Does Euclidian distance work when Location Models are applied in rural areas?

Authors: Kenneth Carling[1], Mengjie Han[1], and Johan Håkansson[1, 2]

This version: 2010-11-25

Abstract: Location Models are used to locate *n* service centers in order to serve a geographically distributed population. A cornerstone of such models is the measure of distance between the service center and a set of demand points, viz, the location of the population (customers, pupils, patients and so on). Evidence support the current practice of using the Euclidian distance. In this paper, we argue and provide empirical evidence that such a measure is misleading once the Location Models are applied to rural areas with heterogeneous transport networks and a uneven distributed population. This paper stems from the problem of finding an optimal allocation of a pre-specified number of hospitals in a large Swedish region with a low population density. We conclude that the Euclidian and the network distances based on a homogenous network (equal travel costs in the whole network) give approximately the same optimums. However network distances calculated from a heterogeneous network (different travel costs in different parts of the network) give widely different optimums when the number of hospitals increases. It is also shown that aggregating the population misplaces the hospitals by on average 25% (10 km).

[1] Kenneth Carling is a professor in Statistics, Mengjie Han is a graduate student in Statistics and Johan Håkansson is a professor in Human Geography at the School of Technology and Business Studies, Dalarna university, SE-781 88 Borlange, Sweden. We are grateful to Bo Zhu who contributed in preparing an early draft of this work.

Key words: Multiple service centers, Optimal location, accessibility, network distance, Euclidian distance, spatial aggregation, P-median

1. Introduction

Consider the problem of allocating *P* service centers to serve a geographically distributed population disaggregated into *Q* demand points. Upon access to extremely detailed data, each individual in the population makes up a demand point, and the point would usually be the individual's residence. In practice, the data is less detailed on residential location and individuals are bunched into a geographical area of some sort such as municipality, parish, statistical sub-areas, blocks or raster. This spatial aggregation of the individuals is known to produce bias (often referred to as the MAUP - Modifiable Areal Unit Problem) in the distance measure thereby introducing the risk of a suboptimal solution to the allocation problem. The problem was discussed early by for instance Hillsman and Rhoda (1978), Openshaw (1984) and Openshaw and Tylor (1979), Holm (1984) and these works are followed by a great number of articles of which many are reviewed by for instance Love, Morris and Wesolowsky (1988), Rushton (1989), Rogers, Plante, Wong and Evans (1991), Hale and Moberg (2003) and Francis, Lowe, Rayco and Tamir (2009).

Location models assist in the allocation problem by locating the service centers to optimize some objective function(s). Conventionally, the objective function is taken to be the minimized distance between the service centers and the demand points, at least if the service is under central control as often is the case in publicly provided services such as kindergartens and schools, museums, hospitals, courts and so on. The rational for this objective function follows from the presumption that the service is tax-funded and that all in the population should have equal access to it. Without loss of generality, we will stick to this objective function in this work. Arguments leading to other objective functions and methods to deal with such can be found elsewhere, see e.g. Berman and Krass (1998).

A crucial measure and input into the location model is the distance between the nearest service center and the demand point. In his seminal paper, Bach (1981) makes a thorough investigation of how to measure the distance. Some competing alternatives are the Euclidian (shortest distance in the plane), the rectilinear (or Manhattan distance), the network distance (shortest distance along existing road or public transport network) and shortest travel time (or cost) along existing network. Intuitively, the travel time (or cost) seems to be the most attractive measure for most settings, yet it is very infrequently employed. One explanation is the difficulty and cost associated with obtaining it and another is the complication that arises in modeling the inherent variation in travel time. The second best would presumably be the network distance while the Euclidian and the rectilinear are the easiest to obtain. Surprisingly, Bach (1981) found that the correlation was close to one for the network and the Euclidian distances when he made an empirical examination of two densely populated German cities. Therefore, one would believe it to be indifferent whether the network or the Euclidian distance is used in Location Models. This argument is also found in Love, Morris and Wesolowsky (1988). They argue that

> Road travel between a pair of cities is seldom along a completely straight path. However, a good approximation of the average total distance between several pairs of cities in a region can often be made by using a weighted straight-line

distance function. (Love, Morris and Wesolowsky 1988, pp.5-6)

This statement I further strengthened from a literature review of location models and distance estimations made by Rushton (1989).

