Single-mode acceleration of matter waves in circular waveguides

O. Dutta, M. Jääskeläinen,* and P. Meystre
Department of Physics and College of Optical Sciences, The University of Arizona, Tucson, Arizona 85721, USA
(Received 13 May 2006; published 17 August 2006)

Ultracold gases in ring geometries hold promise for significant improvements of gyroscopic sensitivity. Recent experiments have realized atomic and molecular storage rings with radii in the centimeter range, sizes whose practical use in inertial sensors requires velocities significantly in excess of typical recoil velocities. We use a combination of analytical and numerical techniques to study the coherent acceleration of matter waves in circular waveguides, with particular emphasis on its impact on single-mode propagation. In the simplest case we find that single-mode propagation is best maintained by the application of time-dependent acceleration force with the temporal profile of a Blackmann pulse. We also assess the impact of classical noise on the acceleration process.

DOI: 10.1103/PhysRevA.74.023609 PACS number(s): 03.75.Dg, 03.75.Be

I. INTRODUCTION

In recent years much effort has been devoted to the trapping and guiding of matter waves in "atom chip" microstructures [1,2], a natural route for applications such as rotation sensors based on the Sagnac effect. Experimental and theoretical work has focused on various trapping schemes using both magnetic and magnetoelectrostatic mechanisms. In several recent developments atomic storage rings operating both at the single-particle level and for quantum-degenerate systems have been realized with magnetic waveguides [3–6].

Exciting new opportunities are also emerging as a result of the availability of ultracold, possibly quantum-degenerate heteronuclear molecular systems generated either by Feshbach resonances [7,8] or by photoassociation [9]. The center-of-mass motion of heteronuclear molecules can be manipulated by relatively modest static or quasistatic inhomogeneous electric fields [10–13]. For example, by bending an hexapole trap into a torus, centimeter-size molecular storage rings can be created [14,15]. Especially when combined with the recent realization of neutral particle detectors [16–18] operating at the single-particle level, these storage rings open up exciting avenues of basic and applied research. In particular, the period of rotation of atoms or molecules can be of the order of milliseconds or longer, providing ample time to exploit feedback techniques to manipulate the properties of their quantum field.

An interferometer based on a centimeter-size molecular storage ring hints at a potential increase in sensitivity of matter-wave gyroscopes by several orders of magnitude. However, a realistic assessment of this possibility, in particular for comparison with possible alternative approaches using, for example, atomic ring traps [3–6], requires the critical study of a number of issues related to their stability, decoherence, and single-mode operation. Coherent large-angle splitting of guided matter waves [19] results in atomic or molecular speeds of the order of their recoil velocities, that is, centimeters per second. These velocities are unacceptably slow for centimeter-size rings, since they correspond to round-trip times of the order of seconds, which is also the typical time scale for the onset of decoherence [28] resulting from three-body recombination losses [20], background gas collisions, and other technical noise. It is therefore necessary to follow the beam splitter by a acceleration stage that will coherently bring the molecules to velocities up to hundreds of meters per second, while keeping them ultracold in their moving frame.

The goal of this paper is to analyze the coherent acceleration of ultracold atoms or polar molecules in a circular waveguide structure and optimize the temporal shape of the accelerating field so as to reduce multimode excitations to a minimum. We find that this goal is achieved for a time-dependent acceleration force that has the temporal profile of a so-called Blackmann pulse. We also assess the impact of noise in the accelerating field on the coherence properties of the molecular matter waves.

Several earlier studies of the propagation of ultracold atoms along waveguides are particularly relevant to this problem: The one-dimensional coherent transport of matter waves in noisy environments, including the role of two-body interactions, was discussed in Refs. [21,22]; the effects of the transverse dynamics on the coherence of atoms propagating along linear magnetic waveguides was considered in Ref. [23]; and the adiabaticity of the atomic propagation in a one-dimensional waveguide structure with smooth changes was considered in Refs. [24,25]. The present paper extends these results to an analysis of matter-wave acceleration in rings and in the presence of classical noise.

The paper is organized as follows. Section II establishes the mathematical framework and describes the coupling between the transverse modes of the waveguide resulting from the longitudinal acceleration of the molecules. It then introduces the Blackmann pulse shape that minimizes the effects of this coupling. In Sec. III we discuss the effect of classical noise on the acceleration process and the resulting decoherence and dephasing of the matter-wave field. We find that depending on the ratio between the target velocity of the atoms or molecules and the radius of the ring, the dominant limitation is either the transfer of population from the ground state to excited states of transverse motion, which causes a loss of visibility, or the dephasing between these states. Finally, Sec. IV is a conclusion and outlook.

