2.5 or lower have energy outputs comparable to that of the flat plate system. With these areas the operating temperatures are so high that the lower
optical efficiency is almost or fully compensated for by the lower heat losses of the LA collectors. This is also indicated by Figure 40 which shows the yearly stagnation time, i.e. the sum of the periods when the collector temperature exceeds 95 °C. There is virtually no overheating for the 10 m² system with LA collectors with a C of 2.5 or greater, indicating that they are almost never able to charge the store completely. With the larger systems there is overheating, but considerably less than for the flat plate system. This is probably mostly due to the longer operating time and lower optical efficiency during the critical periods, as demonstrated in the detailed study of April 13 in section 6.4 and illustrated by Figure 19. The C 1.5 collector has more overheating hours than the FP collector, since its optical efficiency is almost as good for all incidence angles below 10°, while it also has a significantly lower heat loss factor.

Figure 39. Yearly useful collector gain of systems with flat plate and optimized load adapted collectors. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

Figure 40. Yearly collector stagnation duration for systems with flat plate and optimized load adapted collectors. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.
Figure 41. Optical efficiency curves for collectors optimized for energy output. "C ..." refers to a collector with the indicated concentration ratio. In each diagram each curve represents a system with the collector area indicated by the legend for the upper right diagram.

One important aspect of collector design is whether one collector design is suitable for different system sizes. It would be impractical and uneconomical for manufacturers and installers to have to handle several collector models for similar types of applications. Figure 41 shows the beam efficiency curves for all combinations, but now with one diagram for each concentration factor. Each of the collectors with C of 1.5, 2.0 and 2.5 has similar curves for all system sizes. Only the C 2.5 collector is somewhat different, in combination with the 10 m² and 20 m² systems. The C 3.0, C 3.5 and C 4.0 collectors clearly need more optical efficiency in summer with the smaller systems than with the larger systems. Thus it seems probable that for a concentration ratio up to 2.5 it is possible to have one and the same collector shape for the whole range of system sizes. For system sizes of 40 m² and larger the curves for all the collectors are probably sufficiently similar to allow one design per concentration factor to be possible.

Figure 42 shows the maximum temperature $T_{stag}$ at any time during the simulated year of operation for all combinations of collectors and system sizes. The temperatures recorded with collectors optimized for the 10 m² system generally are the highest ones, since the optical efficiency of these collectors is shifted more towards summer solar elevations. When the collector design is optimized for larger areas and more efficiency at lower incidence angles, the stagnation temperature is generally reduced. Thus the C 3.5 and C 4.0 collectors for larger
systems have stagnation temperatures comparable to that of the FP collector, even though their heat loss factors are considerably lower than for the FP collector. The diagram shows that if the optimization is made for energy output only, the stagnation temperature will not be any lower than for the flat plate collector.

![Graph showing stagnation temperatures for different collectors and system sizes.](image)

Figure 42. Maximum possible stagnation temperatures for different collectors and system sizes. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

### 7.5 Optimization for energy output, stagnation temperature and stagnation duration

The collectors in the previous section were optimized with a target function for energy output only. Another set of optimizations was performed in order to investigate the influence of limits on maximum stagnation temperature and duration. The stagnation temperature is relevant for choice of materials depending on their maximum operating temperature, for example the temperature at which a polymer gets soft or is destroyed, or at which a solder melts. But materials are also sensitive to successive degradation that is dependent both on the temperature and the duration of the exposure. For example, rubber can become brittle and components of anti-freeze liquids can break down chemically when exposed to elevated temperatures for longer periods of time. The following target function was used:

\[
F_{t2} = Q_u - W_T \max(0, T_{stag} - 182) - W_t \max(0, t_{stag} - 98)
\]

The target function \(F_{t2}\) effectively limits the stagnation temperature of the collector to 182 °C and the total stagnation duration to 98 hours. These values were chosen because the 10 m² flat plate reference system experienced 98 hours of stagnation in the simulation and the collector had a maximum stagnation temperature of 182 °C. Thus the combined thermal stress, from both temperature and duration, for the optimized collector optics will be similar to the thermal stress on the flat plate 10 m² system. The weighting factors \(W_T\) and \(W_t\) had to be varied by trial and error between 10 and 50, depending on collector type and system size, in order to attain convergence.

The resulting optical efficiency curves with the \(F_{t2}\) setup for the target function is shown in Figure 43. The curves are now more dissimilar than in Figure 41. Only the large systems with high concentration factors have similar curves for optimizations with and without the thermal limits.
Figure 43. Beam efficiency curves for collectors optimized for energy output with limits on stagnation temperature and stagnation duration. "C ..." refers to a collector with the indicated concentration ratio. In each diagram each curve represents a system with the collector area indicated by the legend for the upper right diagram.

The useful output of the systems with collectors optimized for output with limits on stagnation duration and temperature is shown in Figure 44, and the stagnation duration is shown in Figure 45. A comparison between Figure 44 and Figure 39 reveals that the reduction in useful output due to the additional restriction of stagnation duration is small, in fact it is always smaller than 4 %, and the average for all combinations is 2 %. Thus it may be possible to use a load adapted collector with large collector areas, without exposing the system components for more thermal stress than they would suffer in a 10 m² flat plate collector system, with only an almost negligible small loss of useful energy compared to using the load adapted collector optimized for energy output only.
Figure 44. Useful collector output with restrictions on stagnation duration and temperature. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

Figure 45. Stagnation duration for collectors with restrictions on stagnation duration and temperature. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio.

Figure 45 shows that when the system size increases the stagnation duration for the flat plate system increases strongly, whereas the durations for the load adapted designs are limited to 98 hours or less. Some of the combinations never attain 98 hours, because the stagnation temperature limit causes such a distribution of the optical efficiency that the stagnation duration also becomes limited.

### 7.6 Optimization for different stagnation temperatures

Stagnation conditions should be considered a normal mode of operation for a solar thermal system. There are many reasons for why sometimes the pump and store cannot cool the collector, in addition to the case when the store is fully charged, for example a power blackout, a pump failure, a leak, low loads due to holidays, or a control system failure. Most likely any system will experience one or more of these conditions more or less regularly during its lifetime. The collector design may be used in order to set an upper limit for the stagnation temperature. But this would probably cause a loss of useful output. With a higher temperature limit the systems ought to collect more useful energy and suffer more overheating, and with a lower limit it would probably be the other way around.
In order to investigate the effects of different design stagnation temperatures, the 40 m² system with the C3 collector was optimized for six different temperature limits: 100, 120, 140, 160, 180 and 200 ºC. The results are shown in Figure 46 and Table 9. With a maximum stagnation temperature of 140 ºC the collector delivers 14 % less energy than with a design for 180 ºC. For collectors designed for 140 ºC and lower there is very little time spent at stagnation. The lower temperature and the short duration reduce thermal stress on the system components and may make it possible to use less expensive materials for the collector. Depending on the material costs it may be economically favourable to make a collector for a lower stagnation temperature, even though it delivers less useful energy.

Table 9. Simulated useful solar energy for a C 3.0 collector in 40 m² systems, optimized for different stagnation temperatures.

<table>
<thead>
<tr>
<th>Stagnation temperature</th>
<th>100 ºC</th>
<th>120 ºC</th>
<th>140 ºC</th>
<th>160 ºC</th>
<th>180 ºC</th>
<th>200 ºC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Useful solar energy Q_u [kWh]</td>
<td>2878</td>
<td>3561</td>
<td>3994</td>
<td>4382</td>
<td>4619</td>
<td>4737</td>
</tr>
<tr>
<td>Ratio Q_u / (Q_u at 180º C) [-]</td>
<td>0.62</td>
<td>0.77</td>
<td>0.86</td>
<td>0.95</td>
<td>1.00</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Figure 46. Simulated useful solar energy Q_u and stagnation duration versus maximum stagnation temperature for 40 m² system with C 3.0 collectors, optimized for different maximum stagnation temperatures. With the collectors designed for stagnation temperatures of 100 ºC and 120 ºC no stagnation at all occurred during the simulation.
Figure 47. Optical efficiency curves for a C 3.0 collector in a 40 m² system optimized for six different stagnation temperatures.

Figure 47 shows the optical efficiency curves for the C 3.0 collector optimized for different temperatures. The differences between the curves are quite large, which is favourable in two ways: Firstly, the tolerance for imperfections in design or manufacturing, when aiming for a certain collector temperature, is quite large. Secondly, since the collectors for the higher temperatures deliver nearly the same amount of useful energy, the collector output is probably also quite insensitive to imperfections.
8 Optical modelling, ray tracing and performance of practical collectors

In the previous chapters hypothetical collectors have been designed, optimized and analyzed. The results of that work has limited practical value if it is not possible to produce real collectors with similar beam and diffuse efficiencies. In order to compare the performance of ideal and practical collectors in detail, it is necessary to have the full beam efficiency curves available. There is some useful material to be found in the literature, but most is for different versions of symmetrical CPC collectors with the acceptance interval centered about the cover normal. For asymmetrical collectors there are only a few complete curves reported. In a report by Muschaweck et al (2000) there is a diagram with theoretical and measured beam efficiency versus incidence angle for a prototype of an asymmetrical concentrator with parabolic cylindrical mirror and tubular receiver.

Fiedler (2002) has reported calculated and measured data for the incidence angle dependence of two asymmetric collector prototypes with flat absorbers. Helgesson et al. (2002) present measured incidence angle dependence for an asymmetric version of the Swedish Mareco collector. For simple designs similar to the type A collector shown in Figure 11 there is virtually nothing published, except for the curves presented by Fiedler. Perers and Karlsson (1993) have reported on optimization of external flat and paraboloidal booster mirrors for collector fields, but that configuration is too different from the ones in this study to offer material readily usable for comparisons.