That the Euclidian distance nowadays is widely accepted as an adequate distance measure is evident from the survey of Francis et al (2009). They summarize the approach of some 40 published articles of which about half are executed on real data. In these articles, the predominant distance measure is the Euclidian (or rectilinear).

The high correlation observed by Bach (1981) is less surprising when theoretical predictions based on Cristaller's (1966) central place theory, first published in German in 1933, is taken into account. Based on the notion of a homogeneous space the theory is simply based on the recognition that different services need different amounts of costumers, and that there is a cost associated with the costumer's movement in space. This shapes a hierarchical city system where the cities are evenly distributed across the space. Transport network distances within this city system tend to equal Euclidian distances. However modifications of Christaller's work by for instance Lösch (1954) and Isard (1956) lend support for another interpretation. They show that the spatial size of the surrounding market areas for the service activities varies. The uneven distribution of the population and a heterogeneous road network are important explanations for this.

Another important explanation for varying market sizes is that the possibilities offered by the landscape for human activities space differs across space. Therefore in more sparsely populated areas and in areas where the landscape does not offer equal conditions for human activities across space, the distances in the road network and also the travel costs could be expected to deviate more from the Euclidian distances.

This work came about as a desire to investigate whether the recent location of two emergency hospitals in Dalecarlia (a rural region in mid-Sweden) is effective and accessible. The hospitals serve a fairly small and unevenly distributed population. The fact that three emergency hospitals have closed recently, prompted the investigation.

Moreover, the regional, administrative division of the country is currently under revision and one potential outcome is a reconfiguration into six regions instead of the existing 21. Since the regions in Sweden are responsible for providing emergency care and are entitled to collect taxes for this purpose, it is expected that a reformed, regional division would prompt a substantial re-location of emergency hospitals.

In a first phase we followed the literature and used the Euclidian distance. However, Lösch's findings also prompted us to reconsider this choice and collect data for the network. The aim in this paper is to examine whether the Euclidian distance works in rural areas in Location Models. While one would expect that the literature would offer a clear cut answer to this question, our extensive review of the literature makes us conclude that Bach's (1981) paper is the only paper that examines the issue empirically and thoroughly, yet in a non-rural area. This paper is the first to empirically investigate the consequence of distance measures on optimal location of *n* service centers in rural areas, by means of a case study.

The paper is organized as follows. Section 2 discusses the data, its sources and the derived measures and provides descriptive statistics of key variables. The region is also visualized in order to put the model into an empirical context. The third section concerns the optimizing algorithm and the experimental design leading to the counter-factual analysis. The fourth section provides results and the fifth section gives conclusions.

2. Data and descriptive statistics

Figure 1 shows the region Dalecarlia. It is about 31 000 km^2. Figure 1a shows the distribution of the population. The population, as of 2009, amounts to 276,000 residents the majority of whom are located in the south-east corner. The remaining residents are located along the two rivers as well as around the lake Siljan in the middle of the region. Overall, the region is not just uneven but also sparsely populated with an average of 9.5 (21 for Sweden overall) inhabitants per square kilometer.

Figure 1b shows the road network, the topography, and highlights the large rivers and lakes. The main roads are shown as a solid black line, whereas other roads are shown with a thin line. The altitude of the region varies substantially. In the western parts the altitude exceeds 1000 meters above sea level, whereas the altitude is less than 100 meters in the south-east corner. Within this general pattern, there is considerable variation and the natural barriers for a road network are abundant. In the south-east however, the soil is fertile and the flat-land is well suited for agriculture and the natural barriers are fewer. Unsurprisingly, the population here is fairly evenly distributed and the road network is denser and transport surface is more homogenous. However the road segments in the road network vary in quality. There are a couple of roads that connect the bigger cities in the region that in general have a higher quality than the rest of the network (se Figure 1b).