*Corresponding author. Email address: mrq@optics.arizona.edu

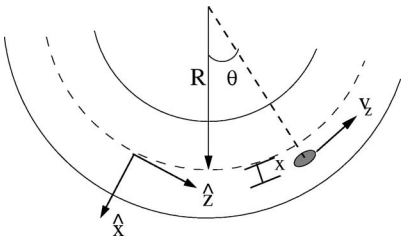

FIG. 1. Particle moving in the $\hat{z}$ direction of the circular waveguide with nonuniform velocity $v_z(t)$ at a distance x from the center of the ring.

II. THEORY

We consider the acceleration of a cloud of ultracold atoms or molecules trapped inside a toroidal waveguide of radius R, or more generally, any guide with constant radius of curvature. The transverse trapping potential is assumed to be harmonic with frequency Ω. The particles are initially located at the center $x=0$ of the waveguide and are to be accelerated along the longitudinal direction $\hat{z}$, see Fig. 1.

To gain some insight into this problem, we first consider the situation where the longitudinal motion is treated classically, in which case the quantized transverse motion is governed by the effective potential

$$V_{\text{eff}}(x) = \frac{1}{2}m\Omega^2 x^2 + \frac{l^2(t)}{2m(R+x)^2}, \tag{1}$$

where $l(t)=mv_z(t)R$ is the magnitude of (classical) angular momentum of the particle.

The acceleration of the particles is assumed to be produced by the application of time-dependent external fields. For instance, in the case of polar molecules these could consist of a Stark accelerator [27] wrapped around the storage ring. As a result, l is an increasing function of time, as is V_{eff}. In order for the particle to follow the minimum of V_{eff}, it must move a distance $x_c(t)$ away from the trap center, thereby causing the centripetal force to balance the transverse trap force. From $dV_{\text{eff}}(t)/dx=0$ we then have

$$\Omega^2 x_c(t) = \frac{l^2(t)}{m^2[R+x_c(t)]^3}. \tag{2}$$

Typical sizes of the transverse ground state of the ring are at most of the order of microns, so that for centimeter-size rings we have $x_c \ll R$. Expanding $V_{\text{eff}}(x)$ around x_c, the transverse motion of the particle is found to be governed by the effective Hamiltonian

$$H_{\text{eff}}[x_c(t)] = \frac{p_x^2}{2m} + \frac{1}{2}m\Omega^2[x-x_c(t)]^2, \tag{3}$$

with $[x,p_x]=i\hbar$.

Turning now to the full quantum problem, the motion of the particle is governed by the Schrödinger equation

$$i\hbar\frac{d\psi(x,\theta)}{dt} = \left[-\frac{\hbar^2}{2m(R+x)^2}\frac{d^2}{d\theta^2} + \frac{p_x^2}{2m} + \frac{1}{2}m\Omega^2x^2\right]\psi(x,\theta), \tag{4}$$

where θ is the azimuthal angle, see Fig. 1, and we have neglected two-body collisions, an approximation expected to be appropriate for the relatively low atomic or molecular densities expected to be achievable in centimeter-size storage rings. The two-dimensional equation (4) couples the radial and longitudinal particle motions. This is different from the situation for the familiar central potential, a consequence of the fact that the center of the guiding potential is not at the center of the ring. We proceed by factorizing the wave function $\psi(x,\theta)$ as

$$\psi(x,\theta) = \phi(\theta)\psi_\perp(x), \tag{5}$$

and introduce a dispersionless Gaussian wavepacket ansatz

$$\phi(\theta,t) = \frac{1}{\pi^{1/4}\Delta\theta^{1/2}}\exp\left[-\frac{\theta^2}{2\Delta\theta^2}\right]\exp\left[-i\frac{l(t)}{\hbar}\theta\right] \tag{6}$$

for the longitudinal wave function $\phi(\theta,t)$ where

$$l(t) = mR\langle v_z\rangle_{\text{cl}}(t), \tag{7}$$

where $\langle v_z\rangle_{\text{cl}}(t)$ is the mean velocity imposed on the particles by the accelerating field and $l(t)$ is the mean angular momentum of the wavepacket. The factor $\exp[-il(t)\theta/\hbar]$ then accounts for the rotation of the center of mass of the particles around the ring, i.e., propagation along the waveguide. We have also neglected the wavepacket dispersion, an approximation justified by the fact that the duration of the acceleration takes place in times of the order of tens of milliseconds, a time during which the dispersion of ultracold atomic or molecular wavepackets remains small.

