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Short Communication: A compelling argument for the gravity *p*-median model

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Abstract: The *p*-median model is used to locate *P* facilities to serve a geographically distributed population. Conventionally, it is assumed that the population always travels to the nearest facility. Drezner and Drezner (2006, 2007) provide three arguments on why this assumption might be incorrect, and they introduce the extended the gravity *p*-median model to relax the assumption. We favour the gravity *p*-median model, but we note that in an applied setting, Drezner and Drezner's arguments are incomplete. In this communication, we point at the existence of a fourth compelling argument for the gravity *p*-median model.

Key words: travel time, spatial aggregation, multi-purpose trips

1. The gravity *p*-median model

Consider the problem of allocating *P* facilities to a population geographically distributed in *Q* demand points such that the population's average or total distance to its nearest facility is minimized (e.g. Hakimi 1964, Handler and Mirchandani 1979, and Mirchandani 1990). The *p*-median objective function is $\sum_{q \in N} w_q \min_{p \in P}\{d_{qp}\}$, where *N* is the number of nodes, *q* and *p* indexes the demand and the facility nodes respectively, w_q the demand at node *q*, and d_{qp} the shortest distance between the nodes *q* and *p*.

The *p*-median model implies that the customer's choice of facility to visit is deterministically given

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by the distance, whereas Drezner and Drezner (2006, 2007) argue, that from an analyst's perspective that the choice is stochastic.

Their first argument is that the population is often spatially aggregated and approximately represented by the area's centre. Their second argument is that customers might act on incomplete information regarding distances to facilities. Their third argument is that facilities vary in attractiveness to customers. They propose $\frac{A_p d_{qp}^{-\lambda}}{\sum_{p \in P} A_p d_{qp}^{-\lambda}}$ to be the probability that a customer located at node q will visit a facility at node p, where A_p is the attractiveness of the facility and λ the power to which the distance is raised. Hence, the gravity *p*-median objective function is $\min_{p \in P} \left\{ \sum_{q \in N} \left[w_q \frac{\sum_{p \in P} A_p d_{qp}^{1-\lambda}}{\sum_{p \in P} A_p d_{qp}^{-\lambda}} \right] \right\}$.

2. The arguments scrutinized in an application

We study the optimal location of facilities in the retail sector in the rural region of Dalecarlia in mid-Sweden (Carling, Han, and Håkansson, 2012 provide detailed descriptions of the region). We focus on location in the network and stipulate that the demand points and facilities are located at the nodes of the network. The location analysis will aid in examining market shares and business opportunities, predicting the resulting spatial re-configuration of major market entrants, and foreseeing shifts in the need for maintenance in the network.

We take an interest in consumer electronics stores, locksmiths, and pet shops. Figure 1 illustrates the current locations of these businesses and the population distribution. A question, therefore, is should the optimal location be investigated by the *p*-median or the gravity *p*-median model?

Turning to back to the arguments for using the gravity *p*-median model employed by Drezner and Drezner (2006, 2007), we find that their first argument does not apply for this setting. The error in location is inconsequential and may, at the most, be 175 metres since a resident is represented by the centre of a square amounting to 250×250 metres. The fact that customers make regular trips to our stores of interest, for example pet owners who repeatedly need to re-stock food for their pets, and the ready availability of route-finders make the second argument farfetched. Nor is it easy to identify a variation in the attractiveness of the stores, for instance picking a lock is simply a matter of picking a lock.