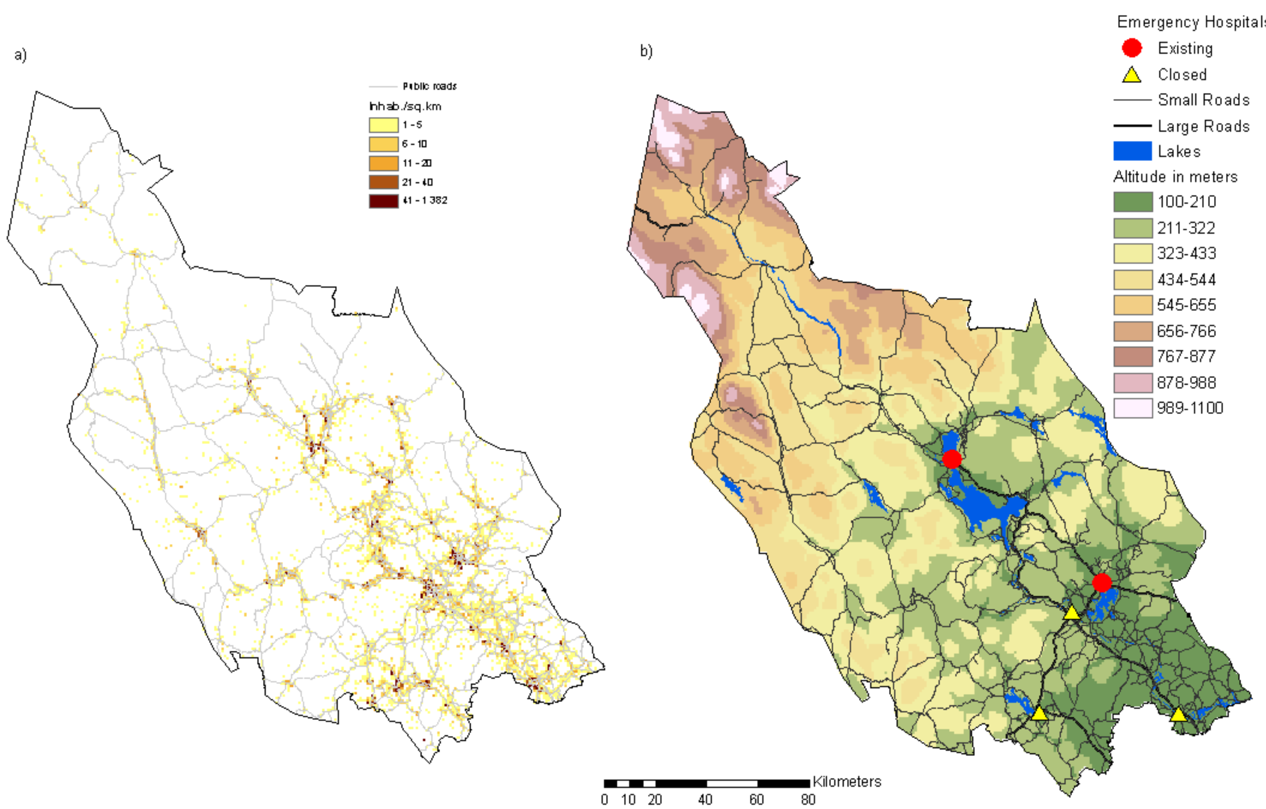

Figure 1a and b: Map of the region Dalecarlia showing the distribution of the population, the hospitals, the public road network and natural barriers.

Figure 1b also shows the two emergency hospitals located in Falun and Mora, as well as the location of the closed emergency hospitals in Avesta, Borlänge and Ludvika.

The population data depicted in Figure 1a comes from Statistiska Centralbyrån (Statistics Sweden). The population data is in raster format and gives the number of residents within each pixel of the region as of the year 2002. The pixels have the size of 250 meters by 250 meters. For the 464,000 pixels that make up the region, there are 15,729 pixels that contain at least one resident. For modeling purpose, the population pixels are transformed into vector format as points which serve as demand points in the Location model. Each point is the center of the pixel. For the ensuing examination of the consequences of spatial aggregation, we note that the spatial aggregation is achieved by collapsing pixels into a greater pixel leading potentially to a different point representation of the demand point. For the problem at hand, there are consequently 15,729 demand points (fewer when the spatial aggregation is increased).