Multiplying Eq. (4) from the left with $\phi^*(\theta,t)$ and integrating with respect to θ results then in the transverse Schrödinger equation

$$i\hbar\frac{d\psi_\perp}{dt} = \left[\frac{1}{2m(R+x)^2}\left(\hbar^2\frac{1}{2\Delta\theta^2} + l^2(t)\right) + \frac{p_x^2}{2m} + \frac{1}{2}m\Omega^2x^2\right]\psi_\perp. \tag{8}$$

As the radius of the storage ring R is typically large compared to the transverse size of the waveguide, we can expand $1/(R+x)^2$ about $x=0$ to obtain

$$i\hbar\frac{d\psi_\perp}{dt} = \left[\frac{p_x^2}{2m} + \frac{1}{2}m\Omega^2x^2 + \frac{1}{2mR^2}\left(\hbar^2\frac{1}{2\Delta\theta^2} + l^2(t)\right)\times\left(1-\frac{2x}{R}\right)\right]\psi_\perp. \tag{9}$$

With $\Delta z \equiv \Delta\theta R$ and $\Delta p_z \equiv \hbar/\Delta z$, and introducing the time-dependent "effective trap center"

$$x_0(t) = x_c(t) + \frac{2\Delta p_z^2}{m^2R\Omega^2}, \tag{10}$$

Eq. (9) finally becomes

$$i\hbar\frac{d\psi_\perp}{dt}=\left[\frac{p_x^2}{2m}+\frac{1}{2}m\Omega^2[x-x_0(t)]^2+\frac{1}{2mR^2}\left(\hbar^2\frac{1}{2\Delta\theta^2}+l^2(t)\right)-\frac{1}{2}m\Omega^2x_0^2(t)\right]\psi_\perp, \tag{11}$$

where the first two terms describe the motion of the particle in a harmonic oscillator with time-dependent potential and the third and fourth terms simply result in a time-dependent phase that can be accounted for through the transformation

$$\psi_\perp\rightarrow\psi_\perp\exp\left\{-\frac{i}{\hbar}\int_0^t\left[\frac{1}{2mR^2}\left(\hbar^2\frac{1}{2\Delta\theta^2}+l^2(t')\right)-\frac{1}{2}m\Omega^2x_0^2(t')\right]dt'\right\}. \tag{12}$$

By comparing Eq. (10) with the corresponding semiclassical expressions (2) and (3) we see that the balance between the centripetal force and transverse trap force is now modified by the "quantum pressure" due to the longitudinal velocity spread of the wavepacket.

The instantaneous eigenstates $\{\phi_n\}_{n=0,1,2,\ldots}$ of the Hamiltonian (11) are time-dependent harmonic oscillator eigenstates that include the effect of this force

$$\phi_n[x,x_0(t)]=\left(\frac{1}{\pi a^2}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}\exp\left[-\frac{[x-x_0(t)^2]}{2a^2}\right]\times H_n\left[\frac{x-x_0(t)}{a}\right], \tag{13}$$

where $a=\sqrt{\hbar/m\Omega}$ is the transverse trap size and $\{H_n\}$ are the Hermite polynomials. The wave function of the accelerated particle can be expanded in this time-dependent basis as

$$\psi_\perp(t)=\sum_{n=0}^{\infty}c_n(t)\phi_n(x,x_0(t))e^{-i(n+1/2)\Omega t}. \tag{14}$$

We assume here that the particles are initially in the ground-state mode $\phi_0(x,x_c(0))$ of transverse motion. The time dependence of the Hamiltonian that governs the transverse motion, see Eq. (11), causes transitions to excited states. To assess the importance of these transitions, we first assume they remain weak and treat them to first order in perturbation theory, thus including only coupling to the first excited state of the time-dependent potential. We find readily

$$c_1(t_f)=\int_0^{t_f}\exp(i\Omega t)\langle\phi_1(x,x_0(t))|\dot{\phi}_0[x,x_0(t)]\rangle dt, \tag{15}$$

where the overdot indicates a time derivative and t_f is the duration of the acceleration pulse. Inserting the expression for the first two eigenstates (13) into Eq. (15) and evaluating the scalar product we find

$$c_1(t_f)=\frac{1}{2}\int_0^{t_f}\exp(i\Omega t)g(t)dt, \tag{16}$$

where

$$g(t)\equiv\frac{d(x_0/a)}{dt}=\frac{d(x_c/a)}{dt} \tag{17}$$

is a coupling constant that depends on the specific temporal shape of the acceleration pulse.

