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Testing Seasonal Unit Roots in Data at Any Frequency, an HEGY approach

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Abstract

This paper generalizes the HEGY-type test to detect seasonal unit roots in data at any frequency, based on the seasonal unit root tests in univariate time series by Hylleberg, Engle, Granger and Yoo (1990). We introduce the seasonal unit roots at first, and then derive the mechanism of the HEGY-type test for data with any frequency. Thereafter we provide the asymptotic distributions of our test statistics when different test regressions are employed. We find that the F-statistics for testing conjugation unit roots have the same asymptotic distributions. Then we compute the finite-sample and asymptotic critical values for daily and hourly data by a Monte Carlo method. The power and size properties of our test for hourly data is investigated, and we find that including lag augmentations in auxiliary regression without lag elimination have the smallest size distortion and tests with seasonal dummies included in auxiliary regression have more power than the tests without seasonal dummies. At last we apply the our test to hourly wind power production data in Sweden and shows there are no seasonal unit roots in the series.

1 Introduction

Data collected periodically usually exhibit seasonality. The data would appear seasonality if the spectrum of the process have peaks at certain frequencies. Modeling seasonality has been a commonly used method for dealing with such kind of data. There are 3 approaches that are most widely used for modeling seasonal time series: deterministic seasonal processes, stationary and nonstationary seasonal processes. The differences lie in how they react to the shocks to the seasonal patterns. In deterministic seasonal processes shocks have no effect on the seasonal pattern, and in stationary seasonal processes they have temporary effect which would diminish with time passes by. But in nonstationary processes the shocks have non-diminishing effect, causing permanent changes to the seasonal pattern and increasing variance of the series. Therefore, the nonstationary seasonal process raises the most concern and testing seasonal unit roots has high priority in the modeling procedure. The misspecification of the type of seasonality would cause severe bias in modeling and forecasting process.

There are many tests that are proposed for testing seasonal unit roots such as Dickey-Hasza-Fuller (DHF) test which is by Dickey, Hasza and Fuller (1984), OCSB test which is proposed by

Osborn, Chui, Smith and Birchenhall (1988). Among these seasonal unit root tests, the HEGY test which is posed by Hylleberg, Engle, Granger and Yoo (1990) has the advantage of testing seasonal unit root at each frequency separately, thus it is widely applied. The HEGY test is firstly proposed for testing seasonal unit roots in quarterly data, Franses (1990), Beaulieu and Miron (1992) extend it to monthly data. However, the HEGY test is not available for testing seasonal unit roots in data at other frequencies such as hourly data and daily data, therefore it is imperative to extend the test to data with other frequencies, which is the focus of this paper. In this paper we propose an HEGY type test for testing seasonal unit root in data with any frequency. Centering on the test we proposed, we provide the test procedure, asymptotic distributions of statistics, and analyze the power and size of our test on hourly data. Based on the power and size properties we compare the performance of different methods of choosing lag augmentations and the performance of the test when deterministic components are included or not. In the end we apply our test to the hourly wind power production data in Sweden and find there are no seasonal unit roots in the series.

The rest of the paper is organized as follows: Section 2 introduces the seasonal unit roots. In section 3 test equations and the procedure for testing seasonal unit roots are presented. The asymptotic distributions of the test statistics are also given in this section. Section 4 provides the finite and asymptotic critical values for HEGY test for hourly and daily data. In section 5 finite sample properties of tests are investigated. In section 6 we apply our test to hourly wind power production data in Sweden. Concluding remarks are given in section 7.

2 Seasonal unit roots

Consider a basic autoregressive polynomial $\varphi(B)$ having a form,

$$\varphi(B) = 1 - B^S \tag{2.1}$$

where B is the lag operator, and "S" is the number of time periods in a seasonal pattern which repeats regularly. For example, S=4 for quarterly data where the seasonal pattern repeats itself every year, and S=24 for hourly data. "S" could also be an odd number, such as S=7 for daily data. The equation $\varphi(z) = 0$ has S roots on the unit circle:

$$z_k = e^{\frac{2\pi i}{S}k} = cos(\frac{2k\pi}{S}) + isin(\frac{2k\pi}{S}), k = 0, 1, 2, ..., (S-1) \tag{2.2}$$

where i is the imaginary unit. Each root z_k in (2.2) is related to a specific frequency $\frac{2k\pi}{S}$. When $k = 0$, the root z_k in (2.2) is called non-seasonal unit root. The other roots z_k in (2.2) are called seasonal unit roots.

Except for roots z_k in (2.2) at frequencies 0 and π, z_k in (2.2) at the others frequencies are pairs of conjugation frequencies. We re-order the S frequencies of z_k by putting their conjugation frequencies together.

a) When S is an even number, the S frequencies are ordered as:

$$\theta_m = \begin{cases} 0 & m = 1 \\ \frac{m-1}{S}\pi & m = 2, 4, ..., (S-2) \\ 2\pi - \frac{m-2}{S}\pi & m = 3, 5, ..., (S-1) \\ \pi & m = S \end{cases} \tag{2.3}$$

In (2.3), θ_m and θ_{m+1} are conjugation frequencies if $m = 2, 4, ..., (S-2)$.

b) When S is an odd number, there is no unit root at frequency π, the S frequencies are ordered as:

$$\theta_m = \begin{cases} 0 & m = 1 \\ \frac{m}{S}\pi & m = 2, 4, ..., (S-1) \\ 2\pi - \frac{m-1}{S}\pi & m = 3, 5, ..., S \end{cases} \tag{2.4}$$

In (2.4), θ_m and θ_{m+1} are conjugation frequencies if $m = 2, 4, ..., (S-1)$.

For both cases (2.3) and (2.4), the unit roots corresponding to frequency θ_m are:

$$u_m = cos\theta_m + isin\theta_m \tag{2.5}$$

The frequencies θ_m of also indicate the number of cycles for u_m in the seasonal pattern, which are derived by $\frac{\theta_m S}{2\pi}$. For example, consider hourly data where S=24, setting m=2, $u_2 = cos\frac{\pi}{12} + isin\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4}i$. Its frequency is $\theta_m = \frac{\pi}{12}$, and it corresponds to $\frac{\theta_m S}{2\pi}$=1 cycle in every 24 hours.

We make the following notations for simplification. For $m = 1$ we still use $m = 1$. For $m = S$ when S is even, denote $m = \pi$. For the rest, when m is even, i.e., $m = 2, 4, ..., (S-2)$ in (2.3), and $m = 2, 4, ..., (S-1)$ in (2.4), denote $m = m_{even}$; when m is odd, i.e., $m = 3, 5, ..., (S-1)$ in (2.3) and $m = 3, 5, ..., S$ in (2.4), denote $m = m_{odd}$. The notations are are used throughout the paper.

As discussed above, the seasonal unit roots in time series would permanently change the seasonal patterns of the series and make the variance of the series increase linearly. Therefore testing seasonal unit roots proceeds modeling seasonality. However the HEGY test is only availiable for data at certain frequencies, such as quarterly and monthly. In order to detect seasonal unit roots in data with any frequency, we extend the HEGY test to data at any frequency in the following section.

3 HEGY-type test

3.1 The HEGY-type testing equations

Let $\{y_t : t \in Z_+\}$ be a univariate time series satisfying a pth-order autoregressive model:

$$\varphi(B)y_t = \varepsilon_t \tag{3.1}$$

where $\varphi(B)$ is the autoregressive polynomial with order p, $S \leq p \leq \infty$, and $\{\varepsilon_t\}$ is a sequence of independent and identically distributed variables with mean 0 and variance σ^2 $(0 < \sigma^2 < \infty)$, denoted as $\varepsilon_t \sim iid(0, \sigma^2)$. Note that the case for $p = \infty$ is discussed in Section 5.

To carry out a seasonal unit root test at any frequency, we begin with the decomposition theory of the polynomial $\varphi(B)$ in (3.1) by following the decomposition technique in HEGY (1990), and it is stated in Lemma 1.

Lemma 1: Consider the autoregressive polynomial $\varphi(u)$ in the model (3.1). Assume that $\varphi(u)$ is expanded at the S unit roots u_m in (2.5), $m = 1, ..., S$. Then $\varphi(u)$ in (3.1) can be decomposed as:

$$\varphi(u) = \sum_{m=1}^{S} \tau_m \varphi_m(u) + \varphi^*(u)(1 - u^S) \tag{3.2}$$

where $\varphi_m(u) = \frac{u}{u_m} \prod_{j=1, j\neq m}^{S} (1 - \frac{u}{u_j})$ for $m = 1,...,S$, and $\varphi^*(B)$ in (3.2) is a remainder polynomial with order $p - S$. For details of the proof of Lemma 1, see Appendix I.

The following testing equation of the HEGY-type test is derived by applying the decomposition in Lemma 1.

Lemma 2: Consider a univariate seasonal time series$\{y_t : t \in Z_+\}$ with frequency S. Assume y_t satisfies the autoregressive model (3.1). Then the model (3.1) has the following form:

$$\varphi^*(B)(1 - B^S)y_t = \sum_{m=1}^{s} \rho_m x_{m,t} + \varepsilon_t \tag{3.3}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$ and $x_{m,t} = \zeta_m(B)y_t$, whereas

$$\zeta_m(B) = \begin{cases} \sum_{j=1}^{S} \cos(j\theta_m)B^j & m = 1, m_{even}, \pi \\ \sum_{j=1}^{S} \sin(j\theta_{m-1})B^j & m = m_{odd} \end{cases} \tag{3.4}$$

Proof: See Appendix I.

The model (3.3) can be used for testing the seasonal unit roots of $\{y_t\}$ in (3.1). For details, we shall discuss as follows:

(a) Misspecification of $\varphi^*(B)$: Assume that the order of $\varphi(B)$ in (3.1) satisfies $S \leq p \leq \infty$. By Lemma 1 the order of the remainder polynomial $\varphi^*(B)$ is $p - S$ and $p - S \geq 0$. If $\varphi^*(B)$ is chosen to be constant while in fact it is not, the residuals of the regression (3.3) are serially correlated. The details of choosing $\varphi^*(B)$ in (3.3) in practical issue will be discussed in the subsection 4.2.

(b) Properties of regressors: It follows from Lemma 2 that the S regressors $x_{m,t}$ in (3.3) are orthogonal to each other, which means $\sum_t x_{m_1,t} x_{m_2,t} = 0$ when $m_1 \neq m_2$. Each regressor x_m is related to the specific frequency θ_m. In practical issue, we have the observed univariate series y_t, and the regressors $x_{m,t}$ are obtained by operating y_t with the filters $\zeta_m(B)$. Noting that in (3.3), we can express $x_{m_{even}}$ by the $x_{m_{odd}}$ which correspond to its conjuagation frequencies, and the relationship could be formulated by $x_{m_{even},t} = x_{m_{odd},,t}(\frac{cos\theta_{m_{even}}}{sin\theta_{m_{even}}} - \frac{B}{sin\theta_{m_{even}}})$. For example, consider the example above where $m = 2$ for hourly data, $x_{2,t} = (2+\sqrt{3})x_{3,t} - (\sqrt{6}+\sqrt{2})x_{3,t-1}$. Thus, we could derive $x_{m_{odd}}$ first and then use them to derive $x_{m_{even}}$.

(c) Testing seasonal unit root. For the polynomial $\varphi(z)$ in (3.1), $\varphi(z) = 0$ has a unit root at frequency θ_m if and only if the parameter of the related regressor $x_{m,t}$ in (3.3) equals to 0.

Thus testing for presence of seasonal unit roots for data at frequency θ_m are equivalent to test if the corresponding parameters ρ_m of $x_{m,t}$ in (3.3) are zero.