The Euclidian distance between the demand point and the nearest service center, that is, emergency hospital can now be calculated. But first we need some notation. The coordinate for the *q*:th pixel is (x_q, y_q) and $q = 1,...,Q$. N_q is the number of residents in the pixel. The coordinate (x_p, y_p) refers to the factual location of the *p*:th service point (where $p = 1,...,P$), whereas uppercase letters refer to the potential location of the service points, that is, (X_p, Y_p). The distance between the demand point and any arbitrary service point is denoted by $d(p, q)$, which equals $\sqrt{(X_q - X_p)^2 + (Y_q - Y_p)^2}$ for the Euclidian distance. The shortest distance is

(1) $$\bar{d}(q) = \min[d(1, q), \ldots, d(P, q)],$$

and the sum of the shortest distances over all demand points for a fixed location of the P service points is,

(2) $$D(p^*) = \sum_{q=1}^{Q} N_q \bar{d}(q).$$

Table 1 shows statistics for the Euclidian distance for the population in Dalecarlia to, on the one hand, the existing two hospitals and, on the other hand, to the five hospitals that would be open had three hospitals not closed.

Table 1: Descriptive statistics for the Euclidian distance (in kilometers) between the two current hospitals and the population to be served, as well as for the location of previous 5 hospitals.

	Percentile						
	5	25	50	75	95	Mean	St. Dev.
2 current hospitals	1	14	28	54	90	32	24
5 previous hospitals	2	5	14	36	66	25	20

The Swedish road system is divided into national roads, local streets and private roads where the local streets are managed by the municipalities. The national roads are public and tax-funded and administered by the Swedish Transport Administration, which is a governmental agency. The national roads are of different quality and in practice distinguished by the speeding limit. Parts of the road network in the cities are local streets often with uniform and low speed limits. Figure 1a shows the public roads in Dalecarlia where the data for the road network comes from the Swedish mapping, cadastral and land registration authority state agency (Lantmäteriet) and gives the situation as of 2001. The road system of the region amounts to 5,437 kilometers.

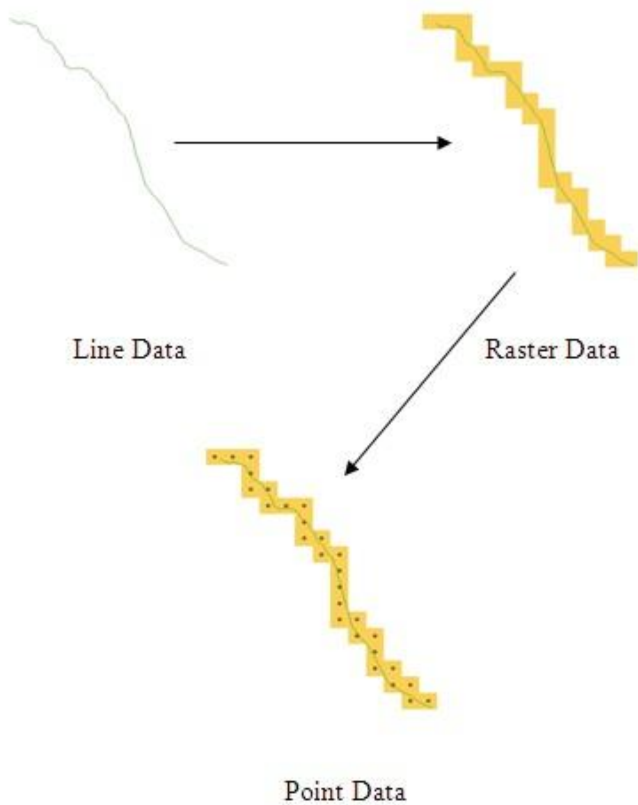

Figure 2: Transformation of the road network of Dalecarlia.

The road network is digitally stored in vector format as line data (Figure 2). The lines, representing a road segment, vary in length and they amount to 1,977 in number and range from a few meters (typically at intersections) to 52 kilometers, although the typical road segment is a couple of kilometers.