The goal of the coherent acceleration is to bring the particles to a final longitudinal angular momentum $l_f\gg l(0)$ while still in a single transverse mode after an acceleration pulse of duration T. It is convenient to express the transition rate $g(t)$ in terms of these experimental parameters. From Eq. (2) we have, for $x_c\ll R$,

$$\frac{x_c(t)}{a}\approx\frac{l^2(t)a^3}{\hbar^2R^3}=\frac{l^2(t)}{\sqrt{\hbar\Omega^3m^3R^3}} \tag{18}$$

so that

$$g(t)=\frac{1}{\sqrt{\hbar\Omega^3m^3R^3}}\frac{dl^2(t)}{dt}=\left(\frac{l_f^2}{\sqrt{\hbar\Omega^3m^3R^3}}\right)\frac{d}{dt}\left(\frac{l^2(t)}{l_f^2}\right). \tag{19}$$

We see, then, that the transitions to excited states of transverse motion depend both on the final value l_f of the mean angular momentum of the particles, and on the shape of the accelerating pulse. Using this insight we rewrite Eq. (16), using Eq. (19), as

$$c_1(t_f)=\frac{l_f^2}{\sqrt{\hbar\Omega^3m^3R^3}}F(\Omega,T), \tag{20}$$

where the shape function

$$F(\Omega,t_f)=\int_0^{t_f}\exp(i\Omega t)\left[\frac{d(l^2/l_f^2)}{dt}\right]dt \tag{21}$$

gives the dependence on the temporal longitudinal velocity profile to the excitation probability. For realistic acceleration processes of finite duration t_f, $F(t_f)$ has the form of a truncated Fourier transform, resulting in side lobes of the excitation probability $|c_1(t_f)|^2$. Optimizing the acceleration process amounts to minimizing $|c_1(t_f)|^2$, that is, terminating it at a time t_f when the possible impact of the side lobes is minimal. One way to achieve this goal is by accelerating the atoms or molecules with a periodic acceleration [26,29] of the general form

$$\frac{d(l^2/l_f^2)}{dt}=\frac{1}{t_f}\left[1+\sum_{j=1}^{\infty}a_j\cos(2j\pi t/t_f)\right], \tag{22}$$

with $\Sigma a_j=-1$. In the following, we use a truncated version of these pulses that contains the first two harmonics only. We further chose the coefficients a_1 and a_2 in such a way that the area of the side lobes of $F(\Omega,t_f)$ is minimized. This is approximately achieved for $a_1=-25/21$ and $a_2=4/21$, the so-called Blackmann pulse. Figure 2 shows the time dependence of $\langle v_z\rangle_{cl}/v_f$ as a function of t/t_f. Inserting the expression for a Blackmann pulse into Eq. (20) and carrying out the integration we find

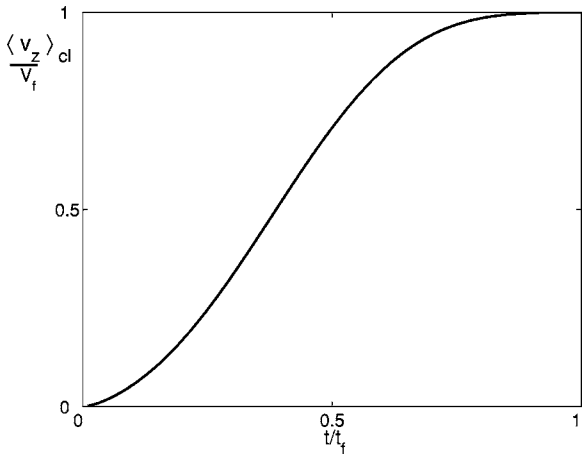

FIG. 2. Longitudinal velocity $\langle v_z\rangle_{\rm cl}/v_f$ as a function of the dimensionless time t/t_f for the Blackmann pulse.

$$|F(\tau)|^2 = \frac{\sin^2\tau}{\tau^2}\left[1 - \frac{25/21}{1-(2\pi/\tau)^2} + \frac{4/21}{1-(4\pi/\tau)^2}\right]^2, \quad (23)$$

where we have introduced the dimensionless time $\tau=\Omega t_f$, a measure of the total number of transverse oscillations during the acceleration process. The transverse mode excitation after the acceleration process is, apart from a constant factor, given by the shape function at $t=t_f$. This is plotted in Fig. 3 for the case of constant acceleration and also for a Blackmann pulse acceleration (23). From Fig. 3, we conclude that the shape function can be made less than 10^{-6} for $\tau=50$. This compares favorably with the case of constant acceleration, with a reduction in the excitation of the atoms or molecules by as many as six orders of magnitude. Figure 4 compares this two-mode perturbative result with a more complete nonperturbative numerical calculation taking 50 transverse modes into account and shows an excellent agreement between the two approaches for the parameters at hand.