In order to make comparisons and provide optical efficiency curves for various simulations a dedicated ray tracing program was created. Certainly there are such programs already on the market and as research tools, but it was necessary for the work to have a complete control over each step in the calculations, which the pre-packaged systems cannot offer. Much of the work was done in a very close conjunction between ray tracing and optimization, bringing optimized curves into the ray tracing tool for comparisons and passing curves from ray traced collector geometries into the optimization environment.

8.1 Ray tracing tool

The ray tracing tool described in this chapter has been used for several tasks in this study, among others:

1) To produce collector parameter data for existing and hypothetical collectors for use with system simulations.
2) To compare the beam efficiency of realistic collector designs with the ideal curves derived from the optimization study in the previous chapter.
3) To derive beam efficiency curves and diffuse efficiencies for validation of the proposed optical efficiencies in Table 3.

The main purpose of the tool is to compare different collector designs and to produce collector parameters for use with simulation programs, not to establish the true optical efficiency with a high degree of precision. The simulations were mainly intended for comparing how different collector designs influence the performances of a certain system. With this in mind some simplifications in the optical calculations were allowed.
The tool was implemented as an Excel worksheet. It was used in two versions, one for 2-dimensional analysis working with rays in a plane perpendicular to the axis of the collector, and one for 3-dimensional analysis where both the transversal and longitudinal components of the ray were considered. Mathematically the 3-dimensional version is only slightly more complicated than the 2-d version, but the main difference is in computation time. For a ray trace with 49 evenly spaced incidence positions on the cover and 37 transversal incidence angles with a step of 5°, a 2-d analysis took about 4 seconds. For the 3-d modelling each of the 49 *37 combinations was also combined with 9 variations of the longitudinal angle, resulting in 16317 rays being traced for one collector. The 3-d version used slightly less than a minute of computation time. During trial-and-error searches for a suitable collector geometry the 2-d version was used, and when a suitable shape was found the 3-d version was used to calculate the diffuse optical efficiency and to produce input data for the system simulation models.

The 3-d ray trace model works as follows:

1. An initial longitudinal incidence angle $\theta_L$ is selected
2. An initial transversal incidence angle $\theta_T$ is selected.
3. An initial incidence position along the cover is selected
4. The reflection losses in the transparent cover are calculated and subtracted
5. It is determined whether a reflector surface or the absorber is next hit by the ray
6. The incidence angle between the ray and the surface is calculated
7. The incidence angle dependent reflectance is calculated
8. If the surface is the absorber the absorbed fraction is recorded
9. If the surface is not the absorber the absorbed fraction is considered lost
10. It is determined which surface is hit next, including the inside of the cover
11. Steps 5 through 10 are repeated an additional 4 times.
12. Steps 3 through 11 are repeated for 49 positions along the cover
13. Steps 2 through 12 are repeated for 37 transversal incidence angles from -85° to 85°
14. Steps 1 through 13 are repeated for 9 longitudinal incidence angles from 0° to 80°

In the 2-d version only one longitudinal angle with the value of 0° is used, otherwise it is identical to the 3-d version.

The optical interactions between the ray and the surfaces are calculated as described in the following sections.

### 8.2 The cover

The cover is considered to be transparent with reflection and absorption losses. The reflection of radiation in the interface between two mediums may be calculated exactly by use of Fresnel's equations for the reflection of radiation on smooth surfaces. They give the reflected fraction of radiation passing from a medium with refractive index $n_1$ to a medium with refractive index $n_2$. Since the reflection is dependent on the polarization, the reflections of the perpendicular component $r_s$ and the parallell component $r_p$ of the radiation must be calculated separately.

$$ r_s = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} $$

(57)
\[ r_p = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} \]  
(58)

\[ r = \frac{I_r}{I_s} = \frac{1}{2} (r_s + r_p) \]  
(59)

\[ \theta_1 \text{ and } \theta_2 \text{ are the angles of incidence and refraction as shown in Figure 48. The angles are related to the indices of refraction by Snell's law:} \]

\[ \frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} \]  
(60)

Using equations 57 and 58, it is possible to calculate the reflection of a single interface. As can be seen from the equations the reflection is the same whether the radiation passes from medium 1 to medium 2 or vice versa. The transmittance through the material \( \tau_a \), disregarding the effects of reflection, can be calculated using the path length in the material \( L \) and the extinction coefficient \( K \), according to equation 61.

\[ \tau_a = e^{-KL/\cos \theta_2} \]  
(61)

The radiation undergoes multiple reflections and suffers additional absorption losses within the material. Using ray tracing methods it is possible to arrive at the following expressions for calculation of the overall transmittance \( \tau_p \), reflectance \( \rho_p \), and absorptance \( \alpha_p \) of a transparent cover, for the parallel polarization, with multiple reflections and absorption accounted for.
\[ \tau_p = \tau_a \left( \frac{1-r_p}{1+r_p} \right) \left( \frac{1-r_p^2}{1-(r_p \tau_a)^2} \right) \]  
(62)

\[ \rho_p = r_p \left( 1 + \tau_a \tau_p \right) \]  
(63)

\[ \alpha_p = (1 - \tau_a) \left( \frac{1-r_p}{1-r_p \tau_a} \right) \]  
(64)

The corresponding expressions for transmittance \( \tau_s \), reflectance \( \rho_s \) and absorptance \( \alpha_s \) for the s-polarized component are analogous to equations 62, 63 and 64 respectively, only with \( r_s \) substituting \( r_p \). For incident unpolarized radiation, the optical properties are found by the average of the \( s \) and \( p \) components:

\[ \tau = \frac{1}{2} (\tau_p + \tau_s) \]

\[ \rho = \frac{1}{2} (\rho_p + \rho_s) \]

\[ \alpha = \frac{1}{2} (\alpha_p + \alpha_s) \]  
(65)

The presentation above of equations 57 through 65 follows the outline given by Duffie and Beckman (1991) where also more detail is provided.

In order to avoid using these rather cumbersome calculations in the ray tracing tool they were approximated by two correlations, developed within this study, one for transmittance and one for reflectance:

\[ \rho_c(\theta) = a + (1-a) \left( \frac{2 \theta}{\pi} \right)^b \]  
(44)

\[ \tau_c(\theta) = c \left( 1 - \left( \frac{2 \theta}{\pi} \right)^d \right) \]  
(45)

where

- \( \theta \) incidence angle expressed in radians
- \( \rho_c \) the incidence angle dependent overall reflectance of the cover
- \( a \) a constant that is close to the reflectance at normal incidence
- \( b \) an exponent describing the angle dependence of the reflectance
- \( \tau_c \) the incidence angle dependent overall transmittance of the cover
- \( c \) a constant that is close to the transmittance at normal incidence
- \( d \) an exponent describing the angle dependence of the transmittance

Both \( a \) and \( b \) in equation 44 are dependent on the refractive index and the extinction coefficient of the medium and they are different for different materials. The optical calculations in this study assume a glass cover with a refractive index of 1.528 for the solar spectrum, a thickness of 4 mm and an extinction coefficient of 4.0 m\(^{-1}\). It represents a typical good quality low-iron glass without anti-reflection treatment.
Figure 49. Incident angle dependent overall transmittance $\tau$ and reflectance $\rho$ for a glass cover, calculated in detail with Fresnel's equations and with correlation methods.

When comparing the results from the detailed calculation with the results from equation 44 for 17 different angles, a best fit was found with values for $a$ and $b$ of 0.0829 and 6.102 respectively. The largest absolute value of the error in reflectance, 0.015 is at 86°. Below 80° the absolute value of the error is always less than 0.01. For the transmittance a best fit was found with values for $c$ and $d$ of 0.9011 and 5.933 respectively. The corresponding curves are shown in Figure 49, and table Table 10. Correlation coefficients and error values for approximated incidence angle dependent transmittance, reflectance and absorptance for a 4 mm glass cover. The error values refer to a comparison with detailed calculations with Fresnel's equations. Table 10 shows the factors and the error values, as well as the error in the calculated absorptance $\alpha_c$. It was calculated as $\alpha_c = 1 - \tau_c - \rho_c$ and compared quite well with the exact value $\alpha = 1 - \tau - \rho$, as shown in the last row of table yy. The absorption adds a little to the efficiency of the collector, by heating the cover, as described in Duffie and Beckman (1991) but the effect is small and is neglected in this study.

<table>
<thead>
<tr>
<th>Correlation constants</th>
<th>Reflectance $\alpha_c$</th>
<th>Transmittance $\tau_c$</th>
<th>Absorptance $\alpha_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.0827</td>
<td>0.9011</td>
<td>0.9144</td>
</tr>
<tr>
<td>$b$</td>
<td>5.816</td>
<td>5.933</td>
<td>0.0021</td>
</tr>
<tr>
<td>$c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10. Correlation coefficients and error values for approximated incidence angle dependent transmittance, reflectance and absorptance for a 4 mm glass cover. The error values refer to a comparison with detailed calculations with Fresnel's equations.

Arthur and Norton (1990) have presented a comparison between the exact solution according to Fresnel and four simplified methods of approximating the transmittance of solar radiation through transparent slabs of materials. Only one of the four simplified methods might offer
less deviation from the exact solution than the method described above. However, since the method found in the literature involved the use of a fifth-order polynomial function with six constants it was considered to be too complicated for use in this project.