First we define 1579 nodes being all the intersections of the region's road network. By converting the line data into point data (see Figure 2) we connect all road segments and work out the $1579 \times (1579 - 1) / 2 = 1{,}245{,}831$ distances between all node-pairs and store them into a distance-matrix. All residents are then assigned to one node being the nearest node in the plane. Similarly the service points are also assigned to the nearest node. Since we did not have digital access to private roads and municipality streets we have approximated the individual distances to the nearest node in the road network by the Euclidian distances. Potentially this might induce a bias in the distance measure particularly for the residence in Falun and Mora. To check the magnitude of this error we examined a sample of 100 demand points and retrieved their network distances to a supply point by using a rout finder program (www.eniro.se). The error was insignificant and almost always less than one percent.

In measuring the distance between a demand point and a service point the line data is fairly awkward, and there is of course a multitude of possible routes to travel between two points. In the first part of the study we assume that the residents opt for the shortest distance between supply and demand points. In the second part of the study we introduce heterogeneity in the road network. We let the travel speed be 65 km per hour on the small roads and 90 km per hour on the large roads (see Figure 1b).

Table 2 gives some statistics for the Network distance for the population in Dalecarlia to, on the one hand, the existing two hospitals and, on the other hand, to the five hospitals that would be open had the hospitals that were closed stayed opened. By comparing the Euclidian distance and the Network distance from Tables 1 and 2, some remarks can be made. First, the Network distance is on average about 30 percent longer than the crew flight making it possible that the distances would yield different results in locating hospitals. Second, the median inhabitant has currently about 36 kilometers to nearest hospital, whereas she would have had only 20 kilometers were all five hospitals operating. Looking at the mean distance on the other hand there is a fairly small increase in distance to the nearest hospital after the removal of the three closed hospitals. Obviously the reduction of the number of hospitals lessened the accessibility to the inhabitants of the densely populated areas whereas the fraction of the population in remote areas suffered comparably less from it.

Table 2: Descriptive statistics for the Network distance (in kilometers) between the two current hospitals and the population to be served, as well for the location of previous 5 hospitals.

	Percentile						
	5	25	50	75	95	Mean	St. Dev.
2 current hospitals	3	18	36	64	116	40	30
5 previous hospitals	3	7	20	45	89	33	25

The correlation[1] is found to be 0.986 and 0.991 if computed as the distance to the current two hospitals

[1] The distribution of $\bar{d}(q)$ is very skewed as can be understood from the large difference in means and medians for both distance measures (see Table 1 and Table 2). Hence, it might be misleading to calculate a correlation coefficient on the

and to the past five hospitals, respectively. Obviously these correlations are high even though the distances are measured in an area with an unevenly distributed population as well as a road network. The correlations were also calculated between the Euclidian distances and the heterogeneous network distances. The Pearson correlation for two hospitals were 0.969 and for five 0.983. As expected, the correlations decrease when heterogeneity into the road network is introduced. The heterogeneity introduced here into the network is modest. Therefore, a large decrease in the correlations was not to be expected.

However, a high correlation does not necessarily imply that the choice of distance measure will not affect the optimal location. In fact, the implication is only strictly true if the population is evenly distributed and served by a homogenous transport surface.

To illustrate this point, the local correlations between different distance measures were calculated for a rectangular area around the twin-cities Falun and Borlänge (figure 3). The area holds for 18% of the demand points and about 34% of the region's population. The correlation between network and Euclidian distance was found to be 0.889, and the correlation decreases to 0.832 when time-distance in the network is compared to Euclidian.

Hence, in total the two distance measures appear to differ just a little. However, there is a significant

---

non-transformed distances. We have confirmed the resistance of the Pearson's correlation by also calculating the Spearman rank correlation.

difference in the distance measures locally, not identified by the correlation measure. It is also obvious that the difference between Euclidian and network distances increase when heterogeneity in the road network is introduced. This point to the need of checking results based on the Euclidian distance with the use of network distance calculated both from a homogenous and a heterogeneous road network.