For the testing procedures, we give the remarks below:

(d) Estimation: Assume that the residuals ε_t in (3.3) are $iid(0, \sigma^2)$ and all roots of $\varphi^*(B)$ lie outside the unit circle. The parameters in auxiliary regression (3.3) can be estimated by the ordinary least squares method.

(e) Testing unit root at frequency 0 and π. The parameter ρ_1 corresponds to the unit root at frequencies 0 and ρ_π corresponds to the unit root at frequencies π. To verify the presence of unit roots, we need to test if the 2 parameters equal to 0. The null hypothesis is

$$H_{0m} : \rho_m = 0$$

against the alternative hypothesis $H_{am} : \rho_m < 0$, where $m = 1\, or\, \pi$. The test statistics used here are t-statistics: $t_m = \hat{\rho}_m / \hat{\sigma}_{\rho m}$, where $\hat{\rho}_m$ is the OLS estimator of ρ_m and $\hat{\sigma}_{\rho m}$ is the sample standard error of $\hat{\rho}_m$. If the null hypothesis is not rejected, the test indicates that the unit root exists at that frequency. When S is odd, we only need to test if the first parameter ρ_1 equals to 0.

(f) Testing complex unit roots. Due to the fact that these pairs of unit roots are conjugates, the regressors appear in pairs and correspond with frequencies in pairs. Thus, only that both parameters are zero could proof the existence of unit roots. This leads to the joint test of each pair. The null hypothesis is:

$$H_{0m} : \rho_m = \rho_{m+1} = 0$$

The alternative hypothesis $H_{am} : \rho_m \neq 0 or \rho_{m+1} \neq 0$, where $m = m_{even}$. The F-statistics:$F_{m,m+1} = \frac{1}{2}(t_m^2 + t_{m+1}^2)$ are used, where the t-statistics t_m and t_{m+1} are derived in the same way as t_1 and t_π. If the null hypothesis is not rejected, the test indicates unit roots exist at the corresponding 2 frequencies. The proof of deriving F-statistics is given in Lemma 3.

Lemma 3:

$$F_{m,m+1} = \frac{1}{2}(t_m^2 + t_{m+1}^2)$$

Proof: The F-statistics could be derived by :

$$F_m = \frac{1}{2\hat{\sigma}^2}(R_m\beta - r)'[R_m(X'X)^{-1}R_m^{'}]^{-1}(R_m\beta - r)$$

where $\beta = [\rho_1, \rho_2, ..., \rho_S]'$, $r = [0, 0]'$, $R_m = [u_m, u_{m+1}]'$ with u_i is an S-vector with 1 in m^{th} element and 0 elsewhere, $X = [x_1, x_2, ..., x_S]$ with $x_i = [x_{i,1}, x_{i,2}, ..., x_{i,T}]'$, $i = 1, 2, ..., S$.

Considering x_i are orthogonal with each other, we have $[R_m(X'X)^{-1}R_m^{'}]^{-1} = diag(\sum_{j=1}^T x_{m;j}^2, \sum_{j=1}^T x_{m+1}^2$

Thus $F_m = \frac{1}{2\hat{\sigma}^2}(\rho_m^2 \sum_{j=1}^T x_{m;j}^2 + \rho_{m+1}^2 \sum_{j=1}^T x_{m+1;j}^2) = \frac{1}{2}(t_m^2 + t_{m+1}^2)$.

Another strategy for testing conjugate unit roots is to test $H_{0(m+1)}^* : \rho_{m+1} = 0$ against $H_{a(m+1)}^* : \rho_{m+1} \neq 0$ by t-type statistics t_{m+1} in (e), where $m = m_{odd}$. If the null hypothesis is not rejected, then examine $H_{0m}^* : \rho_m = 0$ against $H_{am}^* : \rho_m < 0$ by t-type statistics t_m in (e). If the null hypothesis is not rejected again, there are unit roots at the corresponding 2 frequencies.

Compared with this strategy, using F-statistics have simpler procedures, thus we focus on the strategy of using F statistics in this paper.

An advantage of the testing procedure is that we only need to estimate (3.3) one time to test the S unit roots. Based on the test results, we are able to choose appropriate differencing filter to render the series stationary. The existence of unit root at frequency 0 and π indicates the filter $(1 - B)$ and $(1 + B)$, and the existence of unit roots at conjugation frequencies θ_m and θ_{m+1} indicates the filter $(1 - cos\theta_m B)(1 - cos\theta_{m+1} B)$. The operator that we choose to difference the data would be the mutiplication of the filters that corresponding to the extisting unit roots.

(g) Testing seasonal integration at order S. The HEGY-type test could test the presence of all the S unit root as a whole, and this leads to a joint test for all the S parameters. The null hypothesis is:

$$H_0 : \rho_1 = \rho_2 = ... = \rho_S = 0$$

against the alternative H_a :The series is seasonally stationary. An F-type test statistic is used here: $F_{all} = \frac{1}{S}\sum_{m=1}^{S} t_m^2$. The proof of the equation is similar with that of Lemma 3. The overall test is sensitive to the absence of unit roots at certain frequencies, because stationarity at one or a pair of frequencies could lead to invalidity of the null hypothesis while there are unit roots at all the other frequencies. Therefore, one should consider both the F_{all} test in (g) and the joint F test in (f) to decide if the operator $1 - B^S$ is needed to render the series stationary.

(h) Deterministic component included in test equation. Our test also applies when there is deterministic components in the series. In this case test equation (3.3) is amended to contain deterministic components such as constant, time trend and seasonal dummies, which lead to test equation (3.5).

$$\varphi^*(B)(1 - B^S)y_t = \sum_{m=1}^{S} \rho_m x_{m,t} + c_0 + c_1 t + \sum_{i=2}^{S} c_i D_{i,t} + \varepsilon_t \tag{3.5}$$

where c_0 is constant, t is the time trend, $D_{i,t}, i = 2, 3, ..., S$ are seasonal dummy variables which equals to 1 if y_t is at the i^{th} time unit in a seasonal period and 0 elsewhere. When (3.5) is employed in our test, the test procedures (d)-(g) are still the same as the case when (3.3) is employed for the test, but the distributions of the test statistics change, see more discussions in the following subsection.

3.2 Asymptotic distributions of the HEGY type test statistics

This subsection gives the asymptotic distributions of the test statistics in subsection 3.1. The asymptotic distributions of those test statistics are derived by following Beaulieu and Miron (1992), Chan and Wei (1988) and Hamilton (1994). First, Theorems 1 gives the asymptotic distributions of the t-statistics when no deterministic component exits in test regression (3.3). The asymptotic distributions of F-statistics could be derived by the asymptotic distributions of t-statistics. Second, Theorems 3 gives the asymptotic distributions of test statistics when different deterministic components are included in testing equation (3.5).

In order to derive the asymptotic distributions of our test statistics, the following assumptions are needed:

Assumption 1: In (3.3) and (3.5), $\{\varepsilon_t\}$ is martingale differences with respect to an increasing sequence of $\sigma-$fields $\{F_t\}$ satisfying the 2 conditions below:

$$E\{\varepsilon_t^2 \mid F_{t-1}\} = \sigma^2 \; a.s.$$

$$\sup_t E\{|\varepsilon_t|^{2+\delta} \mid F_{t-1}\} < \infty \; a.s., \; for \; some \; \delta > 0$$

where σ^2 is finite.

Assumption 2: Consider the autoregressive polynomial $\varphi(z)$ in (3.1), it is assumed there are no repeated unit roots for $\varphi(z) = 0$ at the frequencies $\theta_m, m = 1, 2, ..., S$.

This assumption ensures that $\varphi(z_m) = 0$ if and only if the parameter of the related regressor $x_{m,,t}$ equals to 0, thus it ensures that testing for presence of seasonal unit roots is equivalent to testing if the corresponding parameters in (3.3) are zero.

Assumption 3: All the roots of $\varphi^*(Z) = 0$ lie outside the unit circle.

In a special case when $\varphi(B) = (1-B^S)(1-\beta_1 B - \beta_2 B^2 - \cdots - \beta_p B^p)$, this assumption ensures all roots of $1 - \beta_1 B - \beta_2 B^2 - \cdots - \beta_p B^p = 0$ lie outside the unit circle. The augmentations do not affect the asymptotic distributions according to Beaulieu and Miron (1992), as long as they are correctly specified.

With the assumptions above, we derive the Theorem 1:

Theorem 1: Consider the regression model (3.3) with assumption 1-3 fulfilled. Under the hypothesis $H_{0m} : \rho_m = 0, m = 1, \pi$, the t-statistics t_m have the asymptotic distributions below;

$$t_m \xrightarrow{L} \frac{\int_0^1 W_m(r)dW_m(r)}{(\int_0^1 W_m(r)^2 dr)^{1/2}}$$

Under the hypothesis $H_{0m} : \rho_m = \rho_{m+1} = 0$, $m = m_{even}$, the t-statistics have the asymptotic distributions below:

$$t_m \xrightarrow{L} \begin{cases} \frac{\int_0^1 W_m(r)dW_m(r) + \int_0^1 W_{m+1}(r)dW_{m+1}(r)}{[\int_0^1 W_m(r)^2 dr + \int_0^1 W_{m+1}(r)^2 dr]^{1/2}} & m = m_{even} \\ \frac{\int_0^1 W_m(r)dW_{m-1}(r) - \int_0^1 W_{m-1}(r)dW_m(r)}{[\int_0^1 W_{m-1}(r)^2 dr + \int_0^1 W_m(r)^2 dr]^{1/2}} & m = m_{odd} \end{cases}$$

where "$\xrightarrow{L}$" stands for converge in distribution, $W_m, m = 1, m_{even}, m_{odd}, \pi$ are mutually independent standard Brownian motions. For details of proof see Appendix II.

We can see that t_1 and t_π have the same asymptotic distribution, and the asymptotic distributions of $t_{m_{even}}$ are not the same with $t_{m_{odd}}$. The F-statistics for joint test $F_{m,m+1} = \frac{1}{2}(t_m^2 + t_{m+1}^2)$, $m = m_{even}$ have the same asymptotic distributions. The asymptotic distribution of F-statistic for integration at order S is retrieved in the similar way as $F_{m,m+1}$ by $F_{all} = \frac{1}{S}\sum_{m=1}^{S} t_m^2$, which varies across different S.

Next we consider the case when there are deterministic components in the test equation, i.e., (3.5) is employed for test. Theorem 2 gives us the asymptotic distributions for the t-statistics.