It is important to model the reflectance of the cover carefully since the internal reflectors at some incidence angles direct radiation towards the inside of the cover so that the incidence angle with the cover becomes quite large. As can be seen in Figure 49 the reflectance actually is larger than the transmittance for angles greater than 79°. After being reflected in the cover the ray may hit the absorber. This contribution to the optical efficiency of the collector may be considerable at certain incidence angle intervals.

### 8.3 The reflector

The reflectors are considered to have a specular reflectance of 0.85 which is assumed to be independent of the incidence angle. The total reflectance of aluminium, a common reflector material, is 0.92 and the difference between 0.85 and 0.92 is the diffuse fraction of the reflected radiation. Some of this may hit the absorber, but this effect is neglected. In a very careful analysis the $s$ and $p$ components of the radiation should be treated separately, and the reflectance should vary with the incidence angle. As Rönnelid (1998) explains, this is normally not done in ray tracing, and it is not worth the effort to do it when the reflector is assumed to be made of aluminium. For an aluminium reflector the absolute of the difference in reflectance between the mean value of the both polarization directions and the value for either direction is less than 2.5 % for incidence angles $\theta < 45$° and less than 5 % for $\theta < 60$°.

Since all collectors of the same type in this study have similar configurations and transmit light rays in approximately the same directions any errors will be similar and probably diminish the validity of the comparisons between the collectors to only a small degree. The radiation absorbed in the reflector is not taken into consideration. This energy may improve the thermal performance of the collector significantly, but the effect has not been investigated in this study.

### 8.4 The absorber

The absorber is modelled with an incidence angle dependent absorptance $\alpha$ as described by Rönnelid (1998). The equation used is

$$\alpha(\theta) = \alpha_0 \left[ 1 - b_{\alpha_o} \left( \frac{1}{\cos(\theta)} - 1 \right)^{c_{\alpha}} \right] \quad (46)$$

where

- $\alpha_0$ Absorptance at normal incidence = 0.95
- $b_{\alpha_o}$ Correlation factor = 0.057
- $c_{\alpha}$ Correlation exponent = 1.2

The values in parenthesis are the ones used in the ray trace, and they represent a sputtered nickel/nickel-oxide absorber surface for which Tesfamichael and Wäckelgård (1992) have presented measurement values. The reflectance $\rho$ of the absorber is calculated as $\rho = 1 - \alpha$. The reflectance is treated as specular, so that the ray gets a second chance of hitting the absorber after reflection in the glass or a reflector. When inspecting different absorber types some look quite shiny and some look more dull, indicating that this assumption may cause significant errors in the calculations. However, even if part of the radiation reflected from the absorber is diffuse, a share of it will find its way back to the absorber.
8.5 Collector geometrical model
The collector geometrical model consists of one cover, one absorber that may be single sided or double sided and up to four reflector surfaces. The reflector surfaces may be planar, circular or parabolic. The absorber is planar. The definition of the geometry is done manually in an Excel sheet, in which all subsequent optical calculations are performed automatically. The collector geometry and the trace of one ray is shown in a diagram, as in Figure 50. The effects of any change in collector geometry or starting point and angle of the ray is immediately visible in the diagram. This makes it easy to visualize the optical function and to observe how the collector geometry influences the optical efficiency (and was initially very helpful for finding faults in the model). Figure 50 shows an example of a diagram from the tool, depicting a collector with \( C = 2 \) and a double sided absorber.

Figure 50. An image of a collector as drawn by the ray tracing tool. The collector has a concentration factor \( C \) of 2.0, a double sided absorber and two planar and one circular reflector surfaces. The ray is reflected on the inside of the glazing.

Figure 50 illustrates the importance of modeling the reflection on the inside of the glass correctly. The ray hits the glass from the inside with an incidence angle such that the reflectance of the glass is 0.76. The optical efficiency of the collector is as high as 0.45 for a beam with location and incidence angle as in the figure, although the efficiency would be zero if the reflectance in the glazing were not accounted for.

8.6 Calculation of optical efficiency for beam radiation
The transversal incidence angle \( \theta_T \) is varied in 37 equal steps of 5\(^\circ\) each, from -85\(^\circ\) to 85\(^\circ\). For each angle combination, 49 rays with an initial intensity \( \mathbf{L}_{b0} \) are traced. The rays are evenly spaced over the width of the cover. For each reflection the intensity of the ray is attenuated as described in the earlier sections. All energy absorbed from a hit on the absorber is recorded and the total absorbed intensity \( \mathbf{L}_{ba} \) is summed. The optical efficiency \( \eta_{ob} \) for the angle combination is calculated as the mean value of \( \mathbf{L}_{ba}/\mathbf{L}_{b0} \) for the 49 rays. Figure 51 shows \( \eta_{ob} \) versus \( \theta_T \) when \( \theta_L = 0 \) for the collector depicted in figure 50. The small peak in efficiency at -35\(^\circ\) is not an error, but rather caused by the reflection on the inside of the glass.
The result of the calculations is a matrix of $\eta_{ob}$ values, as the one exemplified in Table 11. The matrix has a full 180º range of transversal incidence angles $\theta_T$, since $\eta_{ob}(\theta_T)$ is asymmetric about the normal to the collector aperture (the figure and the table have 10º intervals but the calculations yield results for 5º intervals). The longitudinal angles $\theta_L$ (in the matrix column headers) represent only half the longitudinal angular range, since all analyzed collectors are trough-like and symmetrical about the meridian plane.

Table 11. A matrix with optical efficiency values from a ray trace of a C 1.5 collector. The top row contains the values for the longitudinal incidence angle $\theta_L$ and the first column contains the values for the transversal incidence angle $\theta_T$. Each cell value in the matrix represents the mean beam optical efficiency for rays passing through 49 evenly spaced positions along the cross section of the collector aperture, with incidence angle components $\theta_L$ and $\theta_T$.

<table>
<thead>
<tr>
<th>Transversal incidence angle $\theta_T$</th>
<th>90º</th>
<th>80º</th>
<th>70º</th>
<th>60º</th>
<th>50º</th>
<th>40º</th>
<th>30º</th>
<th>20º</th>
<th>10º</th>
<th>0º</th>
<th>Beam optical efficiency $\eta_{ob}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>90º</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>80º</td>
<td>0</td>
<td>0.071</td>
<td>0.102</td>
<td>0.111</td>
<td>0.114</td>
<td>0.116</td>
<td>0.117</td>
<td>0.117</td>
<td>0.118</td>
<td>0.118</td>
<td></td>
</tr>
<tr>
<td>70º</td>
<td>0</td>
<td>0.112</td>
<td>0.167</td>
<td>0.184</td>
<td>0.192</td>
<td>0.197</td>
<td>0.200</td>
<td>0.202</td>
<td>0.203</td>
<td>0.203</td>
<td></td>
</tr>
<tr>
<td>60º</td>
<td>0</td>
<td>0.136</td>
<td>0.204</td>
<td>0.225</td>
<td>0.235</td>
<td>0.241</td>
<td>0.244</td>
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<td>0.248</td>
<td>0.248</td>
<td></td>
</tr>
<tr>
<td>50º</td>
<td>0</td>
<td>0.143</td>
<td>0.214</td>
<td>0.229</td>
<td>0.233</td>
<td>0.236</td>
<td>0.238</td>
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<td>0.241</td>
<td>0.242</td>
<td></td>
</tr>
<tr>
<td>40º</td>
<td>0</td>
<td>0.140</td>
<td>0.212</td>
<td>0.220</td>
<td>0.218</td>
<td>0.219</td>
<td>0.220</td>
<td>0.220</td>
<td>0.220</td>
<td>0.221</td>
<td></td>
</tr>
<tr>
<td>30º</td>
<td>0</td>
<td>0.140</td>
<td>0.208</td>
<td>0.210</td>
<td>0.204</td>
<td>0.201</td>
<td>0.200</td>
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<td>0.199</td>
<td>0.199</td>
<td></td>
</tr>
<tr>
<td>20º</td>
<td>0</td>
<td>0.165</td>
<td>0.274</td>
<td>0.303</td>
<td>0.311</td>
<td>0.313</td>
<td>0.313</td>
<td>0.313</td>
<td>0.313</td>
<td>0.313</td>
<td></td>
</tr>
<tr>
<td>10º</td>
<td>0</td>
<td>0.188</td>
<td>0.323</td>
<td>0.369</td>
<td>0.386</td>
<td>0.391</td>
<td>0.392</td>
<td>0.392</td>
<td>0.392</td>
<td>0.392</td>
<td></td>
</tr>
<tr>
<td>0º</td>
<td>0</td>
<td>0.193</td>
<td>0.347</td>
<td>0.412</td>
<td>0.439</td>
<td>0.450</td>
<td>0.453</td>
<td>0.454</td>
<td>0.454</td>
<td>0.454</td>
<td></td>
</tr>
<tr>
<td>-10º</td>
<td>0</td>
<td>0.218</td>
<td>0.423</td>
<td>0.519</td>
<td>0.563</td>
<td>0.581</td>
<td>0.589</td>
<td>0.591</td>
<td>0.592</td>
<td>0.593</td>
<td></td>
</tr>
<tr>
<td>-20º</td>
<td>0</td>
<td>0.236</td>
<td>0.486</td>
<td>0.610</td>
<td>0.668</td>
<td>0.694</td>
<td>0.705</td>
<td>0.709</td>
<td>0.711</td>
<td>0.712</td>
<td></td>
</tr>
<tr>
<td>-30º</td>
<td>0</td>
<td>0.250</td>
<td>0.520</td>
<td>0.653</td>
<td>0.717</td>
<td>0.747</td>
<td>0.760</td>
<td>0.767</td>
<td>0.769</td>
<td>0.770</td>
<td></td>
</tr>
<tr>
<td>-40º</td>
<td>0</td>
<td>0.261</td>
<td>0.532</td>
<td>0.663</td>
<td>0.727</td>
<td>0.758</td>
<td>0.774</td>
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<td>0.785</td>
<td>0.786</td>
<td></td>
</tr>
<tr>
<td>-50º</td>
<td>0</td>
<td>0.269</td>
<td>0.536</td>
<td>0.661</td>
<td>0.722</td>
<td>0.754</td>
<td>0.771</td>
<td>0.779</td>
<td>0.784</td>
<td>0.785</td>
<td></td>
</tr>
<tr>
<td>-60º</td>
<td>0</td>
<td>0.249</td>
<td>0.482</td>
<td>0.584</td>
<td>0.634</td>
<td>0.659</td>
<td>0.674</td>
<td>0.681</td>
<td>0.685</td>
<td>0.686</td>
<td></td>
</tr>
<tr>
<td>-70º</td>
<td>0</td>
<td>0.212</td>
<td>0.380</td>
<td>0.443</td>
<td>0.471</td>
<td>0.486</td>
<td>0.494</td>
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<td>0.500</td>
<td>0.501</td>
<td></td>
</tr>
<tr>
<td>-80º</td>
<td>0</td>
<td>0.187</td>
<td>0.271</td>
<td>0.294</td>
<td>0.304</td>
<td>0.309</td>
<td>0.311</td>
<td>0.313</td>
<td>0.314</td>
<td>0.314</td>
<td></td>
</tr>
<tr>
<td>-90º</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
8.7 Types of collectors analyzed