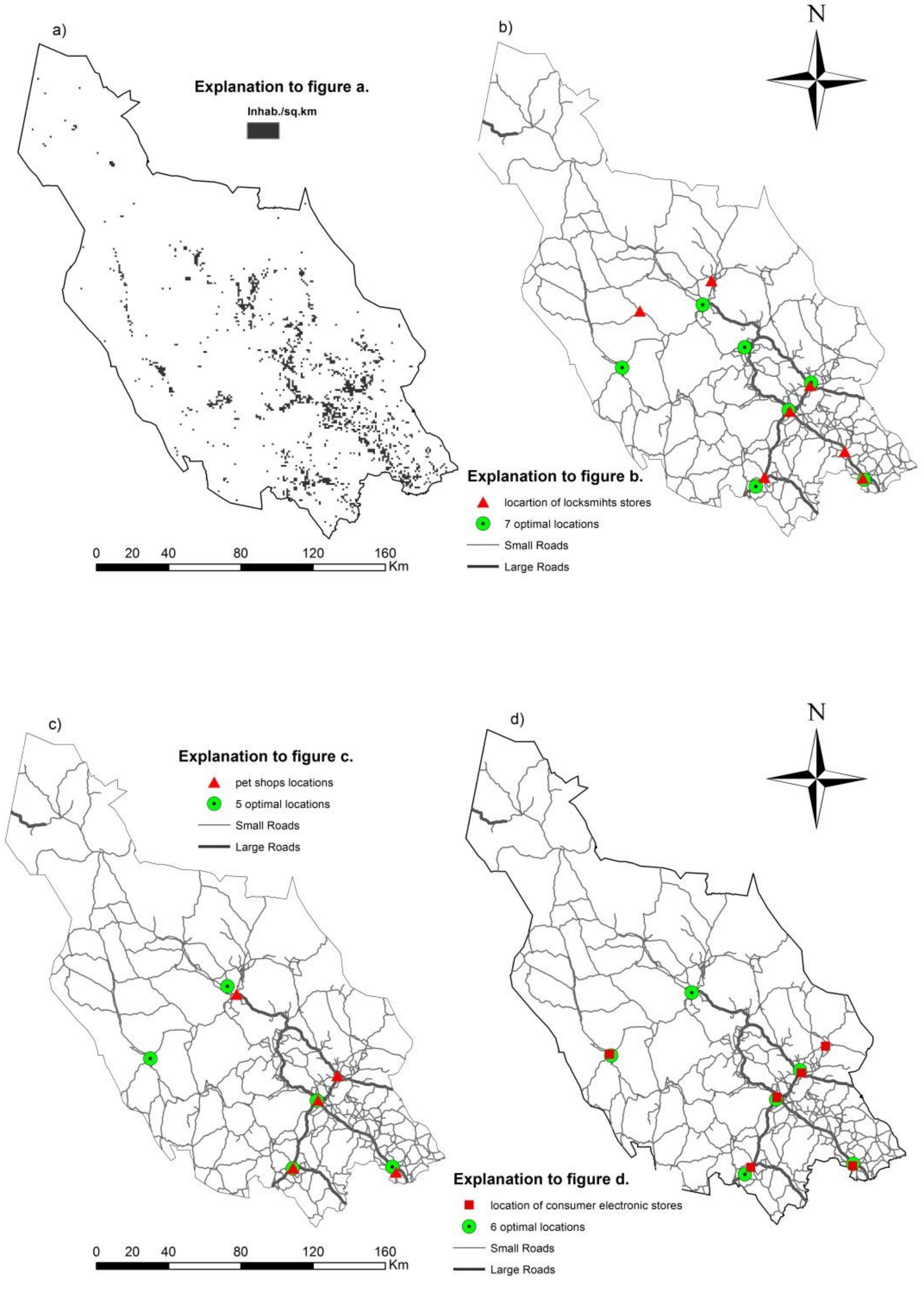

Figure 1 a-d. Map of the Dalecarlia region showing (1a) one-by-one kilometer cells where the population exceeds 5 inhabitants. (The optimal locations were found using the standard *p*-median model on a travel time network.), (1b) important infrastructure and locations of the regions 7 locksmith stores and 7 optimal location of facilities, (1c) important infrastructure, locations of the region’s 5 pet shops and 5 optimal locations of facilities, (1d) important infrastructure and locations of the region’s 6 consumer electronic stores and 6 optimal locations of facilities

Therefore, it appears that the *p*-median model is sufficient to use in these cases. Figure 1 illustrates the resulting optimal location of the stores in a travel-time network. However, the discrepancy between the current location and the *p*-median optimal location, presumably being the market efficient solution, is puzzling.

There is a fourth compelling argument favouring of the gravity *p*-median model. Trips, particularily shopping trips, do not seem to be conducted for a single purpose. Bazzani, Giorgini, Rambaldi, Gallotti, and Giovannini (2010) studied urban vehicle-movements by GPS in Florence, Italy. Jia, Bin, Carling, Bohlin, and Ban (2012) studied, in Borlänge, car-movements by GPS in the most populated part of Dalecarlia. Both studies found clear evidence of multi-purpose trips, and the latter study found the average number of purposes to be about seven. Thus, it is reasonable to view the choice of facility as stochastic, as the analyst is ignorant of the other purposes prompting a trip.

Acknowledgements

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