Theorem 2. Consider the regression model (3.5) with assumptions 1-3 fulfilled. Under the hypothesis $H_{0m} : \rho_m = 0, m = 1, \pi$, the t-statistics t_m have the asymptotic distributions below;

$$t_1 \xrightarrow{L} \frac{\int_0^1 W_m(r)dW_m(r) - W_m(1)\int_0^1 W_m(r)dr + 1^t W_N^*}{[\int_0^1 W_m(r)^2 dr - (\int_0^1 W_m(r)dr)^2 + 1^t W_N^{**}]^{1/2}}$$

$$t_\pi \xrightarrow{L} \frac{\int_0^1 W_m(r)dW_m(r) - 1^\mu W_m(1)\int_0^1 W_m(r)dr}{[\int_0^1 W_m(r)^2 dr - 1^\mu(\int_0^1 W_m(r)dr)^2]^{1/2}}$$

Under the hypothesis $H_{0m} : \rho_m = \rho_{m+1} = 0$, $m = m_{even}$, the t-statistics have the asymptotic distributions below:

$$t_m \xrightarrow{L} \begin{cases} \frac{\int_0^1 W_m(r)dW_m(r) + \int_0^1 W_{m+1}(r)dW_{m+1}(r) + 1^\mu W_{cos}^*}{[\int_0^1 W_m(r)^2 dr + \int_0^1 W_{m+1}(r)^2 dr + 1^\mu W_{cos}^{**}]^{1/2}} & m = m_{even} \\ \frac{\int_0^1 W_m(r)dW_{m-1}(r) - \int_0^1 W_{m-1}(r)dW_m(r) + 1^\mu W_{sin}^*}{[\int_0^1 W_m(r)^2 dr + \int_0^1 W_{m-1}(r)^2 dr + 1^\mu W_{sin}^{**}]^{1/2}} & m = m_{odd} \end{cases}$$

where,

$W_N^* = 3W_1(1)\int_0^1 W_1(r)dr - 6W_1(1)\int_0^1 rW_1(r)dr - 6[\int_0^1 W_1(r)dr]^2 + 12\int_0^1 rW_1(r)dr \int_0^1 W_1(r)dr$
$W_N^{**} = -3(\int_0^1 W_1(r)dr)^2 + 12\int_0^1 W_1(r)dr \int_0^1 rW_1(r)dr - 12(\int_0^1 rW_1(r)dr)^2$
$W_{cos}^* = -W_m(1)\int_0^1 W_m(r)dr - W_{m+1}(1)\int_0^1 W_{m+1}(r),$
$W_{cos}^{**} = -(\int_0^1 W_m(r)dr)^2 - (\int_0^1 W_{m+1}(r)dr)^2$
$W_{sin}^* = -W_m(1)\int_0^1 W_{m-1}(r)dr - W_{m-1}(1)\int_0^1 W_m(r)dr,$
$W_{sin}^{**} = -(\int_0^1 W_{m-1}(r)dr)^2 - (\int_0^1 W_m(r)dr)^2$

1^t and 1^μ are indicator functions, $1^t = 1$ if trend is included in (3.5) and 0 if trend is not included. $1^\mu = 1$ if seasonal dummies are included in (3.4) and 0 elsewhere. For proof of Theorem 2, see Appendix III.

We can see in Theorem 2, different deterministic components included in the testing equation would affect the asymptotic distributions of the t-statistics. The included trend only affect the distributions of t_1, and the dummies included would affect the distributions for all the t-statistics except t_1. Thus the inclusion of seasonal dummies would affect the asymptotic distributions of $F_{m,m+1}$, $m = m_{even}$. Similar with the conclusion from Theorem 1, the F-statistics $F_{m,m+1}$ have the same asymptotic distributions because of the same distribution for the t-statistics when $m = m_{even}$ and $m = m_{odd}$. F_{all} has different distributions for different S.

4 Critical values for hourly and daily data.

In this part we focus on the critical values for the HEGY-type test at different frequencies. In this section we give the asymptotic and finite-sample critical values of the test statistics in section 3 for hourly data first, and then give those for the test statistics of daily data.

4.1 Hourly data.

For hourly data, there are 24 frequencies, $\frac{k}{12}\pi, k = 0, 1, ...24$. We have to test unit root at frequency 0, π and 11 pairs of conjugation frequencies. As stated in Section 2, , the frequencies as following: m=1 for frequency 0, m=2,4,...,22 ($m = m_{even}$) for frequency $\frac{1}{12}\pi, \frac{2}{12}\pi, ..., \frac{11}{12}\pi$, and m=3,5,...,23 ($m = m_{odd}$) for $\frac{23}{12}\pi, \frac{22}{12}\pi, ..., \frac{13}{12}\pi$, m=24 for frequency π. The finite-sample critical values for our test are derived by simulating data from the model $y_t = y_{t-24} + \varepsilon_t$, where $\varepsilon_t \sim N(0, 1)$, then estimate the test regression (3.3) and (3.5) to get the value of statistics. The procedure is repeated 10000 times, yielding the finite-sample critical values. The critical values of $F_{m,m+1}$, $m = m_{even}$ are derived by combining the 11 F-statistics that have the same asymptotic distributions and get quantiles. The critical values of the asymptotic distributions are calculated by letting T=5000 to simulate a Brownian motion $W(r)$ on [0,1], the number of replications are set to 10,000. The finite sample and asymptotic critical values when (3.3) is employed for test are reported in the Table 1-4. Because different deterministic components in (3.5) would lead to different distributions of the statistics, we provide the distributions of 4 situations when different deterministic components are included in (3.5): only intercept included, intercept and trend included, intercept and seasonal dummies included, intercept, trend and seasonal dummies included. For the critical values of these 4 cases see Appendix V.

Table 1: Critical values for t_1 of hourly data. No deterministic part included

Sample size T	Probability that t_1 is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-2.29	-1.96	-1.68	-1.38	0.89	1.28	1.60	2.00
T=240	-2.44	-2.08	-1.81	-1.49	0.90	1.29	1.60	1.95
T=360	-2.43	-2.10	-1.82	-1.50	0.89	1.28	1.59	1.95
T=480	-2.44	-2.12	-1.82	-1.52	0.89	1.27	1.59	1.96
T= ∞	-2.51	-2.20	-1.95	-1.64	0.86	1.25	1.62	1.98

Table 2: Critical values for the t_{24} of hourly data. No deterministic part included

Sample size T	Probability that t_{24} is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-2.35	-1.99	-1.72	-1.42	0.90	1.25	1.58	2.02
T=240	-2.49	-2.13	-1.85	-1.51	0.90	1.26	1.61	2.00
T=360	-2.55	-2.18	-1.89	-1.53	0.89	1.27	1.61	2.00
T=480	-2.61	-2.23	-1.92	-1.57	0.88	1.26	1.62	2.00
T= ∞	-2.52	-2.22	-1.93	-1.60	0.88	1.27	1.64	2.00

Table 3: Critical values for the $F_{m,m+1}, m = m_{even}$ of hourly data. No deterministic part included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.01	0.02	0.05	0.10	2.06	2.67	3.29	4.11
T=240	0.01	0.03	0.05	0.11	2.23	2.88	3.53	4.40
T=360	0.01	0.03	0.05	0.11	2.25	2.90	3.59	4.51
T=480	0.01	0.03	0.05	0.11	2.34	3.01	3.69	4.57
T= ∞	0.01	0.03	0.05	0.12	2.40	3.06	3.74	4.83

Table 4: Critical values for the F_{all} of hourly data. No deterministic part included

Sample size T	Probability that $F_{all} = \frac{1}{24}\sum_1^{24} t_m^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.39	0.45	0.51	0.58	1.28	1.41	1.55	1.75
T=240	0.44	0.50	0.56	0.63	1.36	1.51	1.64	1.79
T=360	0.45	0.51	0.57	0.66	1.40	1.54	1.65	1.85
T=480	0.47	0.53	0.59	0.67	1.43	1.57	1.68	1.87
T= ∞	0.49	0.56	0.63	0.71	1.46	1.59	1.74	1.89

4.2 Daily data.

For daily data, there are 7 frequencies, $\frac{k}{7}\pi, k = 0, 1, ...6$, we have to test unit roots at frequency

0 and 3 pairs of conjugation frequencies. The frequencies are arranged as introduced in Section 2, i.e., m=1 for frequency 0, m=2,4,6 ($m = m_{even}$) for frequency $\frac{2}{7}\pi, \frac{4}{7}\pi, ..., \frac{6}{7}\pi$, and m=3,5,7, ($m = m_{odd}$) for $\frac{5}{7}\pi, \frac{3}{7}\pi, \frac{1}{7}\pi$. The data generating process is the same with that for hourly data, except that there regressors are different according to (3.4) and the model is $y_t = y_{t-7} + \varepsilon_t$, where $\varepsilon_t \sim N(0,1)$. The asymptotic distributions of the test statistics are derived in the same way as in hourly data. The critical values for the first case are given in Table 5-7. The critical values for the other cases are given in Appendix V.

Table 5: Critical values for the t_1 for daily data. No deterministic part included

Sample size T	Probability that $t_1$is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	-2.55	-2.18	-1.88	-1.56	0.91	1.28	1.59	1.99
T=280	-2.47	-2.17	-1.91	-1.58	0.88	1.26	1.60	1.99
T=420	-2.55	-2.24	-1.95	-1.61	0.89	1.26	1.61	2.00
T=560	-2.54	-2.23	-1.94	-1.63	0.89	1.25	1.60	1.97
T= ∞	-2.51	-2.21	-1.94	-1.64	0.86	1.25	1.62	1.97

Table 6: Critical values for the $F_{m,m+1}, m = m_{even}$ for daily data. No deterministic part included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.01	0.03	0.06	0.12	2.31	2.99	3.68	4.50
T=280	0.01	0.03	0.06	0.12	2.38	3.07	3.71	4.66
T=420	0.01	0.03	0.06	0.11	2.38	3.08	3.72	4.68
T=560	0.01	0.03	0.06	0.12	2.40	3.14	3.81	4.70
T= ∞	0.01	0.03	0.06	0.12	2.41	3.14	3.80	4.75

Table 7: Critical values for the F_{all} for daily data. No deterministic part included

Sample size T	Probability that $F_{all} = \frac{1}{7}\sum_1^7 t_m^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.20	0.27	0.34	0.43	1.74	2.02	2.31	2.63
T=280	0.20	0.27	0.34	0.45	1.77	2.06	2.35	2.68
T=420	0.19	0.26	0.33	0.45	1.80	2.08	2.36	2.73
T=560	0.21	0.28	0.35	0.45	1.81	2.10	2.39	2.70
T= ∞	0.22	0.29	0.35	0.46	1.82	2.12	2.40	2.72

5 Size and Power studies for houly data

In this subsection we give the size and power analysis of our test for hourly data. Many papers do research on the size and power of the HEGY test, e.g., Ghysels, Lee and Noh (1994) studied the size and power of HEGY test for quarterly data, Rodrigues and Osborn (1999) studied those for monthly data. In their studies, the HEGY test suffers from size distortions when there is strong seasonal moving average innovation in the series and the lag augmentations could reduce the size distortions. Based on the following size studies, the HEGY-type test for hourly data also suffers from the size distortion due to the negative strong moving average component in the series. To study size of our test, we focus on the size distortion problem due to the moving average components in the series, and find that among the 3 commonly used methods to choose $\varphi^*(B)$, including lags without lag elimination performs the best in reducing size distortion.

Another important practical issue is whether to include deterministic components especially seasonal dummies in application. For power of our test, we investigate the effects of including seasonal dummies in test regression on the power of our test, and find that unless there are evident signs indicating there are no deterministic seasonality in the series, it is prudent to include dummies in the testing equation.

Size of test. The reason of these distortions is easy to figure out. In the test regression (3.3) and (3.5), when the order of $\varphi(B)$ is greater than S, the augmentation $\varphi^*(B)$ is needed to accommodate the serial correlation in the residuals. When there is moving average component in the series, the order of $\varphi^*(B)$ should be infinity to render the residuals white noise. Therefore the finite order $\varphi^*(B)$ in practical issue would cause changes in the distributions of test statistics. Appropriate augmentations would attenuate these biases. There are 2 commonly used methods to choose augmentations:

M1: Include a certain number of lags in the auxiliary regression first and then exclude the lags whose parameters are not significantly different from.

M2: Include lags without eliminating the lags which do not have significantly nonzero parameters, the number of lags included is decided by AIC criteria.

M1 is recommended by HEGY (1990) and Beaulieu and Miron (1992). Ghysels, Lee and Noh (1994) find that M2 has better performance than M1 in monthly data. We study the size of our test with the 2 kinds of augmentations above, and show that M2 is more appropriate to attenuate the size distortions in hourly data.