Three main types of collectors have been analyzed, called type A, type B and type B2, exemplified in Figure 52. Type A has a single sided absorber. Type B and B2 have double sided absorbers but with the difference that the absorber of type B is parallel to the cover and type B2 has the absorber approximately perpendicular to the cover.

![Figure 52. Three types of collectors analyzed in the study. Cover, reflector and absorber surfaces are indicated as "cov", "ref" and "abs" respectively.](image)

8.8 Results for collectors with single sided absorber

The collector of type A in figure 52 has several attractive features. It has a simple shape, making it uncomplicated to design and manufacture. It uses the least amount of reflector material. The whole absorber surface can face the direction where the most useful direct beam radiation comes from, and the collector could thus have a high beam efficiency for radiation in this angle interval.

The ray tracing tool was used for creating collector geometries with beam efficiency curves as similar as possible to the ones created with the optimization process described in chapter 7. The beam efficiency curves that were optimized for energy, stagnation temperature and stagnation duration in a 40 m² system were used. Collector geometries were designed corresponding to the optimized, ideal curves for the concentration ratios of 1.5, 2, 2.5, 3 and 3.5 by trial and error. The result can be seen in Figure 53. It was found that a collector shape like type A in fig 41, with a single planar reflector, was sufficient for achieving $\eta_{ob}$ curves rather similar to the theoretical ones for the collectors with C of 2 and 2.5. The theoretical curve has a smoothly changing slope that is obtained automatically by the gradual decrease in optical efficiency with higher incidence angles, as the reflected radiation increasingly bypasses the absorber, and as the direct radiation on the absorber gets more and more oblique. The ray traced C 1.5 collector came close to the ideal curve for a 10 m² system, but it was impossible to produce the reduction in optical efficiency needed at around -5 ° for the 40 m² system.

With the C 3 and C 3.5 collectors it was not possible to attain the optimum $\eta_{ob}$ in the high end of the desired range of angles, indicating a substantial loss of summer radiation. These collectors might work better with a roof slope of around 20°. One reason for the mismatch problem is that it is difficult to hide the absorber from radiation incident between -90° and -60°. This is where the sun is below the horizon for a 30° tilted south-facing collector, where no direct radiation is present at all and where the contribution from ground reflected radiation mostly is small. It is almost useless to have any optical efficiency in this region. But in order to have a low optical efficiency below -60° the collector would have to be geometrically more complicated, causing other optical losses instead.
Figure 53. Collector geometries and diagrams of beam optical efficiency $\eta_{ob}$ versus incidence angle for collectors with single sided absorber. The bold curves in the diagrams represent efficiencies calculated by ray-tracing of the geometry to the left of each curve. The thinner curves represent theoretically optimized optical efficiencies for collectors with areas of 10 m$^2$ and 40 m$^2$. The notation "C ..." refers to a collector with the indicated concentration ratio.
Table 12 contains numerical values for some important aspects of the collectors of type A in Figure 53.

Table 12. Parameters for collectors with a single sided absorber, derived by ray tracing:

<table>
<thead>
<tr>
<th>Type</th>
<th>C</th>
<th>(\eta_{od,r})</th>
<th>(\eta_{od,t})</th>
<th>(A_r/A_c)</th>
<th>(D/W_a)</th>
<th>(D_{143})</th>
<th>(D_{122})</th>
<th>(D_{70})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.5</td>
<td>0.56</td>
<td>0.51</td>
<td>0.52</td>
<td>0.47</td>
<td>67</td>
<td>57</td>
<td>33</td>
</tr>
<tr>
<td>A</td>
<td>2.0</td>
<td>0.42</td>
<td>0.38</td>
<td>0.70</td>
<td>0.64</td>
<td>92</td>
<td>78</td>
<td>45</td>
</tr>
<tr>
<td>A</td>
<td>2.5</td>
<td>0.33</td>
<td>0.31</td>
<td>0.84</td>
<td>0.82</td>
<td>117</td>
<td>100</td>
<td>57</td>
</tr>
<tr>
<td>A</td>
<td>3.0</td>
<td>0.27</td>
<td>0.25</td>
<td>0.94</td>
<td>0.93</td>
<td>133</td>
<td>113</td>
<td>65</td>
</tr>
<tr>
<td>A</td>
<td>3.5</td>
<td>0.23</td>
<td>0.22</td>
<td>0.95</td>
<td>0.94</td>
<td>134</td>
<td>115</td>
<td>66</td>
</tr>
</tbody>
</table>

A comparison between \(\eta_{od,r}\) and \(\eta_{od,t}\) shows to what degree the assumed relation between concentration factor and diffuse optical efficiency in Table 3 is valid. The difference is 9 % for the C 1.5 collector and decreases with increasing C down to 2 % for the C 3.5 collector. The prediction in section 5.2 that the relation would be most correct for the case where the absorber is perpendicular to the reflector is thus confirmed. The low concentration collectors have diffuse efficiencies that are higher than the theoretically estimated ones since more than half of the diffuse radiation hits the absorber directly, due to the larger angle between reflector and absorber. (The values for \(\eta_{od}\) in Table 3 are about 8 % lower than the corresponding values for \(\eta_{od,t}\) in Table 12, since the former are for the 3-dimensional case and the latter are for the 2-d case, so as to be comparable with the ray traced values, which were obtained from the 2-d version of the ray tracing tool.)

The ratio between reflector and aperture area, \(A_r/A_c\), is useful for cost calculations. The three last columns in Table 12 are useful for an estimation of how well suited the collector is for mounting on a normal roof. If the depth of the collector is around 100 mm it will take up only little more vertical space than a traditional tiled roof, facilitating retrofits on existing houses and making installations on new houses possible with standard roof designs and components. The absorber widths of 143, 122 and 70 mm are chosen because these are the standard widths of a common Swedish commercial absorber strip. With the 70 mm strip width all collectors comfortably fit on the roof, and with a 143 mm strip the collectors with a concentration ratio of 2.0 or less have an acceptable depth.

8.9 Results for collectors with double sided absorbers

Five collectors with double sided absorbers were designed to fit the theoretically optimized curves as closely as possible. The results of the ray trace of these collectors is shown in Figure 54. The Spring/Fall Mareco (SFM in figures and tables) was developed by VUAB and Finsun AB and is described by Helgesson et al. (2002). It is an asymmetric CPC trough collector that
is designed to have a reduced efficiency in summer. It was defined geometrically from data reported by Adsten (2002) and then ray traced in the same manner as the other collectors.

Figure 54. Collector geometries and diagrams of beam optical efficiency $\eta_{\text{ob}}$ versus incidence angle for collectors with double sided absorber. The bold curves in the diagrams represent efficiencies calculated by ray-tracing of the geometry to the left of each curve. The thinner curves represent theoretically optimized optical efficiencies for collectors with areas of 10 m$^2$ and 40 m$^2$. The notation "C ..." refers to a collector with the indicated concentration ratio. The collector denoted SFM is the "Spring/Fall Mareco" version of the Swedish Mareco collector.

The characteristical parameters in Table 13 for the collectors of type B were derived from the ray tracing results. The correspondence between theoretically estimated diffuse optical efficiency and the efficiency derived by ray tracing is excellent for these collectors.
Table 13. Parameters for collectors with double sided absorber derived by ray tracing. SFM refers to the "Spring/Fall" Mareco collector.