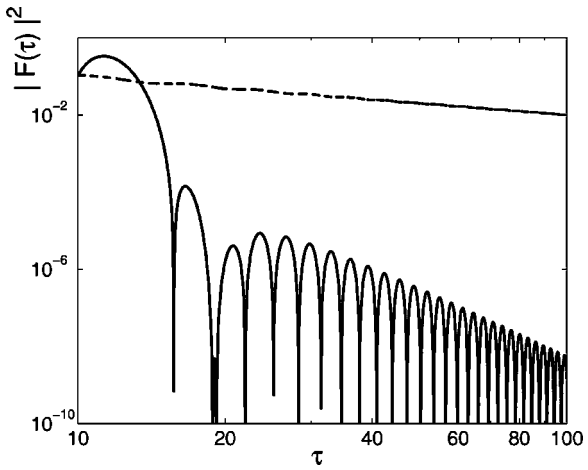

FIG. 3. Shape functions $F(\tau)$ at the end of acceleration process for an optimized Blackman pulse (solid line) and for the case of a constant acceleration (dashed line), as a function of the dimensionless final time $\tau=\Omega t_f$.

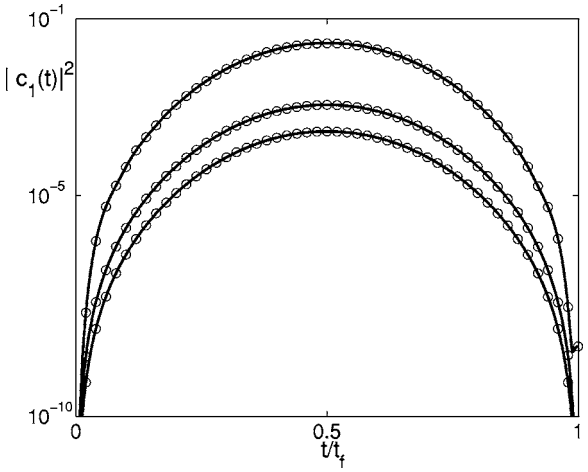

FIG. 4. Transverse excitation as a function of the time t/t_f. Solid lines: perturbative approach including only the first two transverse modes; open circles: nonperturbative calculation including 50 transverse modes. From top to bottom, the curves correspond to $\tau=\Omega t_f=100$, 500, 1000, and $l_f^2/\sqrt{\hbar\Omega^3m^3R^3}=10$.

III. CLASSICAL NOISE

In the previous section we investigated the effect of the accelerating force on the transverse dynamics of wavepackets under fully coherent conditions. Realistic accelerating fields are always somewhat noisy, resulting in fluctuations in the mean longitudinal angular momentum of the particles. In this section we discuss the effect of these fluctuations on the dynamics of the transverse wavepacket. For this purpose we assume that the angular momentum of the particle is of the form

$$l(t) = mR\langle v_z\rangle_{\rm cl}(t) + \delta l(t), \quad (24)$$

where $\delta l(t)$ is a Markovian Gaussian random process with

$$\langle \delta l(t)\rangle = 0, \quad (25)$$

$$\langle \delta l(t)\,\delta l(t')\rangle = \langle \delta l^2\rangle \exp[-(t-t')/t_c], \quad (26)$$

and the average is over an ensemble of realizations of the experiment and t_c is the noise correlation time.

Substituting this form of $l(t)$ into the transverse Schrödinger equation (11) then gives

$$i\hbar\frac{d\psi_\perp}{dt} = \left[\frac{p_x^2}{2m} + \frac{1}{2}m\Omega^2[x-x_0(t)]^2 + \delta V(t)x\right]\psi_\perp, \quad (27)$$

with

$$\delta V(t) = \frac{2l(t)\,\delta l(t)}{mR^3}. \quad (28)$$

The first two terms on the right-hand side of Eq. (27), describe the deterministic effect of the acceleration on the transverse state of the wavepacket, while the third term, which accounts for the classical noise, is readily seen to induce additional couplings between the transverse modes. In the presence of noise the state of the system is not expected to remain pure. We know from Sec. II that for realistic pa-

rameters, exact numerical results are in good agreement with a simplified perturbative analysis including the ground and first excited state of transverse excitation only. We therefore expand the density matrix $\rho(t)$ in terms of the time-dependent basis states (13) as