The data generating process (DGP) is

$$(1 - B^{24})y_t = \varepsilon_t + \theta_i \varepsilon_{t-i}$$

where $\varepsilon_t \sim iid\, N(0,1)$, $i = 1, 24$, and we choose $\theta_1 = \pm 0.5$, ± 0.9 and $\theta_{24} = \pm 0.5$, ± 0.9. The sample size is 480, and the size estimates are based on 5% critical values in the previous subsection. We employ test equation (3.5) for our test with constant included, i.e., $c_i = 0$, $i = 1, 2, ..., S$. 5 different kinds of augmentations are used: no augmentation, 24 lags augmentations, 48 lags augmentations, 48 lags augmentation with lag elimination and prewhitening procedure. In the lag elimination process, we include the lags with parameter significantly different from zero at 0.1 level. These procedures are repeated 10,000 times each, yielding the size estimates in Table 8-11.

Based on the size estimates in Table 8-11, when there are no lag augmentations (Table 8), the size distortions are quite large. When 24 or 48 lag augmentations are included in the auxiliary regression (3.3) (Table 9 and 10), the size estimates are all around 0.05 except when there is strong negative seasonal MA component, i.e., $(1 - B^{24})y_t = \varepsilon_t - 0.9\varepsilon_{t-24}$. When there is significant negative seasonal MA component in the series, M2 also perform better and the distortions are reduced sharply when 48 lags are included. The lag elimination augmentation (Table 11) process have size distortions for all statistics in most DGPs except for the DGP where $\theta_{24} = \pm 0.5$. The size estimations suggest us that M2 performs better to accomadate serial correlations in residuals, and when there is strong negative seasonal MA component in the series, more lag augmentations are needed.

Table 8: Empirical size of HEGY-type test statistics at of 5% level with SARIMA models. No augmentation.

	Frequencies						
	0	π	$\frac{\pi}{12}, \frac{23\pi}{12}$	$\frac{\pi}{6}, \frac{11\pi}{6}$	$\frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{\pi}{3}, \frac{5\pi}{3}$	$\frac{5\pi}{12}, \frac{19\pi}{12}$
$\theta_1 = 0.5$	0.02	0.37	0.17	0.23	0.12	0.12	0.21
$\theta_1 = -0.5$	0.54	0.01	0.29	0.22	0.18	0.14	0.12
$\theta_{24} = 0.5$	0.03	0.01	0.14	0.14	0.14	0.14	0.14
$\theta_{24} = -0.5$	0.31	0.35	0.36	0.35	0.35	0.37	0.35
$\theta_1 = 0.9$	0.02	0.76	0.44	0.56	0.36	0.16	0.54
$\theta_1 = -0.9$	1.00	0.27	0.31	0.17	0.12	0.09	0.08
$\theta_{24} = 0.9$	0.03	0.01	0.17	0.17	0.17	0.18	0.17
$\theta_{24} = -0.9$	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	$\frac{\pi}{2}, \frac{3\pi}{2}$	$\frac{7\pi}{12}, \frac{17\pi}{12}$	$\frac{2\pi}{3}, \frac{4\pi}{3}$	$\frac{3\pi}{4}, \frac{5\pi}{4}$	$\frac{5\pi}{6}, \frac{7\pi}{6}$	$\frac{11\pi}{12}, \frac{13\pi}{12}$	F_{all}
$\theta_1 = 0.5$	0.11	0.22	0.14	0.17	0.30	0.29	0.59
$\theta_1 = -0.5$	0.12	0.11	0.12	0.11	0.12	0.12	0.69
$\theta_{24} = 0.5$	0.14	0.14	0.14	0.14	0.14	0.14	0.36
$\theta_{24} = -0.5$	0.36	0.36	0.34	0.37	0.36	0.35	0.98
$\theta_1 = 0.9$	0.34	0.56	0.19	0.31	0.73	0.73	0.97
$\theta_1 = -0.9$	0.08	0.08	0.07	0.08	0.08	0.08	1.00
$\theta_{24} = 0.9$	0.17	0.17	0.17	0.16	0.17	0.17	0.48
$\theta_{24} = -0.9$	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 9: Empirical size of HEGY-type test statistics at of 5% level with SARIMA models. 24 lag augmentations.

	Frequencies						
	0	π	$\frac{\pi}{12}, \frac{23\pi}{12}$	$\frac{\pi}{6}, \frac{11\pi}{6}$	$\frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{\pi}{3}, \frac{5\pi}{3}$	$\frac{5\pi}{12}, \frac{19\pi}{12}$
$\theta_1 = 0.5$	0.04	0.04	0.05	0.05	0.05	0.04	0.05
$\theta_1 = -0.5$	0.04	0.04	0.05	0.05	0.05	0.04	0.04
$\theta_{24} = 0.5$	0.08	0.07	0.05	0.06	0.05	0.05	0.05
$\theta_{24} = -0.5$	0.08	0.11	0.08	0.08	0.09	0.08	0.08
$\theta_1 = 0.9$	0.03	0.02	0.05	0.04	0.05	0.05	0.05
$\theta_1 = -0.9$	0.07	0.05	0.05	0.05	0.05	0.05	0.05
$\theta_{24} = 0.9$	0.11	0.10	0.07	0.07	0.07	0.06	0.07
$\theta_{24} = -0.9$	0.49	0.77	0.93	0.93	0.91	0.94	0.91
	$\frac{\pi}{2}, \frac{3\pi}{2}$	$\frac{7\pi}{12}, \frac{17\pi}{12}$	$\frac{2\pi}{3}, \frac{4\pi}{3}$	$\frac{3\pi}{4}, \frac{5\pi}{4}$	$\frac{5\pi}{6}, \frac{7\pi}{6}$	$\frac{11\pi}{12}, \frac{13\pi}{12}$	F_{all}
$\theta_1 = 0.5$	0.05	0.05	0.05	0.05	0.04	0.05	0.05
$\theta_1 = -0.5$	0.05	0.04	0.04	0.05	0.05	0.04	0.04
$\theta_{24} = 0.5$	0.05	0.06	0.05	0.06	0.05	0.05	0.08
$\theta_{24} = -0.5$	0.08	0.09	0.08	0.09	0.08	0.08	0.18
$\theta_1 = 0.9$	0.04	0.04	0.04	0.04	0.05	0.05	0.04
$\theta_1 = -0.9$	0.05	0.04	0.05	0.04	0.05	0.04	0.04
$\theta_{24} = 0.9$	0.07	0.07	0.06	0.07	0.07	0.07	0.14
$\theta_{24} = -0.9$	0.93	0.93	0.90	0.93	0.93	0.91	1.00

Table 10: Empirical size of HEGY-type test statistics at of 5% level with SARIMA models. 48 lag augmentations.

	Frequencies						
	0	π	$\frac{\pi}{12}, \frac{23\pi}{12}$	$\frac{\pi}{6}, \frac{11\pi}{6}$	$\frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{\pi}{3}, \frac{5\pi}{3}$	$\frac{5\pi}{12}, \frac{19\pi}{12}$
$\theta_1 = 0.5$	0.03	0.04	0.05	0.05	0.05	0.05	0.04
$\theta_1 = -0.5$	0.04	0.04	0.05	0.04	0.04	0.04	0.05
$\theta_{24} = 0.5$	0.04	0.03	0.05	0.06	0.05	0.05	0.05
$\theta_{24} = -0.5$	0.04	0.06	0.04	0.04	0.04	0.04	0.04
$\theta_1 = 0.9$	0.04	0.03	0.04	0.04	0.05	0.04	0.04
$\theta_1 = -0.9$	0.04	0.04	0.05	0.05	0.04	0.04	0.04
$\theta_{24} = 0.9$	0.03	0.02	0.08	0.07	0.07	0.07	0.07
$\theta_{24} = -0.9$	0.20	0.40	0.62	0.59	0.57	0.62	0.56
	$\frac{\pi}{2}, \frac{3\pi}{2}$	$\frac{7\pi}{12}, \frac{17\pi}{12}$	$\frac{2\pi}{3}, \frac{4\pi}{3}$	$\frac{3\pi}{4}, \frac{5\pi}{4}$	$\frac{5\pi}{6}, \frac{7\pi}{6}$	$\frac{11\pi}{12}, \frac{13\pi}{12}$	F_{all}
$\theta_1 = 0.5$	0.04	0.05	0.04	0.04	0.04	0.04	0.04
$\theta_1 = -0.5$	0.04	0.04	0.04	0.04	0.04	0.04	0.03
$\theta_{24} = 0.5$	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$\theta_{24} = -0.5$	0.04	0.04	0.04	0.04	0.04	0.04	0.03
$\theta_1 = 0.9$	0.04	0.04	0.05	0.05	0.05	0.05	0.04
$\theta_1 = -0.9$	0.04	0.04	0.04	0.04	0.04	0.05	0.03
$\theta_{24} = 0.9$	0.08	0.07	0.06	0.07	0.07	0.07	0.09
$\theta_{24} = -0.9$	0.60	0.62	0.54	0.62	0.59	0.58	1.00

Table 11: Empirical size of HEGY-type test statistics at of 5% level with SARIMA models. 48 lag augmentations with lag elimination.

	Frequencies						
	0	π	$\frac{\pi}{12}, \frac{23\pi}{12}$	$\frac{\pi}{6}, \frac{11\pi}{6}$	$\frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{\pi}{3}, \frac{5\pi}{3}$	$\frac{5\pi}{12}, \frac{19\pi}{12}$
$\theta_1 = 0.5$	0.02	0.27	0.16	0.21	0.12	0.12	0.17
$\theta_1 = -0.5$	0.52	0.02	0.18	0.14	0.12	0.10	0.09
$\theta_{24} = 0.5$	0.04	0.03	0.05	0.05	0.05	0.05	0.05
$\theta_{24} = -0.5$	0.04	0.06	0.04	0.04	0.04	0.04	0.04
$\theta_1 = 0.9$	0.02	0.70	0.43	0.53	0.36	0.15	0.50
$\theta_1 = -0.9$	1.00	0.02	0.23	0.12	0.08	0.06	0.05
$\theta_{24} = 0.9$	0.03	0.02	0.07	0.07	0.07	0.07	0.07
$\theta_{24} = -0.9$	0.22	0.43	0.64	0.60	0.57	0.62	0.61
	$\frac{\pi}{2}, \frac{3\pi}{2}$	$\frac{7\pi}{12}, \frac{17\pi}{12}$	$\frac{2\pi}{3}, \frac{4\pi}{3}$	$\frac{3\pi}{4}, \frac{5\pi}{4}$	$\frac{5\pi}{6}, \frac{7\pi}{6}$	$\frac{11\pi}{12}, \frac{13\pi}{12}$	F_{all}
$\theta_1 = 0.5$	0.09	0.16	0.11	0.12	0.20	0.18	0.40
$\theta_1 = -0.5$	0.10	0.10	0.10	0.11	0.12	0.12	0.55
$\theta_{24} = 0.5$	0.06	0.06	0.06	0.05	0.06	0.06	0.05
$\theta_{24} = -0.5$	0.05	0.05	0.05	0.05	0.04	0.04	0.04
$\theta_1 = 0.9$	0.32	0.50	0.15	0.26	0.65	0.65	0.94
$\theta_1 = -0.9$	0.06	0.06	0.07	0.07	0.07	0.08	1.00
$\theta_{24} = 0.9$	0.13	0.07	0.08	0.07	0.07	0.07	0.10
$\theta_{24} = -0.9$	0.65	0.66	0.58	0.66	0.63	0.62	1.00

Power of test. For the power analysis, we focus on the influence of including seasonal dummies in test equation (3.5) on power of our test statistics. The data generating processes is Seasonal ARIMA(1,0,0)$_{24}$ model with different seasonal intercepts:

$$(1 - 0.9B^{24})y_t = \sum_{i=1}^{24} \alpha_i D_{it} + \varepsilon_t$$

where $\alpha_i, i = 1, 2, ..., 24$ are (0.9,-0.6,0.1,0.5,-0.3,0,0.8,-0.3,-0.5,0.6,0.9,-0.7,0,0.5,0,0.1,0.2,-0.4,0.7,-0.2,-0.8,0,0.1,0). First we include only constant in (3.5) i.e., $c_i = 0$, $i = 1, 2, ..., S$, denote as Case A. Next we include constant and seasonal dummies in (3.5), i.e., $c_1 = 0$, denoted as Case B. The augmentations used are 0 lags and 24 lags without lag elimination. The sample size is 480, nominal size is 5%, and each process is replicated for 10,000 times, yielding the results in Table 12.