<table>
<thead>
<tr>
<th>Type</th>
<th>C</th>
<th>η_{od,r}</th>
<th>η_{od,t}</th>
<th>A_r/A_c</th>
<th>D/W_a</th>
<th>D_{143}</th>
<th>D_{122}</th>
<th>D_{70}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[-]</td>
<td>[-]</td>
<td>[-]</td>
<td>[-]</td>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
</tr>
<tr>
<td>B</td>
<td>1.5</td>
<td>0.50</td>
<td>0.51</td>
<td>1.37</td>
<td>1.14</td>
<td>163</td>
<td>139</td>
<td>80</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>0.38</td>
<td>0.38</td>
<td>1.23</td>
<td>1.14</td>
<td>163</td>
<td>139</td>
<td>80</td>
</tr>
<tr>
<td>B</td>
<td>2.5</td>
<td>0.30</td>
<td>0.31</td>
<td>1.17</td>
<td>1.14</td>
<td>163</td>
<td>139</td>
<td>80</td>
</tr>
<tr>
<td>B (SFM)</td>
<td>1.85</td>
<td>0.41</td>
<td>0.41</td>
<td>1.26</td>
<td>1.11</td>
<td>159</td>
<td>135</td>
<td>78</td>
</tr>
</tbody>
</table>

Collectors with double sided absorbers have some advantages compared to ones with single sided absorbers. The absorber is less expensive per unit area of absorbing surface, since only half as much material for the fins and the liquid channel are needed. The only extra cost is for creating the absorbing surface on the back. Furthermore, with the configurations of Figure 54 the heat losses may be expected to be lower than for a single sided design with the same concentration ratio. This is mainly due to the convection mechanisms. Since the absorber in the double sided case is located near the uppermost part of the collector, the fraction of the cover that can take part in convective heat transfer is smaller than for the single sided collector, where the level of the lowest part of the absorber is located closer to the low end of the cover. This rather intuitive reasoning is illustrated in Figure 55. From the figure one might be tempted to think that the loss coefficient from absorber to cover for the left hand collector would be larger because of the small distance. However, the combined convective and transmission heat transfer has a minimum at a distance of approximately 10 mm (Duffie and Beckman 1991, p.264). For larger distances the convective and radiative losses dominate over the transmission losses, and then the heat transfer is almost independent of the distance.

Figure 55. Illustration of possible convective heat transfer from absorber to cover for two collectors with concentration factor 2.0. The left collector has a double sided absorber and convection is only possible in the upper part of the collector. The right collector has a single sided absorber and most of the cover can take active part in the convective cooling of the absorber.
As discussed in section 5.2 collectors with double sided absorbers ought to have larger optical losses than those with single sided absorbers, due to the difference in the fraction of the radiation that undergoes reflection before reaching the absorber. This is confirmed by a comparison between Figure 53 and Figure 54. The difference between the theoretical and the ray traced curves is greater for the double sided ones. The collectors of design B and similar have half their absorber area almost fully exposed for radiation from all directions, so much of the optical efficiency of the collector is necessarily found at higher angles. This limits the amount of optical efficiency available at lower angles, which is evident, for example, in the curve for the C 2.5 collector of type B in Figure 55. The C 2.5 collector of type A in Figure 53 has a better optical performance in the important angle range, partly due to this effect, and partly due to the fact that a larger fraction of the radiation hits the absorber without reflections.

A number of collector designs of type B2, shown in Figure 52, with a double sided absorber positioned approximately perpendicular to the cover were also designed and ray traced. However, their optical performance was scarcely better than that of the type B, and they may be expected to have worse thermal performance, so they were not included in any further analyses.
9 Simulated performance of realistic, ray traced collector designs

A number of the ray traced collector designs presented in sections 8.8 and 8.9 were selected for calculation of useful solar output and stagnation performance with the Excel simulation model described in section 7.3. The simulations were exactly the same ones as performed during the optimization of the ideal collectors with the only difference that these simulations were made with optical efficiency curves obtained from ray tracing of realistic collector designs. The collectors and the parameters used were according to the following table:

Table 14. Collector parameters used for simulation of realistic, ray traced, collector designs. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio. "SFM" refers to the "Spring/Fall Mareco" collector with a concentration ratio of 1.85.

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Concentration ratio C [-]</th>
<th>First order heat loss coefficient c1 [Wm⁻²K⁻¹]</th>
<th>Second order heat loss coefficient c2 [Wm⁻²K⁻²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>Flat plate</td>
<td>1.0</td>
<td>3.50</td>
<td>0.01</td>
</tr>
<tr>
<td>C 1.5 A</td>
<td>A</td>
<td>1.5</td>
<td>2.75</td>
<td>0.01</td>
</tr>
<tr>
<td>C 2.0 A</td>
<td>A</td>
<td>2.0</td>
<td>2.37</td>
<td>0.01</td>
</tr>
<tr>
<td>C 2.5 A</td>
<td>A</td>
<td>2.5</td>
<td>2.15</td>
<td>0.01</td>
</tr>
<tr>
<td>C 3.0 A</td>
<td>A</td>
<td>3.0</td>
<td>2.00</td>
<td>0.01</td>
</tr>
<tr>
<td>SFM</td>
<td>B</td>
<td>1.85</td>
<td>2.00</td>
<td>0.01</td>
</tr>
<tr>
<td>C 1.5 B</td>
<td>B</td>
<td>1.5</td>
<td>2.14</td>
<td>0.01</td>
</tr>
<tr>
<td>C 2.0 B</td>
<td>B</td>
<td>2.0</td>
<td>1.95</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The Mareco collector and the C 1.5 B collector were modelled with measured heat loss coefficients derived from Adsten (2002) and Fiedler (2002) respectively and the C 2.0 B collector was modelled with a coefficient that was extrapolated from these two values. The resulting heat loss coefficients are close to those estimated in section 5.5 and comparable to the ones used for the optimizations of ideal collectors. The collectors of type A, however, may be expected to perform worse than the estimated relation, as discussed in section 8.9 and indicated in Figure 55. In order to take this into account, the following, modified version of equation (20)

\[ F'U_L = 1.55 + 2.25/C \] \[ [Wm^{-2}K^{-1}] \] (66)

was used for collectors of type A. Using this relation results in a \( F'U_L \) that is 0.42 Wm⁻²K⁻¹ higher for a C 2.0 type A collector than for a C 2.0 type B collector. The resulting \( F'U_L \) values are in accordance with measured data reported by Rönnelid et al. (1996) and Fiedler (2002) and consistent with the assumed value for the flat plate collector.

9.1 Collector useful output and stagnation performance

The yearly useful solar output calculated with the simulations is shown in figure 56. In systems with smaller collector areas the flat plate collectors deliver more energy than any of the load adapted types, but as the collector area increases the difference gets gradually
smaller. The difference in output between the A and B types is not so great, indicating that the better optical performance of type A is balanced by the lower heat losses of type B.

The simulated maximum maximum stagnation temperatures $T_{stag}$ are shown in Figure 57. No load adapted system has a stagnation temperature that is significantly higher than that of the flat plate system. The $T_{stag}$ for the C 1.5 A and the C 2.0 A collectors are so low that it may be possible to use less expensive materials for parts of the collectors. These stagnation temperatures are lower than the ones recorded during the optimizations in chapter 7 since the optical efficiencies of the practical, ray traced collectors are lower than those for the ideal ones.

The yearly stagnation duration $t_{stag}$ is shown in Figure 58. The SFM collector has quite moderate $t_{stag}$, even though it has the highest $T_{stag}$ of all collectors, which at first may seem surprising. It is explained by a look at Figure 54. There is a sharp drop in the optical efficiency just below a $\theta_T$ of -10°. On the high side of this drop the SFM collector has the highest efficiency of all type B collectors, causing the high $T_{stag}$. Most of the stagnation danger occurs in summer, at higher solar elevations, and in this region the SFM collector has a lower
optical efficiency than most of the other collectors, whereby the stagnation duration is kept low. The other collectors perform as expected.

![Figure 58. Yearly stagnation duration for simulated systems. Missing bars mean that no stagnation occurred. "FP" refers to a flat plate collector. "C .." refers to a collector with the indicated concentration ratio. "SFM" refers to the "Spring/Fall Mareco" collector with a concentration ratio of 1.85.]

### 9.2 Collector cost performance

The details of the cost calculations presented in this section are given in Appendix A. The material cost of the collectors described in the previous section was calculated from factory prices for collector materials, excluding VAT. The estimated materials cost $P_{cm}$, expressed in SEK per unit collector aperture area, for the different collectors is shown in figure 59.

![Figure 59. Estimated material cost $P_{cm}$ in SEK per m$^2$ collector area for different collectors, excluding VAT. "FP" refers to a flat plate collector. "C .." refers to a collector with the indicated concentration ratio. "SFM" refers to the "Spring/Fall" version of the Swedish Mareco collector with a concentration ratio of 1.85.]

A calculation of a cost performance index, CPI, was made. The CPI expresses how much investment is needed in collector materials in order to save one kWh of auxiliary energy annually, compared to a reference system without collectors. The result is presented in figure 60, as CPI versus $f_{sav}$, the fractional energy savings, which expresses how much auxiliary energy a solar system saves compared to a reference system without collectors. At a lower $f_{sav}$, around 20 %, which is a typical for a Swedish combisystem, the flat plate and the load adapted collectors of type A with concentration ratios of 1.5 have the lowest CPI. At a $f_{sav}$ of 50 % the best load adapted collectors have CPI values which are 13 % lower than the CPI for
the flat plate collector, indicating a potential for a lower total energy cost than with a flat plate system.

Figure 60. Collector cost performance index $\text{CPI}_{\text{cm}}$ based on collector material cost, versus fractional energy savings $f_{\text{sav}}$ for systems with different collectors. "FP" refers to a flat plate collector. "C ..." refers to a collector with the indicated concentration ratio. "SFM" refers to the "Spring/Fall Mareco" collector with a concentration ratio of 1.85.