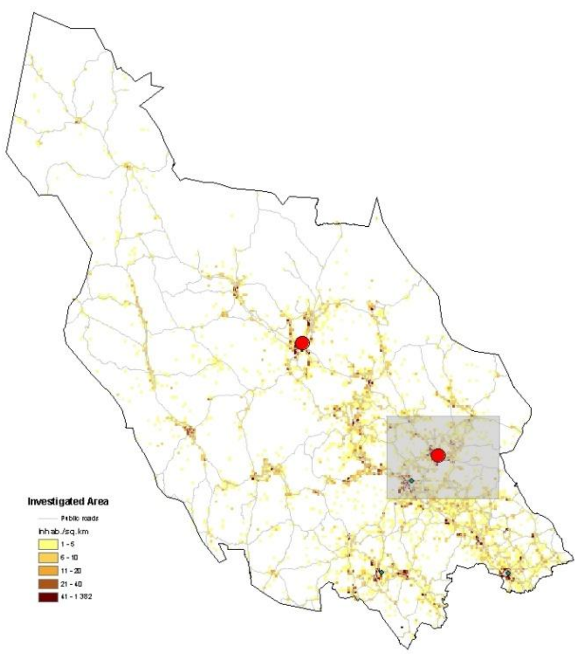

Figure 3:Sub-area (in grey) for investigation of local correlations between network and Euclidian distance to nearest current hospital.

3. Experiments and optimization

In this paper we are concerned with the choice of distance measure and its effect on the allocation problem, and particularly so for the issue of allocating hospitals. For this reason we will vary the two factors, distance measure and the number of hospitals, in the experiment. The demand point is simply a point in the plane, but it represents a potentially heterogeneous population which raises the issue of whether attributes such as number of residents, average income, educational level and so on should be assigned to each demand point. For the problem at hand, involving a tax-funded service, it seems sufficient to control for the number of residents at each demand point.

The MAUP has been widely documented in quantitative geography (Fotheringham and Wong, 1991). In fact, the issue of aggregation is thoroughly investigated and Francis et al (2009) and cited articles therein give an extensive overview of the issue, both for the Location Models as well as for other types of spatial models. Here we content ourselves to note that one would naturally choose the lowest level of aggregation compatible with the spatial data at hand as long as it is computationally feasible. In a regional setting, the pixels are of 250 by 250 meters in this study, which is notable. However we also elaborate with pixels of 5000 by 5000 meters to analyse the aggregation problem.

Before presenting the results, some details about the optimization technique is required[2]. The population weighted summed Euclidian distance, S, in the case of P hospitals can be expressed as

[2] Although the algorithm identifies the optimal location, it is very computationally heavy. Had we known at the onset of the work that we would end up running the algorithm repeatedly as done in the experimental part, we would have opted for and implemented one of the many effective algorithms available.

(3) $$S = \sum_{q=1}^{Q} N_q \cdot \min\left[\sqrt{(x_i - X_1)^2 + (y_i - Y_1)^2}, \ldots, \sqrt{(x_i - X_P)^2 + (y_i - Y_P)^2} \right]$$

where the uppercase letters refer to the coordinates of the hospitals and lowercase letters refer to the Q demand points. While it is easy to derive an analytical solution to the optimization problem for one hospital, it is infeasible to find an analytical solution to a problem involving multiple hospitals. Instead we consider a simulation approach.

In the simulations, we generate a pair of uniform random numbers on the map for each of a given number P of hospitals. These P points represent the possible locations of the hospitals. We repeat the procedure R times. The optimal location is then said to be the smallest of all $S(R)$ and corresponding P locations. To find the optimum under the restriction that the citizens must follow the road system to the hospital, we proceed in an identical fashion, only replacing the Euclidian distance with the Network distance. Based on prior, extensive testing, we set R=150 000 as default.

4. Results

Table 3 gives the distribution of the residents' distances to the hospital for a number of experiments. It shows that there is a substantial difference in optimal minimal distances to any number of hospitals depending on whether the Euclidian or the network distances are used. For instance, the Euclidian mean distance for one optimal located hospital is 40 km while it is 52 km when it is optimized by road network.

Table 3: The population's distance (in kilometers) to the nearest hospitals under optimal locations.