$$\rho = \sum_{i,j=0,1} \rho_{ij}\phi_i^*[x,x_0(t)]\phi_j[x,x_0(t)]. \tag{29}$$

This results in a Bloch vectorlike description of the transverse motion

$$\frac{d\mathbf{U}}{dt} = \left[g(t)\mathbf{A} - i\Omega\mathbf{C} - \frac{ia}{\hbar}\mathbf{B}\delta V(t) \right]\mathbf{U}, \tag{30}$$

where $\mathbf{U}=[\mathbf{w},\mathbf{u},\mathbf{v}]$ and

$$w = \rho_{00} - \rho_{11},$$

$$u = 2\,\mathrm{Re}(\rho_{01}),$$

$$v = 2\,\mathrm{Im}(\rho_{01}), \tag{31}$$

and the matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ matrices are given by

$$\mathbf{A} = \begin{bmatrix} 0 & \sqrt{2} & 0 \\ -\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \\ -\sqrt{2} & 0 & 0 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{32}$$

Averaging the stochastic differential equation (30) over a large number of realizations of the fluctuating potential $\delta V(t)$ with $\langle \delta V\rangle = 0$, we have [30]

$$\frac{d\langle \mathbf{U}\rangle}{dt} = \left[g(t)\mathbf{A} - i\Omega\mathbf{C} - \frac{a^2}{\hbar^2}\int_0^\infty \langle \delta V(t)\,\delta V(t-t')\rangle \times \mathbf{B}\mathcal{S}(t')\mathbf{B}\mathcal{S}^{-1}(t')dt' \right]\langle \mathbf{U}\rangle, \tag{33}$$

where

$$\mathcal{S}(t') = \exp\left(\int_0^{t'} [g(t)\mathbf{A} - i\Omega\mathbf{C}]dt \right). \tag{34}$$

The formal solution of Eq. (33) is of the general form

$$\langle U\rangle = \mathcal{M}\langle U(0)\rangle, \tag{35}$$

but for the initial condition $\mathbf{U}(0)=[1,0,0]$ appropriate for the problem at hand only the matrix element $\mathbf{M}_{11}$ is required. To evaluate the noise correlation function appearing in Eq. (33) we assume the noise to be broadband,

$$\langle V(t)V(t-t')\rangle = \langle \delta V^2\rangle \exp(-t'/t_c), \tag{36}$$

where t_c is the noise correlation time. After evaluating the integral, we find that to lowest order in perturbation theory the probability of transverse excitation $\epsilon = 1-\rho_{00}$ resulting from the classical noise $\delta V(t)$ is

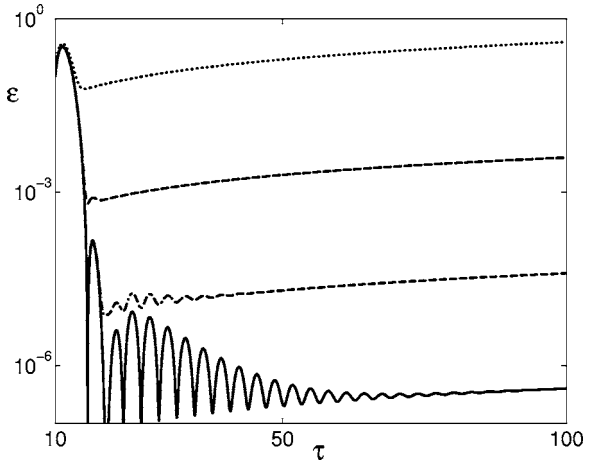

FIG. 5. Excitation probability in presence of classical noise for various values of $(\Omega t_c)\langle \delta v^2\rangle / v_f^2 = 10^{-4}$(dotted line), 10^{-6}(dashed line), 10^{-8}(dash-dotted line), 10^{-10}(solid line), and $mv_f^4/2\hbar\Omega^3R^2 = 10$.

$$\epsilon = \frac{1}{2}\left\{ 1 - \frac{e^{-\eta}}{1+\xi^2} - \frac{\xi^2 e^{-\eta/2}}{1+\xi^2}\left[\cos\left(\frac{\tau}{2}\sqrt{4(1+\xi^2) - (\eta/\tau)^2} \right) - \frac{\eta/\tau}{\sqrt{4(1+\xi^2) - (\eta/\tau)^2}} \sin((\tau/2)\sqrt{4(1+\xi^2) - (\eta/\tau)^2}) \right] \right\}, \tag{37}$$

where the noise coefficient η is

$$\eta = \frac{2a^2 t_c}{\hbar^2 m^2 R^6}\langle \delta l^2\rangle \int_0^{t_f} l^2(t')dt',$$

$$= \frac{l_f^2 \pi t_c}{\hbar\Omega^2 m^3 R^6}\langle \delta l^2\rangle \tag{38}$$