An earlier study by Nordlander and Lorentz (2003) gave similar relations between the cost performance of flat plate systems and load adapted systems, also when the total installation cost, including all system components and labor, were taken into account.
10 TRNSYS simulation study

In this chapter the size of the storage tank and the capacity of the solar loop heat exchanger are cost optimized for different system sizes, and it is determined whether there is any significant difference in optimal size of these components between systems with flat plate collectors and systems with load adapted collectors. For a certain load and collector area and storage design it is mainly the volume of the store and the capacity of the heat exchanger that are interesting to optimize. The dimensioning of other major parts of the systems, such as the control system and the hot water preparation components is almost independent of the collector area. The solar loop pump and piping dimensions may have to be larger for a larger system, but the additional cost is small, whereas the cost for a store or a heat exchanger is almost linear with capacity. The TRNSYS (Klein at al, 2000) programme was used since it is well suited for detailed study of how variations in component parameters affect the system output.

10.1 Heating system simulation model

A model of a solar combisystem, configured as in Figure 1, was created for simulation with TRNSYS. The model was designed according to the results of Lorentz et al (1998). The system had a domestic hot water load of 3100 kWh and a space heating load of 8000 kWh, typical for a Swedish family in a modern single family house. All systems were simulated at a collector slope of 30°. A flat plate collector system and a system with load adapted collectors with single sided absorbers and a concentration factor of 2.0 were simulated with different system sizes. The collectors had parameters according to Table 12 and Table 14.

10.2 Optimization of store volume and heat exchanger capacity

An economical optimization was performed in order to determine suitable store volumes and solar loop heat exchanger capacities for different systems. The objective was twofold, firstly the results were used for setting the parameters in all the other simulations made for this study, and secondly it is interesting to learn whether the same dimensioning criteria can be used for flat plate systems as for load adapted.

The following cost functions, representing consumer prices, excluding sales tax, were used for the optimizations.

<table>
<thead>
<tr>
<th>Component</th>
<th>Cost Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat plate collector</td>
<td>SEK 2000 A_c</td>
</tr>
<tr>
<td>Load adapted collector</td>
<td>SEK 1680 A_c</td>
</tr>
<tr>
<td>Storage tank</td>
<td>SEK 1737 + 7869 V_st + 2000 V_st</td>
</tr>
<tr>
<td>Heat exchanger</td>
<td>SEK 1718 + 0.9268 UA_hx</td>
</tr>
</tbody>
</table>

where $A_c$ is the collector area in m$^2$, $V_{st}$ is the store volume in m$^3$ and $U_{A_hx}$ is the solar loop heat exchanger capacity in W K$^{-1}$. The costs are based on price information from large component suppliers in Sweden. The factor SEK 2000 in the equation for the storage tank accounts for the cost of the building space. It corresponds to a floor area requirement of 0.5 m$^2$ per m$^3$ of storage volume and a valuation of the building area of SEK 4000 / m$^2$.

The optimization was performed with successive simulations and adjustments of component sizes in order to find the size where the marginal component cost divided by the marginal...
energy collection equalled the system cost divided by the total output. The results are presented in Table 15 and Figure 61.

Table 15. Economically optimal component sizes and costs for stores and heat exchangers. FP denotes flat plate collectors and LA denotes load adapted collectors with a concentration ratio of 2.0.

<table>
<thead>
<tr>
<th>Collector area</th>
<th>Store volume</th>
<th>Heat exchanger capacity</th>
<th>Cost for store and heat exchanger</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c [m^2]$</td>
<td>$V_{st,FP} [m^3]$</td>
<td>$U_{A,FP} [W K^{-1}]$</td>
<td>Cost FP [SEK]</td>
</tr>
<tr>
<td>10</td>
<td>0.73</td>
<td>589</td>
<td>11165</td>
</tr>
<tr>
<td>20</td>
<td>1.14</td>
<td>1121</td>
<td>15735</td>
</tr>
<tr>
<td>40</td>
<td>1.78</td>
<td>2011</td>
<td>22925</td>
</tr>
<tr>
<td>60</td>
<td>2.30</td>
<td>2714</td>
<td>28668</td>
</tr>
<tr>
<td>80</td>
<td>2.52</td>
<td>3518</td>
<td>31584</td>
</tr>
</tbody>
</table>

The difference in optimal component sizes between FP and LA systems with identical collector area is quite small. The difference in cost is also small, especially in relation to the total system cost shown in Figure 61.

10.3 Collector output and overheating at different roof inclinations

All calculations and optimizations presented above are made for a fixed roof inclination of 30º. Even if most Swedish roofs have a slope near this angle, there are many houses with other roof slopes. Roof slopes less than 20º and larger than 45º are very uncommon though. In order to find out how sensitive the collector useful output is to the roof slope, a number of TRNSYS runs with different slopes of the collector aperture were made. The main difference in performance is due to the difference in solar irradiance and in beam efficiency for different slopes. The optical efficiency for diffuse sky radiation does also vary with the slope. The diffuse efficiency for sky radiation was calculated according to equation 31, resulting in the values in Table 16.
Table 16 Diffuse efficiency for sky radiation at different collector slopes.

<table>
<thead>
<tr>
<th>Collector slope [°]</th>
<th>Diffuse optical efficiency $\eta_{od}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.373</td>
</tr>
<tr>
<td>25</td>
<td>0.359</td>
</tr>
<tr>
<td>30</td>
<td>0.342</td>
</tr>
<tr>
<td>35</td>
<td>0.322</td>
</tr>
<tr>
<td>40</td>
<td>0.300</td>
</tr>
<tr>
<td>45</td>
<td>0.276</td>
</tr>
</tbody>
</table>

The result of the simulations is presented in Figure 62. The difference in useful solar output for different slopes is quite small. The smaller systems collect relatively more energy with the flatter roofs, since with these angles the optimal angle range of the collector is directed towards the higher solar elevations during summer. With the large systems the collector collects more energy with a steeper slope, since the summer collection is accomplished even with a lower optical efficiency, due to the large area, and more additional energy is collected during the darker seasons when the collector's optimal angular interval is directed towards a sun, which then has a lower elevation.

Figure 62. Useful collector output with $C = 2.0$ load adapted collectors in systems with different collector areas and slopes.

The yearly heat losses during stagnation periods for collectors with different slopes is shown in Figure 63. The difference between systems is large. With flatter roofs there is much more overheating than with steeper roofs. This is to be expected, since the incidence angle onto the load adapted collectors when mounted with flatter slope is such that they accept more summer radiation than they are designed for.
Figure 63. Yearly thermal losses per collector unit area during stagnation for systems with C 2.0 load adapted collectors, in systems with different collector areas and slopes.

10.4 Comparison between simulations with TRNSYS and Excel

A comparison between the results from detailed simulations with TRNSYS and with the simplified, faster, Excel model described in section 7.3 was made. 12 calculations, six with each method, and with system and collector parameters according to were made.

The Excel model was initially calibrated by trial and error adjustments of the upper and lower limits for the average store operating temperature, as to give values for the yearly collector outputs close to the TRNSYS results on average. The best correspondence was found for a maximum store average temperature of 88 ºC and a minimum store average temperature of 34º C. These values seem reasonable, since the high value is close to what may be expected when a system goes into stagnation, and the lower one is reasonable for a store that is rather well stratified and barely able to deliver domestic hot water at 50 ºC. The Excel simulations were finally made with the above mentioned settings for the store temperatures for all six variations in Table 17. The biweekly averages of the store temperature $T_s$ and the biweekly sums of the useful collector output $Q_u$ for the Excel and TRNSYS simulations were recorded and are shown in Figure 64 and Figure 65.

Table 17. Parameters used for comparison of results from simulations with TRNSYS and with a simplified Excel model.

<table>
<thead>
<tr>
<th>Collector type</th>
<th>First order heat loss coefficient [\text{Wm}^{-2}\text{K}^{-1}]</th>
<th>Diffuse optical efficiency [-]</th>
<th>Collector area [\text{m}^2]</th>
<th>Store volume [\text{m}^3]</th>
<th>Heat exchanger capacity [\text{Wm}^{-2}\text{K}^{-1}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat plate</td>
<td>3.5</td>
<td>0.72</td>
<td>10</td>
<td>0.75</td>
<td>600</td>
</tr>
<tr>
<td>Flat plate</td>
<td>3.5</td>
<td>0.72</td>
<td>20</td>
<td>1.1</td>
<td>1100</td>
</tr>
<tr>
<td>Flat plate</td>
<td>3.5</td>
<td>0.72</td>
<td>60</td>
<td>2.4</td>
<td>2700</td>
</tr>
<tr>
<td>C 2.0 A</td>
<td>2.25</td>
<td>0.34</td>
<td>10</td>
<td>0.75</td>
<td>600</td>
</tr>
<tr>
<td>C 2.0 A</td>
<td>2.25</td>
<td>0.34</td>
<td>20</td>
<td>1.1</td>
<td>1100</td>
</tr>
<tr>
<td>C 2.0 A</td>
<td>2.25</td>
<td>0.34</td>
<td>60</td>
<td>2.4</td>
<td>2700</td>
</tr>
</tbody>
</table>
The store temperatures calculated with different methods in Figure 64 reveal that in some cases the TRNSYS calculation gives higher temperatures, but in most cases the Excel method gives higher temperatures. The difference is almost never greater than 7 ºC. The discrepancy between the models may generally be explained by the very crude modeling of the heat store in the Excel model. The TRNSYS model uses an economically optimized solar loop heat exchanger that probably tends to increase the temperature drop over the exchanger in summertime, whereas the Excel model does not assume any temperature difference between collector and store at all.
Figure 65. Useful collector output $Q_u$, for a flat plate collector (FP) and a load adapted collector of concentration 2.0 (C 2.0) in systems with different collector areas, simulated with TRNSYS and with a simplified Excel model.