From the estimates in Table 12, the F_{all} statistics have poor power estimates in all the data generating processes, and when there is seasonal intercept in the DGP but not in the test equation, the F_{all} statistics have very low power. When Case B is employed for testing seasonal unit root, the power of F_{all} have better performance than Case A. The F-statistics for conjugation frequencies have power close to 1, and they have similar power estimates among different frequencies. When Case A is used where there are no seasonal dummies in the regressions, the power for $F_{m,m+1}$ are smaller than those when Case B is employed for test. The power of the t-statistics for testing unit root at frequency 0 and π are about the same for the two cases. Therefore the inclusion of the seasonal dummies is a prudent decision in practical issue, especially for F_{all}. Similar conclusion is derive by Rodrigus and Osborn (1999). The results also suggests that one should combine the test results of $F_{m,m+1}$ statistics and the F_{all} statistic to decide whether the seasonal differention $(1 - B^S)$ is needed to render the series stationary.

Table 12: Empirical power of HEGY-type test statistics at 5% level for SARIMA(1,0,0) with seasonal dummies. DGP with seasonal intercepts a_i

	Frequencies						
	0	π	$\frac{\pi}{12}, \frac{23\pi}{12}$	$\frac{\pi}{6}, \frac{11\pi}{6}$	$\frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{\pi}{3}, \frac{5\pi}{3}$	$\frac{5\pi}{12}, \frac{19\pi}{12}$
Case A 0 lag	0.94	0.88	0.85	0.85	0.86	0.85	0.86
Case A 24 lag	0.95	0.90	0.88	0.89	0.88	0.89	0.89
Case B 0 lag	0.92	0.92	0.95	0.96	0.96	0.95	0.96
Case B 24 lag	0.94	0.94	0.97	0.97	0.97	0.97	0.97
	$\frac{\pi}{2}, \frac{3\pi}{2}$	$\frac{7\pi}{12}, \frac{17\pi}{12}$	$\frac{2\pi}{3}, \frac{4\pi}{3}$	$\frac{9\pi}{12}, \frac{15\pi}{12}$	$\frac{5\pi}{6}, \frac{7\pi}{6}$	$\frac{11\pi}{12}, \frac{13\pi}{12}$	F_{all}
Case A 0 lag	0.86	0.87	0.89	0.86	0.85	0.86	0.37
Case A 24 lag	0.89	0.90	0.91	0.89	0.88	0.88	0.55
Case B 0 lag	0.96	0.96	0.96	0.95	0.96	0.96	0.65
Case B 24 lag	0.97	0.97	0.97	0.97	0.97	0.97	0.77

6 Testing seasonal unit roots in hourly wind power production data in Sweden

Our test is applied to hourly wind power production data in Sweden. The wind condition in Sweden are different in winter and summer time, so we separate the year into warm season and cold season. Warm season covers the time from April to September and cold season covers the rest. We test seasonal unit roots in the production data in warm season and cold season separately. For cold season, the data used starts from 0 o'clock Nov 1, 2008, ends at 23 o'clock Feb 28, 2009, and the data has 2880 observation for 120 days. For warm season, the data used starts from 0 o'clock May 1, 2009, ends at 23 o'clock Aug 31, 2009, which have 2952 observation for a 123 days period. The data are plotted in Figure 1. We include constant and seasonal dummies in (3.5) considering there is no evident trend in both series. "p" is chosen to be 33 for cold season and 30 for warm season without lag elimination based on AIC criteria. Considering the F statistics for conjugate unit roots have better performance than the t-statistics, we only provide the F statistics for conjugate unit roots in Table 13.

From the results in Table 13 we can see that the F statistics for conjugate unit roots are all significantly nonzero at 5% level, so is t_π, therefore there is no seasonal unit root in both series. The estimate of t_1 for the 2 series are not significant at 5% level, therefore there is unit root at

0 frequency. The differencing filter $(1-B)$ is needed to render both series stationary.

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Figure 1: Hourly wind power production data in Sweden 2008-2009.

Table 13: Testing for seasonal roots in wind production data

	Frequency						
	0	π	$\frac{\pi}{12},\frac{23\pi}{12}$	$\frac{\pi}{6},\frac{11\pi}{6}$	$\frac{\pi}{4},\frac{7\pi}{4}$	$\frac{\pi}{3},\frac{5\pi}{3}$	$\frac{5\pi}{12},\frac{19\pi}{12}$
coldseason	0.02	-12.75*	128.66*	117.36*	114.88*	127.23*	111.08*
warmseason	0.49	-12.23*	122.67*	119.60*	123.18*	145.36*	138.15*
	$\frac{\pi}{2},\frac{3\pi}{2}$	$\frac{7\pi}{12},\frac{17\pi}{12}$	$\frac{2\pi}{3},\frac{4\pi}{3}$	$\frac{3\pi}{4},\frac{5\pi}{4}$	$\frac{5\pi}{6},\frac{7\pi}{6}$	$\frac{11\pi}{12},\frac{13\pi}{12}$	F_{all}
coldseason	124.61*	127.75*	99.35*	117.02*	124.68*	114.31*	115.68*
warmseason	131.01*	150.14*	141.52*	142.37*	161.67*	116.91*	130.62*

"*" stands for significant at 5% level

7 Concluding remarks

In this paper we propose an HEGY type test for testing seasonal unit roots in data with any frequency. Seasonal unit roots in a univariate time series would make the series nonstationary, and misspecification of the seasonal patterns in the modeling process would lead to seriously biased results. Among the many approaches in detecting seasonal unit roots, the HEGY type test has the advantage of testing presence of seasonal unit roots at different frequencies separately, therefore it could detect certain types of nonstationarity in application. We use the technique of HEGY test to derive the test regression in data with any frequency, and then provide the testing procedure. The finite-sample and asymptotic distributions of the test statistics are also given. Considering the inclusion of deterministic component like constant, trend and seasonal dummies would affect the finite-sample and asymptotic distribution of the test statistics, we provide the distributions under different cases.

The analysis of power and size of our test for hourly data is provided in this paper, giving suggestions on how to choose augmentations and deterministic components in test regression. According to the simulation results, we find including lags without lag elimination has better performance, and it is better to include more lag augmentations when there are strong negative seasonal moving average component in the series. Also we find that the inclusion of seasonal dummies are prudent unless there are strong indication saying no deterministic seasonality exists.

Our decomposition is similar with Chan and Wei (1988), specifically $x_{1,t}$ and x_π are the same with u_t and v_t in their paper. In their paper, Chan and Wei pointed out that the limiting distributions of the estimators for model $y_n = \varphi_1 y_{n-1} + \varphi_2 y_{n-2} + \ldots + \varphi_p y_{n-p} + \varepsilon_t$ do not have a closed form when there are (non-) seasonal unit roots in the series y_t, and the distributions maybe "extremely complicated". So they gave up deriving such expressions and instead derive the asymptotic properties of their decomposition factors such as u_t and v_t. Thus they do not provide a strategy for testing seasonal unit roots. The HEGY-type test bypass the asymptotic distributions of the estimators in the autoregressive model and consider factorizing the autoregressive polynomial $(1-\varphi_1 B - \varphi_2 B^2 - \ldots - \varphi_p B^p)$, and then use the factors to test for the existence of seasonal unit roots. Our results still could not lead to the asymptotic distributions for estimators of an autoregressive model containing seasonal unit roots. But they may be helpful to derive them.

8 Appendix

Appendix I: Decomposition process.

Proof of Lemma 1:

To carry out a seasonal unit root test in quarterly data, HEGY (1990) make decomposition of $\varphi(B)$. The decomposition is based on the mathematic approximation theory below:

Lagrange Interpolation Polynomial: Given a polynomial $f(x)$ and a set of n points $x_1, ..., x_n$ which lie in (a, b), $-\infty < a < b < \infty$. The Lagrange Interpolation Polynomial is defined by

$$L(x) = \sum_{k=1}^{n} f(x_k) l_k(x)$$

where $l_k(x) = \frac{\omega(x)}{(x-x_k)\omega'(x_k)}$, $\omega(x) = (x - x_1)(x - x_2) \cdots (x - x_n)$, and $\omega'(x)$ is the derivative of $\omega(x)$.

$L(x)$ is used to approximate $f(x)$, and its deviation is given by $f(x) - L(x) = \frac{f^{(n)}(\varepsilon)}{n!}\omega(x)$, where $f^{(n)}(\varepsilon)$ is the n^{th} derivative of $f(x)$ and $\varepsilon \in (a, b)$. With the Lagrange Interpolation Polynomial and its deviation, the polynomial $f(x)$ has the decomposition:

$$f(x) = \sum_{k=1}^{n} f(x_k) l_k(x) + \frac{f^{(n)}(\varepsilon)}{n!}\omega(x)$$

We give the proof for the case when S is even first. Expand the autoregressive polynomial $\varphi(B)$ above at the S unit root $u_m, k = 1, 2, ..., S$ which are defined in section 2. Let $\omega(B) = \prod_{m=1}^{S}(B - u_m) = \prod_{m=1}^{S}(1 - \frac{B}{u_m})$, $-\frac{f(u_m)}{\omega'(u_m)u_m} = \tau_m$, we could get:

$$\varphi(u) = \sum_{m=1}^{S} \tau_m \frac{1 - u^S}{1 - \frac{u}{u_m^e}} + \frac{f^{(S)}(B)}{S!}(1-u^S) = \sum_{m=1}^{S} \tau_m (1-u^S)(\frac{1}{1 - \frac{u}{u_m^e}} - 1) + (\frac{f^{(S)}(B)}{S!} + \sum_{m=1}^{S} \tau_m)(1-u^S)$$

The last equation is derived by subtracting and adding $\sum_{m=1}^{S} \tau_m(1 - u^S)$. Denote $\varphi_m(u) = \frac{u}{u_m^e} \prod_{j=1, j\neq m}^{S}(1 - \frac{u}{u_j})$, for $m = 1, 2, ..., S$ and $\varphi^*(u) = \frac{f^{(S)}(B)}{S!} + \sum_{m=1}^{S} \tau_m$, we could get (3.2).

When S is odd, let $\omega(B) = -\prod_{m=1}^{S}(1 - \frac{B}{u_m})$, $\frac{f(u_m)}{\omega'(u_m)} = \tau_m$, $\varphi^*(u) = -\frac{f^{(S)}(B)}{S!} + \sum_{m=1}^{S} \tau_m$, with the same procedure we could get (3.2).

Proof of Lemma 2:

For (3.2), it is obvious to see that, for $m = 1$, $\varphi_1(B) = B(1 + B + B^2 + ... + B^{S-1})$, and $m = \pi$, $\varphi_\pi(B) = (-B)[1 - B + B^2 + ... + (-B)^{S-1}]$.

Next we consider $\varphi_m(B)$ that is related to the frequencies other than 0 and π. These frequencies are pairs of conjugation frequencies, therefore $\varphi_m(B)$ need to be considered in pairs. For $m = m_{even}$,

$$\begin{aligned}\tau_m\varphi_m(B) &= \tau_m u_{m+1}B(1-u_mB)\prod_{j=1,j\neq m,m+1}^{S}(1-\frac{B}{u_j})\\ &= \tau_m B(cos\theta_m - isin\theta_m - B)\prod_{j=1,j\neq m,m+1}^{S}(1-\frac{B}{u_j})\end{aligned}$$

where i is the imaginary unit.