and the dimensionless function ξ is given by

$$\xi = \sqrt{2}\frac{\int_0^{t_f} g(t)dt}{\tau} = \frac{\sqrt{2}}{\tau}\frac{l_f^2}{\sqrt{\hbar\Omega^3 m^3 R^3}}, \tag{39}$$

and accounts for the transverse excitation resulting from the deterministic part of the acceleration. In the limit of large τ which typically results in a low excitation (see Fig. 3) we have that $\xi \ll 1$, $\eta/\tau \ll 1$ and Eq. (37) reduces to

$$\epsilon = \frac{l_f^4}{2\hbar\Omega^3 m^3 R^6}\left[2(\Omega t_c)\frac{\langle \delta l^2\rangle}{l_f^2}\tau + \frac{\sin^2\left(\frac{\tau}{2}\right)}{(\tau/2)^2} \right],$$

$$= \frac{mv_f^4}{2\hbar\Omega^3 R^2}\left[2(\Omega t_c)\frac{\langle \delta v^2\rangle}{v_f^2}\tau + \frac{\sin^2(\tau/2)}{(\tau/2)^2} \right], \tag{40}$$

where we have introduced the final velocity v_f and its fluctuations $\langle \delta v^2\rangle$. Figure 5 shows the excitation probability ϵ for various values of $(\Omega t_c)\langle \delta v^2\rangle / v_f^2$. The first term in Eq. (40) is due to the velocity fluctuations and increases with time, while the second term due to coherent dynamics, i.e., nona-

diabaticity, decreases with time as is expected from the adiabatic theorem of quantum mechanics. As a result the total excitation exhibits a minimum as a function of acceleration time, as can be seen in Fig. 5, which shows $\epsilon(\tau)$ for various values of the velocity fluctuations. The level of single-mode propagation that can be achieved is thus limited both by adiabaticity and velocity fluctuations.

IV. CONCLUSIONS

In this paper we have analyzed the influence of an accelerating classical, possibly noisy, force on ultracold atoms or molecules moving in a circular waveguide. A simplified two-state model for velocity dependent trap modes was derived and solved in the perturbative limit of weak coupling between these states. The excited state population was found to be minimized by choosing a Blackman pulse accelerating field. For such a choice the acceleration of molecules from initial velocities around the recoil velocity up to a speed of 10 m/s in a ring of 0.3 m radius over a time interval of 100 ms and with a radial trap frequency of 10 krad/s results in an excitation probability $\epsilon < 10^{-3}$. A similar excitation is obtained for an atomic storage ring with a radius of 5 cm, a target velocity of 20 cm/s, an acceleration time of 100 ms and a radial frequency of 1 krad/s. We remark that since Eqs. (16)–(18) hold whether considering a gradual increase or a decrease in longitudinal velocity, our analysis can also be applied to the (classical) deceleration of particles.

The effects of noise in the acceleration pulse can be modeled by incorporating the fluctuations in angular momentum, or equivalently longitudinal velocity, of the particle. We found that in this case a dominant contribution to the departure from single-mode dynamics arises from transitions to neighboring excited states due to velocity fluctuations. Generally speaking, the excitation probability consists of two contributions, one due to the fluctuations and one due to coherent nonadiabaticity. Their combined effect results in the appearance of an optimum excitation time, after which classical fluctuations dominate and lead to an increasing departure from single-mode operation.

As a concluding remark, we note that for classical systems the noise description of Sec. III is identical to that of thermal fluctuations. If one assumes for simplicity that the velocity fluctuations to be of thermal origin—although this is not strictly valid for quantum degenerate systems—this analogy can be used to estimate the effects of nonzero temperature on single-mode operation. For a temperature of 1 μK a final longitudinal velocity of 10 m/s, and $(\Omega t_c)\langle \delta v^2 \rangle / v_f^2 = 10^{-8}$, this approximate argument implies that the optimal single-mode acceleration of the system reached for $\tau_f \sim 20$.

ACKNOWLEDGMENTS

This work is supported in part by the U.S. Office of Naval Research, by the National Science Foundation, by the U.S. Army Research Office, by the Joint Services Optics Program, and by the National Aeronautics and Space Administration.