No of hospitals	Percentile					Mean	St. Dev.
	5	25	50	75	95		
			Euclidian Distance				
1	2	15	37	57	97	40	33
2	3	13	28	41	57	29	21
3	2	8	19	36	58	24	20
4	2	9	14	27	58	20	18
5	2	5	14	20	59	17	18
			Homogenous network Distance				
1	4	19	45	69	137	52	43
2	5	15	35	50	80	36	26
3	3	9	24	46	78	31	25
4	5	12	19	34	80	27	24
5	4	10	17	27	65	23	23

Table 3 in combination with Table 2 reveals that there is quite an improvement in access to the population to be made if the current hospitals were relocated. The mean network distance for the population could be reduced by 10%.

Another conclusion to be drawn from Table 3 is that dramatic improvements in the population's average access to a hospital are to be made when the number of hospitals increases. For instance, it is possible to reduce the populations' average mean network distance to a hospital by more than 35% if the number of hospitals increases from two to five optimally located hospitals.

Table 4: The population's time, in minutes, to drive to hospitals under optimal locations.

No of hospitals	Percentile						
	5	25	50	75	95	Mean	St. Dev.
1	3.5	14.8	33.0	52.9	106.9	39.9	34.8
2	3.5	11.4	25.3	39.7	71.0	27.7	21.3
3	2.5	7.9	19.1	32.8	69.0	23.7	20.4
4	2.9	6.1	14.4	25.6	69.0	20.1	20.2
5	2.5	8.4	13.1	24.1	63.9	19.2	19.1

Table 4 shows that the improvement in accessibility stops at four hospitals. This is in contrast to Table 3 and the Euclidian and homogenous network distances in which case there is a steady improvement in accessibility by adding hospitals.

The optimal locations of the hospitals are of interest in themselves. However, the large amount of experiments renders it difficult to present them on one map. We highlight some locations in map (see Figure 4) and present aggregated statistics on the difference in location in Table 5. Figure 4a show the optimal locations of the four hospitals. Besides the most northern located hospital, optimal locations are quit close to where the previous emergency hospitals were located. In Figure 4a the remaining two hospitals as well as two optimally located hospitals are shown. One seems to be well located while the other is misplaced.

The Euclidian mean distance (in kilometers) deviation between actual locations of two and five hospitals and the optimally located hospitals is shown in Table 5. The table also shows the Euclidian mean distance (in kilometers) deviation between optimal locations of two, three, four and five hospitals based on different distance measures. Table 5 confirms that the current two hospitals are in optimally located (compare Figure 4a). The mean deviations are between 10.3 and 13.6 km which is about one fourth of the population's average network distance to the factual located hospitals (see Table 2). When five hospitals are compared the misplacement is even bigger, up to 50%. It is also clear that the deviation tends to increase when the road network becomes more and more detailed fashion.

Table 5: Average difference in km between different numbers of actual and optimally located hospitals.

Distance measure	No of hospitals			
	2	3	4	5
Actual vs Euclidian	13.6			8.2
Actual vs Homogenous network	10.5			11.4
Actual vs Heterogeneous network	10.9			15.5
Actual vs Aggregated Heterogeneous network	10.3			11.5
Euclidian vs Homogenous network	3.9	6.9	8.8	5.6
Euclidian vs Heterogeneous network	3.5	7.9	7.0	10.1
Euclidian vs Aggregated Heterogeneous network	4.1	3.0	7.4	7.7
Homogenous network vs Heterogeneous network	0.4	4.1	5.5	9.6
Homogenous network vs Aggregated Heterogeneous network	0.3	6.3	7.8	11.2
Heterogeneous network vs Aggregated Heterogeneous network	0.6	7.0	4.7	9.5

In Table 5 it is also shown that the use of Euclidian distances for optimal location of three and more hospitals generates large misplacements. The differences between Euclidian distances and network distances is about 7 km which is approximately 30% of the distance that the population has to travel on average when the hospitals are optimally located with Euclidian distance (compare Table 3).

It is also revealed by Table 5 that there is just a small difference in locations between the homogenous and the heterogeneous network when two, three and four optimal hospitals are located. However when the number of hospitals increases to five the deviation becomes large (11.2 km). This is more or less 50 % of the average distance to an optimally located hospital based on homogenous network distances. The geographical misplacement between the homogenous network distances based optimal locations for five hospitals and for the heterogeneous based locations is shown in Figure 4b. Clearly, the main difference is in locating a hospital in the south eastern part of the county. When the network becomes heterogeneous the optimum suggests another hospital closer to the regional population center.

a) b)

Figure 4: actual location of existing emergency hospitals as well as optimal locations based in different levels of aggregation and distance measures.