Similarly we get $\tau_{m+1}\varphi_{m+1}(B) = \tau_{m+1}B(cos\theta_m + isin\theta_m - B)\prod_{j=1,j\neq m,m+1}^{S}(1-\frac{B}{u_j})$

Make the substitution: $2\tau_m = \rho_m + i\rho_{m+1}, 2\tau_{m+1} = \rho_m - i\rho_{m+1}$. Then we have:

$$\tau_m\varphi_m(B) + \tau_{m+1}\varphi_{m+1}(B) = [\rho_m B(cos\theta_m - B) + \rho_{m+1}Bsin\theta_m]\prod_{j=1,j\neq m,m+1}^{S}(1-\frac{B}{u_j})$$

$$= \rho_m\zeta_m(B) + \rho_{m+1}\zeta_{m+1}(B) \tag{8.1}$$

We give the two triangle equations below:

$$(cos\theta_m - x)\prod_{j=1,j\neq m,m+1}^{S}(1-\frac{x}{z_j}) = \sum_{j=1}^{S}cos(j\theta_m)x^{j-1} \tag{8.2}$$

$$sin\theta_m\prod_{j=1,j\neq m,m+1}^{S}(1-\frac{x}{z_j}) = \sum_{j=1}^{S}sin(j\theta_m)x^{j-1} \tag{8.3}$$

(8.2) is equivalent to: $(cos\theta_m - x)(1-x^S) = \sum_{j=1}^{S}cos(j\theta_m)x^{j-1}(1-2cos\theta_m x + x^2)$. Using the triangle equation $cosa + cosb = \frac{1}{2}[cos(a+b) + cos(a-b)]$, and we could easily proof the equation.

For (8.3), with similar process as above, (8.3) is equivalent to $sin\theta_m(1-x^S) = (1-2cos\theta_m x + x^2)\sum_{j=1}^{S}sin(j\theta_m)x^{j-1}$, and we could proof the equation.

With (8.2) and (8.3), we could express the two polynomials in (8.1):

$\zeta_m(B) = \sum_{j=1}^{S}cos(j\theta_m)B^j$, $\zeta_{m+1}(B) = \sum_{j=1}^{S}sin(j\theta_m)B^j$, where $m = m_{even}$

When $m = 1$, we could also write $\zeta_1(B) = \varphi_1(B) = \sum_{j=1}^{S}cos(j\theta_1)B^j$ where $\theta_1 = 0$. Similarly when $m = \pi$, $\zeta_\pi(B) = \varphi_\pi(B) = \sum_{j=1}^{S}cos(j\pi)B^j$. So when $m = 1, \pi$, we have the same form as $m = m_{even}$.

With the process above, Lemma 2 is proofed.

Appendix II Asymptotic distributions of test statistics when (3.3) is employed for test.

A2.1 Derivation of t-statistics and F-statistics

The t-statistics in this paper have

A2.1 Matrix Decomposition

In the process of deriving the asymptotic distributions of the test statistics, the following matrixes are used:

$$A_m = \begin{pmatrix} cos(0*\theta_m) & cos(\theta_m) & cos(2\theta_m) & ... & cos((S-1)\theta_m) \\ cos((S-1)\theta_m) & cos(0*\theta_m) & cos(\theta_m) & ... & cos((S-2)\theta_m) \\ cos((S-2)\theta_m) & cos((S-1)\theta_m) & cos(0*\theta_m) & ... & cos((S-3)\theta_m) \\ ... & & & & \\ cos(\theta_m*1) & cos(\theta_m*2) & cos(\theta_m*3) & ... & cos(0*\theta_m) \end{pmatrix}$$

$$C_m = \begin{pmatrix} sin(0*\theta_{m-1}) & sin(\theta_{m-1}) & sin(2\theta_{m-1}) & ... & sin((S-1)\theta_{m-1}) \\ sin((S-1)\theta_{m-1}) & sin(0*\theta_{m-1}) & sin(\theta_{m-1}) & ... & sin((S-2)\theta_{m-1}) \\ sin((S-2)\theta_{m-1}) & sin((S-1)\theta_{m-1}) & sin(0*\theta_{m-1}) & ... & sin((S-3)\theta_{m-1}) \\ ... & & & & \\ sin(\theta_{m-1}*1) & sin(\theta_{m-1}*2) & sin(\theta_{m-1}*3) & ... & sin(0*\theta_{m-1}) \end{pmatrix}$$

We provide the decomposition theory of the matrix A_m and C_m in this part. The decomposition is used to lower the dimensions of A_m and C_m because they are not full rank. The technique used is singular value decomposition (SVD), which factorizes a matrix A_m and C_m in to 3 matrixes, $A_m = U^m D^m (V^m)'$, $C_m = U^{m*} D^{m*} (V^{m*})'$. Consider A_m, the matrix D^m is a diagonal matrix with diagonal elements $\lambda_1, ..., \lambda_S$ which are square roots of eigen values of $A_m^{'} A_m$, and they are ordered form largest to smallest (Called singular values). Both U^m and V^m are unitrary matrixes. Denote the j^{th} column of U^m and V^m as u_j^m and v_j^m. The decomposition of C_m follows the same way. Similarly, denote j^{th} column of U^{m*} and V^{m*} as u_j^{m*} and v_j^{m*}. The columns of U^m are the eigen vectors of $A_m A_m^{'}$ and the columns of V^m are the eigen vectors of $A_m^{'} A_m$. They have the relationship $u_j^{m*} = \lambda_j^{-1} C_m v_j^m$.

The following Lemma is used in getting the eigen values and eigen vectors of the matrixes above. The lemma could be found in many books about circulant matrixes, so we directly give it.

Property of circulant matrix: Suppose matrix A is a circulant matrix in a form:

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & ... & a_{s-1} \\ a_{s-1} & a_0 & a_1 & ... & a_{s-2} \\ a_{s-2} & a_{s-1} & a_0 & ... & a_{s-3} \\ & & & ... & \\ a_1 & a_2 & a_3 & ... & a_0 \end{pmatrix}$$

The eigen values for A is $\lambda_k = \sum_{k=0}^{S-1} a_j e^{\frac{2\pi jk}{S} i}, k = 0, 1, ..., S-1$ and the corresponding eigen vector is $w_k = \frac{1}{\sqrt{S}}(1, e^{\frac{2\pi k}{S} i}, e^{\frac{4\pi k}{S} i}, ..., e^{\frac{2(n-1)\pi k}{S} i})'$, where i is the imaginary unit.

I) First we consider θ_m equal to 0 and π. The matrixes needed are A_1 and A_π, and we have the factorization $A_m = U^m D^m (V^m)'$, $m = 1, \pi$. We could derive that there is only 1 non-zero singular value for both A_1 and A_π which is $\lambda_1 = S$. Thus only the first column of U and V matter. Because A_1 and A_π are symmetric, $A_1^{'} A_1$ and $A_\pi^{'} A_\pi$ are symmetric, so $U^m = V^m$.

It is also easy to get that the eigen vector corresponding to singular value λ_1. For $A_1^{'} A_1$, $u_1^1 = v_1^1 = \frac{1}{\sqrt{s}}(1, 1, ..., 1)'$, and for $A_\pi^{'} A_\pi$, $u_1^\pi = v_1^\pi = \frac{1}{\sqrt{s}}(1, -1, ..., 1, -1)'$. Therefore we get the equation, $(u_1^1)' u_1^1 = (v_1^1)' v_1^1 = 1$, $(u_1^2)' u_1^2 = (v_1^2)' v_1^2 = 1$.

II) Next we consider the matrix A_m for m_{even}.

D: We could get $A_m^{'} A_m = \frac{S}{2} A_m$ which is also a circulant matrix. Denote $\beta_k = \frac{2\pi}{S}(k-1), k =$

$1, 2, ..., S$. The eigen values of $\frac{S}{2}A_m$ are

$$\begin{aligned}
\lambda_k &= \frac{S}{2}\sum_{j=0}^{S-1} cos(j\theta_m)e^{j\beta_k i} = \frac{S}{4}\sum_{j=0}^{S-1} cos[j(\theta_m+\beta_k)] + \frac{S}{4}\sum_{j=0}^{S-1} cos[j(\theta_m-\beta_k)] \\
&+ i\frac{S}{4}\sum_{j=0}^{S-1} sin[j(\theta_m+\beta_k)] + i\frac{S}{4}\sum_{j=0}^{S-1} sin[j(\theta_m-\beta_k)] \\
&= \frac{S}{4}\sum_{j=0}^{S-1} cos[j(\theta_m+\beta_k)] + \frac{S}{4}\sum_{j=0}^{S-1} cos[j(\theta_m-\beta_k)]
\end{aligned}$$

It can be seen that for a specific θ_m, λ_k are non-zero only when $\beta_k = \theta_m$ and $2\pi - \theta_m$, and in that case $\lambda_k = \frac{S^2}{4}$. Thus we get the singular values for A_k are $\lambda_1 = \lambda_2 = \frac{S}{2}$.

V: The corresponding eigen vectors for λ_1 and λ_2 are $\varrho_1^m = \frac{1}{\sqrt{S}}[1, e^{i\theta_m}, e^{2i\theta_m}, ..., e^{(S-1)i\theta_m}]'$, $\varrho_2^m = \frac{1}{\sqrt{S}}[1, e^{-i\theta_m}, e^{-2i\theta_m}, ..., e^{-(S-1)i\theta_m}]'$. Make linear transformation:

$$v_1^m = \frac{\varrho_1^m + \varrho_2^m}{\sqrt{2}} = \sqrt{\frac{2}{S}}[1, cos\theta_m, cos(2\theta_m), ..., cos((S-1)\theta_m]'$$

$$v_2^m = \frac{\varrho_1^m - \varrho_2^m}{\sqrt{2}i} = \sqrt{\frac{2}{S}}[0, sin\theta_m, sin(2\theta_m), ..., sin,\,'\,'((S-1)\theta_m]'$$

and we get the first two columns of V^m.

U: The columns of U are derived with the relationship $u_j = \lambda_j^{-1}A_m v_j^m$, so we can get:

$$u_1^m = \sqrt{\frac{2}{S}}[1, cos\theta_m, cos(2\theta_m), ..., cos((S-1)\theta_m]'$$

$$u_2^m = \sqrt{\frac{2}{S}}[0, sin\theta_m, sin(2\theta_m), ..., sin((S-1)\theta_m]'$$

Therefore we get the relationship $(u_1^m)'v_1^m = (u_2^m)'v_2^m = 1$, $(u_1^m)'v_2^m = (u_2^m)'v_1^m = 0$.

III) Then we consider decomposition of C_m when $m = m_{odd}$.

We could get $C_m^T C_m = \frac{S}{2}A_{m-1}$, therefore we have the same singular values as A_{m-1} which are $\lambda_1 = \lambda_2 = \frac{S}{2}$, therefore we get that the first two columns of V^{m*} are the same with those of V^{m-1}, i.e., v_1^{m-1} and v_2^{m-1}. With the relationship $u_j^{m*} = \lambda_j^{-1}C_m v_j^{m-1}$, we could get $u_1^{m*} = -\sqrt{\frac{2}{S}}[0, sin\theta_{m-1}, sin(2\theta_{m-1}), ..., sin((S-1)\theta_{m-1}]'$, $u_2^{m*} = \sqrt{\frac{2}{S}}[1, cos\theta_{m-1}, cos(2\theta_{m-1}), ..., cos((S-1)\theta_{m-1}]'$.