[1] R. Folman, P. Krüger, J. Denschlag, C. Henkel, and J. Schmiedmayer, Adv. At., Mol., Opt. Phys. **48**, 263 (2002); J. Reichel, Appl. Phys. B: Lasers Opt. **74**, 469 (2002).
[2] Eur. Phys. J. D **35**, 1 (2005), special issue on atom chips, edited by C. Henkel, J. Scmiedmayer, and C. Westbrook.
[3] J. A. Sauer, M. D. Barrett, and M. S. Chapman, Phys. Rev. Lett. **87**, 270401 (2001).
[4] S. Wu, W. Rooijakkers, P. Striehl, and M. Prentiss, Phys. Rev. A, **70**, 013409 (2004).
[5] S. Gupta, K. W. Murch, K. L. Moore, T. P. Purdy, and D. M. Stamper-Kurn, Phys. Rev. Lett. **95**, 143201 (2005).
[6] A. S. Arnold, C. S. Garvie, and E. Riis, Phys. Rev. A **73**, 041606(R) (2006).
[7] C. A. Stan, M. W. Zwierlein, C. H. Schunck, S. M. F. Raupach, and W. Ketterle, Phys. Rev. Lett. **93**, 143001 (2004).
[8] S. Inouye, J. Goldwin, M. L. Olsen, C. Ticknor, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **93**, 183201 (2004).
[9] A. J. Kerman, J. M. Sage, S. Sainis, T. Bergeman, and D. DeMille, Phys. Rev. Lett. **92**, 033004 (2004); **92**, 153001 (2004); T. Bergeman, A. J. Kerman, J. Sage, S. Sainis, and D. DeMille, Eur. Phys. J. D **31**, 179 (2004).
[10] Topical issue on Ultracold Dipolar Molecules, Eur. Phys. J. D **31**, 149 (2004), edited by J. Doyle, B. Friedrich, R. V. Krems, and F. Masnou-Seeuws.
[11] S. Kotochigova, P. S. Julienne, and E. Tiesinga, Phys. Rev. A **68**, 022501 (2003).
[12] See, e.g., F. M. H. Crompvoets, R. T. Jongma, H. L. Bethlem, Andr J. A. van Roij, and G. Meijer, Phys. Rev. Lett. **89**, 093004 (2002); M. R. Tarbutt, H. L. Bethlem, J. J. Hudson, V. L. Ryabov, V. A. Ryzhov, B. E. Sauer, G. Meijer, and E. A. Hinds, Phys. Rev. Lett. **92**, 173002 (2004); F. M. H. Crompvoets, H. L. Bethlem, J. Küpper, A. J. A. van Roij, and G. Meijer, Phys. Rev. A **69**, 063406 (2004).
[13] T. Junglen, T. Rieger, S. A. Rangwala, P. W. H. Pinkse, and G. Rempe, Phys. Rev. Lett. **92**, 223001 (2004).
[14] D. P. Katz, J. Chem. Phys. **107**, 8491 (1997).
[15] F. M. H. Crompvoets, H. L. Bethlem, R. T. Jongma, and G. Meijer, Nature (London) **411**, 174 (2001).
[16] A. Öttl, S. Ritter, M. Köhl, and T. Esslinger, Phys. Rev. Lett. **95**, 090404 (2005).
[17] H. Mabuchi, Q. A. Turchette, M. S. Chapman, and H. J. Kimble, Opt. Lett. **21**, 1393 (1996).
[18] P. Münstermann, T. Fischer, P. W. H. Pinske, and G. Rempe, Opt. Commun. **159**, 63 (1999).
[19] O. Dutta, M. Jääskeläinen, and P. Meystre, Phys. Rev. A **71**, 051601(R) (2005).
[20] W. Ketterle, D. S. Durfee, and D. M. Stamper-Kurn, e-print cond-mat/9904034.
[21] C. Henkel and S. Pötting, Appl. Phys. B: Lasers Opt. **72**, 73 (2001).

[22] C. Henkel, P. Krüger, R. Folman, and J. Schmiedmayer, Appl. Phys. B: Lasers Opt. **76**, 173 (2003).
[23] C. Schroll, W. Belzig, and C. Bruder, Phys. Rev. A **68**, 043618 (2003).
[24] M. Jääskeläinen and S. Stenholm, Phys. Rev. A **66**, 023608 (2002).
[25] M. W. J. Bromley and B. D. Esry, Phys. Rev. A **68**, 043609 (2003).
[26] W. Hänsel, J. Reichel, P. Hommelhoff, and T. W. Hänsch, Phys. Rev. A **64**, 063607 (2001).
[27] B. Friedrich, Eur. Phys. J. D **31**, 313 (2004).
[28] E. Joos, H. D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, and I.-O. Stamatescu, *Decoherence and the Emergence of the Classical World* (Springer, Berlin, 2003).
[29] R. B. Blackmann and J. W. Tukey, *The Measurement of Power Spectra from the Point of View of Communications Engineering* (Dover, New York, 1959).
[30] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1981).