The last result that is shown in Table 5 is that spatial aggregation in rural generates large misplacements. This is especially true when the number of hospitals increases. This is shown in Figure 4 b. When the aggregation increases the suggested optimal locations are closely connected to the road not just with higher speed limit but also with a larger population along it.

5. Conclusions

The question we have addressed in this study was whether the Euclidian distance works in rural areas in Location Models. The analysis show it doesn't work, although correlation between Euclidian and network distance was found to be high.

The study also shows the importance of what kind of network distance to use when the optimal number of service centers is decided. When using a heterogeneous network it was clear that the decrease in mean distance diminish when going from four to five hospitals, while this was not the case when a homogenous network was used. In the study we tested the importance of the level of aggregation. It was shown that it didn't matter that much when the number of located service centers was low. However when the number increased it was clear that by using a more spatial aggregated level of analysis resulted in less satisfying results.

Moreover, we find that it is possible to drastically decrease the population mean distance to hospitals, ie increase the accessibility of the hospitals. This illustrates the importance of developing useful and realistically optimal allocation models in social and economic planning. The question is whether or not all location model based planning is misplaced due to the large extant shown in this study . To answer this question we need to see similar studies in other areas.

In this study we elaborated with the different travel speeds in the road network to a limited extent. However by doing so interesting results have emerged. F urther research would therefore seek to take the full step and use the whole heterogeneity that exists in the actual road network to further scrutinize these questions of optimal locations of service centers. By also using past heterogeneity in the road network the stability over time in the optimal location pattern could also be scrutinized.

6. Reference

Bach, L., 1981, The problem of aggregation and distance for analyses of accessibility and access opportunity in location-allocation models, Environment and Planning A, 13: 955-978.

Berman, O., and Krass, D., 1998, Flow intercepting spatial interaction model: a new approach to optimal location of competitive facilities, Location Science, 6:41-65.

Christaller, W., 1966: Central Places in central Germany. C. W. Baskin, trans. Englewood Cliffs, N.J.: Prentice-Hall.

Fotheringham, A.S. and Wong, D.W.S. 1991: The modifiable areal unit problem in multivariate statistical analysis. Environment and Planning A 23: 1025-44.

Francis, R.L., Lowe, T.J., Rayco, M.B., and Tamir, A., (2009), Aggregation error for location models: survey and analysis, Annals of Operations Research, 167: 171-208.

Hale, T.S. and Moberg, C.R. 2003: Location Science Research: A Review, Annals of Operations Research, 32:21-35.

Hillsman, E. L., and Rhoda, 1978, Errors in measuring distances from population to service centers, Annals of Regional Science, 12:74-88.

Holm, E. 1984: Att lokalisera utbildning, sysselsättning och boende (To locate education, work and residence.), GERUM RAPPORT B:9, Umeå University, Umeå.

Isard, W., 1956: Location and Space Economy, Wiley.

Love, R.F. Morris, J.G. and Wesolowsky, G.O. 1988: Facilities location –Models & Methods, North-Holland, New York.

Lösch, L. J., 1954: The Economics of Location, Yale University Press, New Haven, Conn..

Openshaw, S. 1984: The Modifiable Areal Unit Problem. Norwich: Geo Books. ISBN 0-86094-134-5.

Openshaw, S. and Tylor, P.J. 1979: A million or so correlation coefficients: three experiments on the modifiable areal unit problem. In Wriglry, N., editor, Statistical applications in spatial science, London: Pion, 127-44.

Rogers, D.F., Plante, R.D., Wong, R.T. and Evans, J.R 1991: Aggregation and Disaggregation techniques and methodology in optimization, Operations Research, 18:25-42

Rushton, G 1989: Applications of location Models, Annals of Operations Research, 39:553-582..

[2] Corresponding author. E-mail: jhk@du.se. Phone: +46-23-778573.