Thus we have the relationship $(u_1^{m*})'v_1^{m*} = (u_2^{m*})'v_2^{m*} = 0$, $(u_1^{m*})'v_2^{m*} = -1$, $(u_2^{m*})'v_1^{m*} = 1$.

A2.2 Proof of Theorem 1:

We give the proof for the case when S is even, and the case when S is odd is proofed in the same way. The asymptotic distribution of the t-statistics for hourly data have been proofed by Beaulieu and Miron (1992), we restated with general S. Under the assumption 1, we give the convergences below. "J" denotes the periods, i.e., $J = \frac{T}{S}$, and $p = 1, 2, ..., S$. Denote $B(r) = [B_1(r), B_2(r), ..., B_S(r)]'$, where $B_i(r)$ are mutually independent standard Brownian motions.

$$\frac{1}{\sigma}T^{-1/2}\sum_{j=0}^{J}\varepsilon_{Sj+p} \xrightarrow{L} \frac{1}{\sqrt{S}}B_p(1) \tag{8.4}$$

$$\frac{1}{\sigma}J^{-3/2}\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+p} \xrightarrow{L} \int_0^1 B_p(r)dr \tag{8.5}$$

$$\frac{1}{\sigma^2}J^{-1}\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q}\varepsilon_{Sj+p} \xrightarrow{L} \int_0^1 B_q(r)dB_p(r) \tag{8.6}$$

$$W_1(1) = \frac{1}{\sqrt{S}}\sum_{p=0}^{S}B_p(1) \tag{8.7}$$

where "$\xrightarrow{L}$" means converge in distribution. (8.4) is proofed by Chan and Wei (1988) under the assumption 1. (8.5) could be found in White (See White, 2001, Page 179) under assumption 1 and (8.4). (8.6) is derived by Chan and Wei (1988) with the same assumptions as (8.5).

For the OLS estimators for regression (3.3),

$$\hat{\rho} = [\sum_{1}^{T} X_t X_t^{'}]^{-1}[\sum_{1}^{T} X_t\varepsilon_t] \tag{8.8}$$

where $\hat{\rho} = [\hat{\rho}_1, \hat{\rho}_2, ..., \hat{\rho}_S]'$, $X_t = [x_{1,t}, x_{2,t}, ..., x_{S,t}]'$

For $m = 1, \pi, m_{even}$, the following expression is used:

$$\begin{aligned} x_{m,t} &= \sum_{i=1}^{S} cos(i\theta_m)B^i y_t = \sum_{i=1}^{S} cos(i\theta_m)\sum_{h=0}^{[t/S]}\varepsilon_{Sh+p-i} \\ &= \sum_{q=p-1}^{p-S} cos(\theta_m(p-q))\sum_{h=1}^{[t/S]}\varepsilon_{Sh+q} \end{aligned}$$

The last step comes from the substitution $i = p - q$. Thus we have:

$$\begin{aligned} \sum_{t=0}^{T} x_{m,t}\varepsilon_t &= \sum_{j=0}^{J-1}\sum_{p=1}^{S}\sum_{q=p-1}^{p-S} cos(\theta_m(p-q))\sum_{h=0}^{j}\varepsilon_{Sh+q}\varepsilon_{Sj+p} \\ &= \sum_{p=1}^{S}\sum_{q=p-1}^{p-S} cos(\theta_m(p-q))\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q}\varepsilon_{Sj+p} \\ &= \sum_{p=1}^{S}\sum_{q=1}^{S} cos(\theta_m(p-q))\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{S(h-1)+q}\varepsilon_{Sj+p} + \sum_{p=2}^{S}\sum_{q=1}^{p-1} cos(\theta_m(p-q)\sum_{j=0}^{J-1}\varepsilon_{Sj+q}\varepsilon_{Sj+p} \end{aligned}$$

The second part would converge in probability to 0 when divided by J under assumption 1, then with (8.5), $\sum_{t=0}^{T} x_{m,t}\varepsilon_t$ converge to

$$\begin{aligned} J^{-1}\sum_{t=0}^{T} x_{m,t}\varepsilon_t &\xrightarrow{L} \sigma^2\sum_{p=0}^{S}\sum_{q=1}^{S} cos(\theta_m(p-q))\int_0^1 B_q(r)dB_p(r) \\ &= \sigma^2\int_0^1 B^T A_m dB = \sigma^2\int_0^1 B^T U^m D^m V^m dB \end{aligned}$$

For the elements in matrix $\sum_1^T X_t X_t^{'}$:

$$
\begin{aligned}
J^{-2}\sum_{t=0}^{T}x_{m,t}^{2} &= J^{-2}\sum_{j=0}^{J-1}\sum_{p=1}^{S}[\sum_{q=p-1}^{p-S}cos(\theta_m(p-q))\sum_{h=1}^{j}\varepsilon_{Sh+q}]^2 \\
&= J^{-2}\sum_{j=0}^{J-1}E_j^{'}A_m^{'}A_mE_j = J^{-2}\sum_{j=0}^{J-1}E_j^{'}A_mA_m^{'}E_j \\
&\xrightarrow{L} \sigma^2\int_0^1 B^{'}(r)U^mD^mD^mU^mB(r)dr
\end{aligned}
$$

where $E_j = [\sum_{h=0}^{j}\varepsilon_{Sh+q-1}, \sum_{h=0}^{j}\varepsilon_{Sh+q-2}, ..., \sum_{h=0}^{j}\varepsilon_{Sh+q-S}]'$.

For the elements that are not in the diagonal, $m_1 \neq m_2$, similar with above: $J^{-2}\sum_{t=0}^{T}x_{m_1,t}x_{m_2,t} = J^{-2}\sum_{j=0}^{J-1}E_j^{'}A_{m_1}^{'}A_{m_2}E_j$, ($A_{m_1}$ and A_{m_2} are changed to C_{m_1} and C_{m_2} for $m = m_{odd}$). Because $A_{m_1}^{'}A_{m_2} = A_{m_1}^{'}C_{m_2} = C_{m_1}^{'}A_{m_2} = C_{m_1}^{'}C_{m_2} = 0$, so $J^{-2}\sum_{t=0}^{T}x_{m_1,t}x_{m_2,t} = 0$. Thus the regressors are orthogonal, and the estimators becomes $\hat{\rho}_m = \frac{\sum_1^T x_{m,t}\varepsilon_t}{\sum_1^T x_{m,t}^2}$, and $\hat{\sigma}_{\rho_m}^2 = \frac{s_T^2}{\sum_{t=0}^{T}x_{m,t}^2}$, where "$s_T^2$" is the estimator of σ^2.

We have $t_m = \frac{\hat{\rho}_m}{\hat{\sigma}_{\rho_m}} = \frac{\sum_{t=0}^{T}x_{m,t}\varepsilon_t}{s_T(\sum_{t=0}^{T}x_{m,t}^2)^{1/2}}$. The convergences of the numerator and denominator are given above. The asymptotic distributions of t-statistics are derived by the continuous mapping theorem:

$$
t_m \xrightarrow{L} \frac{\sigma^2 J\int_0^1 B'U^mD^mV^mdB(r)}{\sigma^2 J(\int_0^1 B'(r)U^mD^mD^mU^mB(r)dr)^{1/2}}, m = 1, \pi, m_{even} \tag{8.9}
$$

In the decomposition in A2.1, when m=1, we get $u_1^1 = v_1^1 = \frac{1}{\sqrt{s}}(1, 1, ..., 1)'$ so we could write $B'(r)u_1^1 = (v_1^1)'B(r) = W_1(r)$. When $m = \pi$, we have $u_1^\pi = v_1^\pi = \frac{1}{\sqrt{s}}(1, -1, ..., 1, -1)'$, so we could get $B'(r)u_1^\pi = (v_1^\pi)'B(r) = W_\pi(r)$, where $W_1(r)$ and $W_\pi(r)$ are mutually independent Brownian motions. Then for the numerator and denominator of the t-statistics in (8.9), when $m = 1, \pi$,

$$
B'(r)U^mD^mV^mdB(r) = B'(r)u_1^mS(v_1^m)'dB(r) = SW_m(r)dW_m(r)
$$

$$
B'(r)U^mD^mD^mU^mB(r) = B'(r)u_1^mS^2(u_1^m)'B(r) = S^2W_m(r)^2
$$

When $m = m_{even}$, we have $v_1^m = u_1^m = \sqrt{\frac{2}{S}}[1, cos\theta_m, cos(2\theta_m), ..., cos((S-1)\theta_m]'$, $v_2^m = u_2^m = \sqrt{\frac{2}{S}}[0, sin\theta_m, sin(2\theta_m), ..., sin((S-1)\theta_m]'$, then we get $B'(r)u_1^m = (v_1^m)'B(r) = W_m(r)$, $B'(r)u_2^m = (v_2^m)'B(r) = W_{m+1}(r)$. Because v_1^m and u_1^m are orthogonal, $W_m(r)$ and $W_{m+1}(r)$ are independent. We have the following equation:

$$
\begin{aligned}
B'(r)U^mD^mV^mdB(r) &= B'(r)u_1^m\frac{S}{2}(v_1^m)'dB(r) + B'(r)u_2^m\frac{S}{2}(v_2^m)'dB(r) \\
&= \frac{S}{2}W_m(r)dW_m(r) + \frac{S}{2}W_{m+1}(r)dW_{m+1}(r)
\end{aligned}
$$

$$\begin{aligned} B'(r)U^mD^mD^mU^mB(r) &= B'(r)u_1^m\frac{S^2}{4}(u_1^m)'B(r)+B'(r)u_2^m\frac{S^2}{4}(u_2^m)'B(r) \\ &= \frac{S^2}{4}W_m(r)^2+\frac{S^2}{4}W_{m+1}(r)^2 \end{aligned}$$

Substitute the terms above into (8.9), and we could get the asymptotic distributions for t_1, t_π and $t_{m_{even}}$.

For $m = m_{odd}$, the asymptotic distributions of the t-statistics are derived in the same way. We could get (8.10):

$$t_m \xrightarrow{L} \frac{\sigma^2 J\int_0^1 B'(r)U^{m*}D^{m*}V^{m*}dB(r)}{\sigma^2 J(\int_0^1 B'(r)U^{m*}D^{m*}D^{m*}U^{m*}B(r)dr)^{1/2}}, m = m_{odd} \tag{8.10}$$

From the decomposition derived in A2.1, we get $-B'(r)u_1^{m*} = (v_2^{m*})'B(r) = W_m(r)$, $B'(r)u_2^{m*} = (v_1^{m*})'B(r) = W_{m-1}(r)$

$$\begin{aligned} B'(r)U^{m*}D^{m*}V^{m*}dB(r) &= B'(r)u_1^{m*}\frac{S}{2}(v_1^{m*})'dB(r)+B'(r)u_2^{m*}\frac{S}{2}(v_2^{m*})'dB(r) \\ &= -\frac{S}{2}W_m(r)dW_{m-1}(r)+\frac{S}{2}W_{m-1}(r)dW_m(r) \end{aligned}$$

$$\begin{aligned} B'(r)U^{m*}D^{m*}D^{m*}U^{m*}B(r) &= B'(r)u_1^{m*}\frac{S^2}{4}(u_1^{m*})'B(r)+B'(r)u_2^{m*}\frac{S^2}{4}(u_2^{m*})'B(r) \\ &= \frac{S^2}{4}W_{m-1}(r)^2+\frac{S^2}{4}W_m(r)^2 \end{aligned}$$

Substitute the terms above into (8.10), and we could get the asymptotic distributions of $t_{m_{odd}}$. Theorem 1 is proofed.

Appendix III. Asymptotic distribution of HEGY type test statistics when (3.5) is employed for test.