The differences in collector output calculated with the two methods, shown in Figure 65, are quite small. The distribution over the year of the collected energy is very similar for all the cases. Table 18 shows a numerical evaluation of the differences between the results with TRNSYS and Excel simulations. The relative error in yearly collector output is 3 % or less for the six systems studied.
Table 18. Comparisons of results for calculated solar output $Q_u$ and store temperatures $T_{st}$ with detailed TRNSYS simulations and simplified simulations with Excel. "LA C 2 A" is a load adapted collector with single sided absorber and a concentration ratio of 2.0. Subscripts TRN and Ex refer to TRNSYS and Excel respectively.

<table>
<thead>
<tr>
<th>Collector type</th>
<th>Area $A_s$ [m²]</th>
<th>Yearly average of store temperature $T_{st,TRN}$ [°C]</th>
<th>Difference between yearly average store temperatures $T_{st,Ex} - T_{st,TRN}$ [°C]</th>
<th>Yearly useful output $Q_u,TRN$ [kWh/year]</th>
<th>Ratio of yearly $Q_u,Ex$ to yearly $Q_u,TRN$</th>
<th>Standard deviation of difference in biweekly $Q_u,Ex - Q_u,TRN$ [kWh/(14 days)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat plate 10</td>
<td>10</td>
<td>46</td>
<td>2.7</td>
<td>2,560</td>
<td>1.01</td>
<td>9</td>
</tr>
<tr>
<td>Flat plate 20</td>
<td>20</td>
<td>51</td>
<td>3.3</td>
<td>3,462</td>
<td>1.00</td>
<td>11</td>
</tr>
<tr>
<td>Flat plate 60</td>
<td>60</td>
<td>58</td>
<td>4.1</td>
<td>5,336</td>
<td>0.99</td>
<td>13</td>
</tr>
<tr>
<td>LA C 2 A 10</td>
<td>10</td>
<td>41</td>
<td>0.3</td>
<td>2,131</td>
<td>1.03</td>
<td>11</td>
</tr>
<tr>
<td>LA C 2 A 20</td>
<td>20</td>
<td>49</td>
<td>1.0</td>
<td>3,208</td>
<td>1.00</td>
<td>9</td>
</tr>
<tr>
<td>LA C 2 A 60</td>
<td>60</td>
<td>58</td>
<td>2.9</td>
<td>5,243</td>
<td>1.01</td>
<td>13</td>
</tr>
</tbody>
</table>

Since using the Excel method is significantly faster than TRNSYS simulations, 0.3 seconds versus 5 minutes for simulating a year of operation, it is very useful for all calculations where it is not necessary to study the performance of the system components in detail.
11 Results and discussion

11.1 Results

The following list summarizes the most important results of the study.

- One and the same model of a load adapted collector is suitable for wide ranges of system sizes and roof slopes.
- Stagnation duration is strongly reduced with load adapted collectors.
- It is possible to design collectors for different maximum stagnation temperatures: Concentrating collectors with a maximum stagnation temperature of 200º C produce slightly more energy than collectors with a 180 ºC stagnation temperature. A collector with a maximum of 140º produces approximately 14 % less energy than a 180 ºC collector in a 40 m² system.
- It is impossible to achieve maximum energy collection and at the same time avoid overheating completely by optical design of a stationary collector, since the relations between solar irradiation, ambient temperature and the heating load are different in the spring than in the fall.
- Realistic load adapted collectors have lower specific material costs (invested material cost per yearly saved kWh) than flat plate collectors at system sizes from 10 m² and up. At 50 % useful solar fraction the specific material cost is 13 % lower.
- Designs with single sided absorbers and designs with double sided absorbers have similar energy collection and cost efficiency at equal concentration ratio.
- The most cost efficient load adapted collector has a concentration ratio around 2.0 and is approximately 100 mm thick with an absorber width of 143 mm.
- A system with load adapted collectors may be designed according to the same general design principles as a conventional system. Only the collector differs.
- In a typical combisystem, realistic load adapted collectors of various designs have a useful energy output per unit area on level with that of flat plate collectors at system sizes of 40 m² and larger.
- The system average and maximum thermal power is slightly lower for a load adapted system than for a flat plate system with the same solar fraction. The difference in store and heat exchanger sizes and costs are relatively small.
- The storage tank need not be bigger than 2.5 m³, even for a system with 80 m² collector area.
- A fast Excel simulation tool developed for the study delivers results that are very close to those from detailed TRNSYS simulations.

The following points summarize preparatory achievements and methods that have been developed in order to make possible the analyses in this study.

- A consistent estimation of collector parameters for comparative studies on flat plate and concentrating collectors
- An analytical method for calculation of diffuse optical efficiency from beam optical efficiency values at discrete incidence angles with arbitrary angle intervals
- A method for calculating a desired beam optical efficiency for a concentrating collector, using biweekly climate data, biweekly operating temperatures and the concept of critical radiation
• An extremely fast Excel tool for calculation of yearly performance with hourly time steps for systems with collectors with arbitrary incidence angle dependence, arbitrary loads, climate and storage characteristics.
• A method for semi-automatic optimization of the incidence angle dependence of the beam optical efficiency for concentrating collectors. The optimization is performed for an arbitrarily weighted combination of useful collector output, stagnation temperature and stagnation duration.
• A ray-tracing tool dedicated for comparative studies of concentrating collectors with internal reflectors. The tool also produces optical parameters for use as input data for simulation programs.
• An amendment to the TRNSYS collector model used in the IEA task 26 work, with routines for a complete treatment of incidence angle effects.

As a side effect of the work, a well-known relation for the maximum concentration for ideal radiation concentrators has been generalized as to be valid for asymmetrical as well as for symmetrical concentrators.

11.2 Discussion
In the present study ways have been found to reduce the overheating stress and lower the cost of solar combisystems by means of appropriate optical design of stationary concentrating collectors. It has been shown that with certain preconditions, like collector slope and a typical relation between thermal load and collector area, it is indeed possible to strongly decrease the stagnation duration and the stagnation temperature of the system. However, the calculated energy output per collector unit area of the load adapted collectors is typically lower or, more seldom, only slightly higher than that of a system with standard flat plate collectors. As for the cost, it seems probable that a load adapted collector may be produced to a significantly lower per unit area cost, on the order of a 15% reduction. The load adapted collectors exhibit their full potential only at higher solar fractions, from approximately 40% and up. At these solar fractions the output per unit area is lower, due to a falling marginal gain as additional energy collection needs to occur at less favourable climatic conditions, so the total investment per unit energy output is markedly higher than for small systems, albeit to a lesser degree than for a flat plate system. Nevertheless, the load adapted collector opens up the possibility to use large collector areas, reaching high solar fractions, without the common problems of overheating. By careful design in order to lower the stagnation temperature it may also be possible to use less expensive materials for the collector, and to increase the life of the heat carrier fluid and to ease the requirements on the stagnation protection functions of the system as a whole.

11.3 Further work
In order to develop the concept further, at least two objectives need to be met. One is to increase the energy collection. The optical performance of the collector has been extensively explored in this study, and may now only be marginally improved, but the study of the thermal performance of the collector has been rudimentary. There ought to be a potential for reducing the heat losses from concentrating collectors further, utilizing anti-convection barriers optimally, or, possibly, using recently introduced, new versions of vacuum tubes. Both practical experiments and theoretical studies of the heat balance of the collector are needed. The complex interaction between components, especially by combined thermal
radiation and convection, will probably require quite advanced models to be used. The other objective is to lower the cost. Much of the work needed is common industrial development, but research work may be needed in order to find suitable materials for a medium temperature range, between 120º and 150º, that are less expensive than present materials, or can offer production advantages. It is possible to optically design a load adapted collector that has its maximum stagnation temperature in this region but still has a respectable energy production. There are already polymer low cost absorbers, but they do not yet have selective surfaces and the stagnation performance needs more study.

Implicit in the practical and cost related considerations taken in this study has been that a load adapted collector should have the same dimensions as a flat plate collector, for example a thickness of approximately 100 mm. From an optical point of view this is not necessary. The collector could be 1 mm thick or 2 meters thick, and still have the desired optical function. A very thin collector would have problems with the heat losses though, since distances on the order of 10 mm or more is needed to keep conduction losses low, unless vacuum technology is applied. On the other hand, it is quite conceivable to use the whole southern section of a roof as one single optical unit, with a reflector of about 4 meter's width and an absorber of about one or two meter's height, hanging vertically. The collector would occupy the southern half of the attic space, but this is hardly a restriction since the attic of a house with a 30º roof slope is normally not used for any purpose, due to the low height of the space. Thus one could use full sizes of reflector and absorber materials, straight off the roll, and build large collector areas in short time with a minimum of material processing. The new developments in full-plate absorbers would fit well in this concept. The possibility of simply hanging very thin, full-width transparent convection barriers like curtains in front of the absorber is also very tempting, thereby avoiding the sagging-down problems common in sloping flat plate collectors with teflon films. The physical dimensions of the collector may not be very theoretically interesting, but this vision of a large scale, low-cost load adapted collector, with due production and cost advantages, may well serve to inspire and enhance the relevance of the related scientific work.

11.4 Future challenges for collector materials development

An inherent limitation in the present concept of the load adapted stationary collector is the impossibility of both having very low heat losses and avoiding high temperatures, if high energy collection is desired. In order to overcome this limitation, it would be very useful to have materials that change optical properties with temperature. The prime choice would be an absorber surface that changes its solar absorptance or thermal emittance with temperature, so that the optical or thermal efficiency could be radically lower at, for example, temperatures above 95º C. Also reflectors or convection barriers with temperature dependent optical properties could be of help. There exist transparent materials that change transmittance with temperature, or by other means, but so far they have not been applied in collectors, maybe due to technical or economical reasons. If there were materials of these kinds, especially the temperature sensitive absorber, the advantages of transparent insulation materials could be fully utilized without the adverse effects of overheating. A collector thus equipped, in combination with a basic optical load adaption by means of reflectors, might be a very interesting development.
Acknowledgements

I want to thank my main supervisor Professor Ewa Wäckelgård for her valuable guidance and eminent support. I am very indebted to my local assistant supervisor at Högskolan Dalarna, Dr. Mats Rönnelid. He has been a constant source of inspiration and constructive criticism, as well as of encouragement during the darker moments. I also want to thank all my other colleagues at the Solar Energy Research Centre (SERC) at Högskolan Dalarna for their support and interest in the project, especially Professor Lars Broman, the founder of SERC, who initiated my academic career in solar energy and started my interest in non-imaging optics. Dr. Bengt Perers has provided me with many pieces of good advice and insights from his vast experience of solar systems. I also want to thank Frank Fiedler and Fares Mustafa, students at the European Solar Engineering School of Högskolan Dalarna 2001-2002, for a great job with building, modelling and testing two load adapted collector prototypes.

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Appendix A

Cost performance of solar collectors

A number of concentrating collector designs, see figure 1, were selected for calculation of useful solar output and for materials cost calculations. The simulations were made with a simplified Excel model and the needed optical efficiency curves were obtained from ray tracing of realistic collector designs. Systems with collectors parameters according to Table 1 were simulated:

Table 1. Collector parameters used for simulation of practical, ray traced, collectors.

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Concentration ratio C [-]</th>
<th>First order heat loss coefficient c1 [W/m²K]</th>
<th>Second order heat loss coefficient c2 [W/m²K²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>Flat plate</td>
<td>1.0</td>
<td>3.50</td>
<td>0.01</td>
</tr>
<tr>
<td>C 1.5 A</td>
<td>A</td>
<td>1.5</td>
<td>2.75</td>
<td>0.01</td>
</tr>
<tr>
<td>C 2.0 A</td>
<td>A</td>
<td>2.0</td>
<td>2.37</td>
<td>0.01</td>
</tr>
<tr>
<td>C 2.5 A</td>
<td>A</td>
<td>2.5</td>
<td>2.15</td>
<td>0.01</td>
</tr>
<tr>
<td>C 3.0 A</td>
<td>A</td>
<td>3.0</td>
<td>2.00</td>
<td>0.01</td>
</tr>
<tr>
<td>SFM</td>
<td>B</td>
<td>1.85</td>
<td>2.00</td>
<td>0.01</td>
</tr>
<tr>
<td>C 1.5 B</td>
<td>B</td>
<td>1.5</td>
<td>2.14</td>
<td>0.01</td>
</tr>
<tr>
<td>C 2.0 B</td>
<td>B</td>
<td>2.0</td>
<td>1.95</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Figure 1. Two designs of concentrating collectors with internal reflectors. Type A has an absorber that is illuminated on one side only, and the absorber of type B is illuminated on both sides.

The result of the simulations is shown in Figure 2. With smaller collector areas the flat plate collectors deliver more energy than any of the load adapted types, but as the collector area increases the difference gets gradually smaller. The difference in output between the A and B types is not so great, indicating that the better optical performance of type A may be more or less balanced by the lower heat losses of type B.

The material cost of the collectors was calculated, with dimensions of absorbers and reflectors from the geometrical definitions used in the ray tracings and manufacturer's prices for commonly used commercial collector materials. The prices for the materials, per unit material area (not collector unit area) are presented in Table 2. The crosses in the "Type" columns indicate which materials are used in the different collectors. The amounts of material is based on relations like the ones in Table 12, and on assumptions of how much of the material is
needed as a minimum. As an example, the insulation for the type A collectors is assumed to have the same width as the absorber, but half the thickness of the insulation in a flat plate collector. The manufacturer's prices are chosen because only the flat plate collector is commercially available; the Spring/fall Mareco and the C 3.0 A exist only as prototypes and the other ones are only proposed, ray traced designs. The material cost should be a relevant measure, because all the collector designs are simple, and because with a modern, automated production process the material cost is the dominant factor of the production cost, also for solar collectors. The additional costs that make up the difference to the customer price are mostly related to storage, capital, handling and other costs that ought to be similar for all the collector types, provided that they are of comparable module sizes and sold in similar quantities.

Table 2. Material costs per material unit area. The cost for the 122 mm absorber strip refers to the active surface area, which is twice the strip area, because the strip has a sputtered selective surface on both sides. The cost of the 70 mm absorber strip is included for reference. FP stands for flat plate collector, Type A is a concentrating collector with single sided absorber and type B has double sided absorber. SFM refers to the “Spring/Fall” version of the Swedish Mareco collector with a concentration ratio of 1.85 and double sided absorber.

<table>
<thead>
<tr>
<th>Material</th>
<th>Cost SEK/m²</th>
<th>Type FP</th>
<th>Type A</th>
<th>Type B, SFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorber 143 mm, single</td>
<td>223</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Absorber 122 mm, double</td>
<td>159</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Absorber 70 mm, double</td>
<td>196</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflector</td>
<td>70</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Insulation</td>
<td>50</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Back side cover</td>
<td>30</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Glass cover</td>
<td>130</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Side profiles</td>
<td>150</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Screws, sealings etc</td>
<td>50</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

The estimated materials cost $P_{cm}$, expressed in SEK per unit collector aperture area, for the different collectors is shown in the diagram in Figure 3. For the flat plate and type A
collectors the cost decreases with the concentration ratio. The C 1.5 B and C 2.0 B types are less costly than the corresponding type A collectors because the double sided absorber is less expensive for the same concentration ratio, and because type B does not need any insulation.

![Graph showing material cost comparison](image)

Figure 3. Estimated material cost $P_{cm}$ in SEK m$^{-2}$ for different solar collectors. FP stands for flat plate collector and "C ..." refers to a collector with the indicated concentration ratio. Type A has single sided absorber and type B has double sided absorber. SFM refers to the "Spring/Fall" version of the Swedish Mareco collector with a concentration ratio of 1.85.

In order to compare the material costs fairly the cost should be related to how much auxiliary energy is saved by the systems, compared with a similar system with the same load but without solar collectors. Since different collectors have different yearly outputs per unit area for different system sizes the cost comparisons ought to be made for equal energy savings, not for equal area. These issues have been covered in some depth in the work of the International Energy Association Solar Heating and Cooling Task 26 on combisystems, and a recommended practice is reported by Weiss (2003). Following this practice, a fractional thermal energy savings $f_{sav}$ was calculated as

\[ f_{sav} = \frac{Q_{aux,0} - Q_{aux}}{Q_{dhw} + Q_{sh}} \]

where

- $Q_{aux}$ Annual auxiliary energy used
- $Q_{aux,0}$ $Q_{aux}$ for a reference case without collectors
- $Q_{dhw}$ Annual domestic hot water load
- $Q_{sh}$ Annual space heating load

The $f_{sav}$ value thus calculated represents an energy saving. The corresponding amount of energy is typically smaller than the amount of solar energy delivered by the collectors because some of the solar energy is lost as storage losses, due to high storage temperatures during sunny periods. A specific investment cost, $CPI_{cm}$ (Cost Performance Index for the collector materials), was defined for the collector materials cost:

\[ CPI_{cm} = P_{cm} A_e / (Q_{aux,0} - Q_{aux}) \text{ [SEK (kWh)$^{-1}$ a$^{-1}$]}. \]

The $CPI_{cm}$ expresses how much investment is needed in collector materials in order to save one kWh annually. The CPI was used for comparisons of cost efficiency between the simulated collectors. The result is presented in Figure 4, as CPI versus $f_{sav}$, for the simulated systems. At the lower values of $f_{sav}$, around 20 %, which is a good value for a traditional
Swedish combisystem, the flat plate and the load adapted collectors of type A with concentration ratios of 1.5 have the lowest CPIcm. With increasing values of $f_{sav}$ the CPIcm for the flat plate collector increases more rapidly than the CPIcm for the other collectors. At a $f_{sav}$ of 50% the best load adapted collectors have CPIcm values which are 13% lower than the CPIcm for the flat plate collector, indicating a potential for a significantly lower energy cost than with a flat plate system. An earlier study by Nordlander and Lorentz (2003) gave a similar relation between a flat plate system and a load adapted system, even when the total system cost including all components and labor were included in the calculation.

![Figure 4. Collector cost performance index CPIcm based only on material cost, versus fractional energy savings $f_{sav}$ for systems with different collectors.](image)

The possibility of lowering the cost of the collectors by using less expensive materials, permitted by designing the collectors for lower stagnation temperatures, has not been investigated. The present cost calculations are made for materials which withstand temperatures over 200 ºC.

**References**