In this section we consider the asymptotic distributions of the t-statistics when there are deterministic components in the auxiliary regression (3.5). We derive the asymptotic distributions of the numerators for t-statistics first. The asymptotic distributions of the denominators could be derived in the same way. Then with continuous mapping theorem, we could derive the asymptotic distributions of t-statistics. The following lemma is needed in our proof.

Lemma 4:

(a) $\frac{1}{\sigma}T^{-3/2}\sum_{t=0}^T x_{m,t} \xrightarrow{L} \begin{cases} \int_0^1 W_1(r)dr & m=1 \\ 0 & elsewhere \end{cases}$

(b) $\frac{1}{\sigma}T^{-5/2}\sum_{t=0}^T tx_{m,t} \xrightarrow{L} \begin{cases} \int_0^1 rW_1(r)dr & m=1 \\ 0 & elsewhere \end{cases}$

(c) $\frac{1}{\sigma^2}S^{-1}\sum_{p=1}^S \bar{x}_{m,p}\bar{\varepsilon}_p \xrightarrow{L} \begin{cases} W_m(1)\int_0^1 W_m(r)dr & m=1,\pi \\ \frac{1}{2}W_m(1)\int_0^1 W_m(r)dr+\frac{1}{2}W_{m+1}(1)\int_0^1 W_{m+1}(r)dr & m=m_{even} \\ \frac{1}{2}W_m(1)\int_0^1 W_{m-1}(r)dr-\frac{1}{2}W_{m-1}(1)\int_0^1 W_m(r)dr & m=m_{odd} \end{cases}$

$(d)\ \frac{1}{\sigma^2}(TS)^{-1}\sum_{p=1}^{S}\bar{x}_{m,p}^2 \xrightarrow{L} \begin{cases} (\int_0^1 W_m(r)dr)^2 & m=1,\pi \\ \frac{1}{4}(\int_0^1 W_m(r)dr)^2+\frac{1}{4}(\int_0^1 W_{m+1}(r)dr)^2 & m=m_{even} \\ \frac{1}{4}(\int_0^1 W_{m-1}(r)dr)^2+\frac{1}{4}(\int_0^1 W_m(r)dr)^2 & m=m_{odd} \end{cases}$

where $\overline{\varepsilon}_p = J^{-1}\sum\limits_{j=0}^{J-1}\varepsilon_{Sj+p}$, $\overline{x}_{m,p} = J^{-1}\sum\limits_{j=0}^{J-1}x_{m,Sj+p}$.

Proof (a): When $m = 1, \pi, m_{even}$, the left side of (a) could be rewritten in the form:

$$\begin{aligned}\frac{1}{\sigma}T^{-3/2}\sum_{t=1}^{T}x_{m,t} &= \frac{1}{\sigma}T^{-3/2}\sum_{p=1}^{S}\sum_{j=0}^{J-1}x_{m,Sj+p} \\ &= \frac{1}{\sigma}T^{-3/2}\sum_{p=1}^{S}\sum_{j=0}^{J-1}\sum_{q=p-1}^{p-S}\sum_{h=0}^{j}cos((p-q)\theta_m)\varepsilon_{Sh+q} \\ &= \frac{1}{\sigma}T^{-3/2}\sum_{p=1}^{S}\sum_{q=p-1}^{p-S}cos((p-q)\theta_m)\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q}\end{aligned}$$

With the convergence in (8.5), we have

$$\frac{1}{\sigma}T^{-3/2}\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q} \xrightarrow{L} \frac{1}{S\sqrt{S}}\int_0^1 B_i(r)dr$$

where $i = q$ when $q \geq 0$ and $i = S + q$ when $q < 0$. We also have the sum of the trg part, for any u_q

$$\sum_{p=1}^{S}cos((p-q)\theta_m)u_q = \begin{cases} Su_q & m=1 \\ 0 & m=\pi, m_{even}\end{cases}$$

With the convergence above and the sum of the cosine part:

$$\frac{1}{\sigma}T^{-3/2}\sum_{p=1}^{S}\sum_{q=p-1}^{p-S}cos((p-q)\theta_m)\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q} \xrightarrow{L} \begin{cases}\frac{1}{S}\sum_{i=1}^{S}\int_0^1 B_i(r)dr & m=1 \\ 0 & m=\pi, m_{even}\end{cases}$$

When $m = m_{odd}$, follow the similar procedure, and consider $\sum_{p=1}^{S} sin(p-q)\theta_{m-1}u_q = 0$, we get

$$\frac{1}{\sigma}T^{-3/2}\sum_{t=1}^{T}x_{m,t} = \frac{1}{\sigma}T^{-3/2}\sum_{p=1}^{S}\sum_{q=p-1}^{p-S}sin((p-q)\theta_{m-1})\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q} = 0$$

Note that $\frac{1}{\sqrt{S}}\sum_{p=1}^{S}\int_0^1 B_p(r)dr = \int_0^1\sum_{p=1}^{S}\frac{1}{\sqrt{S}}B_p(r)dr$, and substitute the convergences above with (8.7), (a) is proofed.

Proof (b): We directly apply the results in Park and Phillips (1988) and get the convergence when $m = 1$:

$$T^{-5/2}\sum_{t=0}^{T}tx_{m,t} = T^{-5/2}\sum_{t=0}^{T}\sum_{i=0}^{t-1}t\varepsilon_i \xrightarrow{L} \int_0^1 rW_1(r)dr$$

When $m = \pi, m_{even}$,

$$\begin{aligned} T^{-5/2}\sum_{t=0}^{T} tx_{m,t} &= T^{-5/2}\sum_{p=1}^{S}\sum_{j=0}^{J-1}(Sj+p)\sum_{q=p-1}^{p-S} cos(\theta_m(p-q))\sum_{h=0}^{j}\varepsilon_{Sh+q} \\ &= T^{-5/2}J\sum_{p=1}^{S} p\overline{x}_{m,p} + T^{-5/2}S\sum_{p=1}^{S}\sum_{j=0}^{J-1} j\sum_{q=1}^{S} cos(\theta_m(p-q))\sum_{h=0}^{j}\varepsilon_{S(h-1)+q} \\ &\quad + T^{-5/2}S\sum_{p=2}^{S}\sum_{j=0}^{J-1} j\sum_{q=1}^{p-1} cos(\theta_m(p-q))\varepsilon_{Sh+q} \end{aligned}$$

The first and third term above vanish when T grows large. For the second term, rewrite as:

$$\sum_{p=1}^{S}\sum_{j=0}^{J-1} j\sum_{q=1}^{S} cos(\theta_m(p-q))\sum_{h=0}^{j}\varepsilon_{S(h-1)+q} = \sum_{j=0}^{J-1}\sum_{q=1}^{S}\sum_{p=1}^{S} jcos(\theta_m(p-q))\sum_{h=0}^{j}\varepsilon_{S(h-1)+q}$$

Similar with (a): $\sum_{p=1}^{S} jcos(\theta_m(p-q))\sum_{h=0}^{j} u_{Sh+q} = 0$, then the second term equals to 0.

When $m = m_{odd}$, follow the same procedure, we get $T^{-5/2}\sum_{t=0}^{T} tx_{m,t} = 0$

Proof (c): When $m = \pi, m_{even}$,, we get the expression below:

$$\begin{aligned} \sum_{j=0}^{J-1} x_{m,Sj+p} &= \sum_{j=0}^{J-1}\sum_{q=p-1}^{p-S} cos(\theta_m(p-q))\sum_{h=0}^{j}\varepsilon_{Sh+q} \\ &= \sum_{q=p-1}^{p-S} cos(\theta_m(p-q))\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q} \end{aligned}$$

Then we have the right part of (c):

$$\begin{aligned} T^{-1}J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p &= T^{-1}J\sum_{p=1}^{S}(J^{-1}\sum_{j=0}^{J-1} x_{m,Sj+p})(J^{-1}\sum_{j=0}^{J-1}\varepsilon_{Sj+p}) \\ &= \frac{1}{S}\sum_{p=1}^{S}[\sum_{q=p-1}^{p-S} cos(\theta_m(p-q))J^{-\frac{3}{2}}\sum_{j=0}^{J-1}\sum_{h=0}^{j}\varepsilon_{Sh+q}](J^{-\frac{1}{2}}\sum_{j=0}^{J-1}\varepsilon_{Sj+p}) \end{aligned}$$

With (7.6), let $Z = [\int_0^1 B_1(r)dr, \int_0^1 B_2(r)dr, ..., \int_0^1 B_S(r)dr]$, $B = [B_1(r), B_2(r), ..., B_{23}(r)]$, the convergence of the term above is obtained.

$$T^{-1}J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p \xrightarrow{L} \frac{\sigma^2}{S}\sum_{p=1}^{S}\sum_{q=1}^{S} cos(\theta_m(p-q))Z_qB_p = \frac{\sigma^2}{24}Z'A_mB$$

With the decomposition in A2.1, similar with the proof of Lemma 1, we derive the convergence when $m = 1, \pi, m_{even}$.

With the same procedure, when $m = m_{odd}$, we derive the following convergence:

$$T^{-1}J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p \xrightarrow{L} \frac{\sigma^2}{S}\sum_{p=1}^{S}\sum_{q=1}^{S} sin(\theta_{m-1}(p-q))Z_qB_p = \frac{\sigma^2}{24}Z'C_mB$$

Similar as $m = 1, \pi, m_{even}$, we could derive the convergence when $m = m_{odd}$. (c) is proofed.

Proof (d): For the proof in (c), replace $\overline{\varepsilon}_p$ with $\overline{x}_{m,p}$, and we could get

$$T^{-2}J\sum_{p=1}^{S}\overline{x}_{m,p}^2 = X'A_mA_m^{'}X\;(or\;X'C_mC_m^{'}X\;for\;m_{odd})$$

Follow the similar step with the proof of (c), also with the decomposition theory in A2.1, the proof would be finished.

Proof of Theorem 2:

Regress y_t, $x_{m,t}$ and ε_t on the deterministic component included in (3.5), we could re express (3.5) as

$$\varphi^*(B)(1-B^S)\widetilde{y}_t = \sum_{m=1}^{S}\rho_m\widetilde{x}_{m,t} + \widetilde{\varepsilon}_t \tag{8.11}$$

where $\widetilde{y}_t = y_t - c_{0,y} - c_{1,y}t - \sum_{i=2}^{S}c_{i,y}D_{i,t}$, $\widetilde{x}_{m,t} = x_{m,t} - c_{0,m} - c_{1,m}t - \sum_{i=2}^{S}c_{i,m}D_{i,t}$, $\widetilde{\varepsilon}_t = \varepsilon_t - c_{0,\varepsilon} - c_{1,\varepsilon}t - \sum_{i=2}^{S}c_{i,\varepsilon}D_{i,t}$.

Asymptotically the vector of seasonal dummies is orthogonal to the vector of trend terms (Beaulieu and Miron, 1992), therefore we could express the terms above with trend and seasonal means:

$\widetilde{x}_{m,t} = x_{m,t} - c_{1,m}(t-\overline{t}) - d_{m,t}$, $d_{m,t} = c_{0,m} + \sum_{i=2}^{S}c_{i,m}D_{i,t} + c_{1,m}\overline{t}$ is the seasonal mean of $x_{m,t}$, i.e., $\overline{x}_{m,p}$ when it is the p^{th} observation in a circle.

$\widetilde{\varepsilon}_t = \varepsilon_t - c_{1,\varepsilon}(t-\overline{t}) - d_{\varepsilon,t}$, $d_{\varepsilon,t} = c_{0,\varepsilon} + \sum_{i=2}^{S}c_{i,\varepsilon}D_{i,t} + c_{1,\varepsilon}\overline{t}$.

Note (i) When (3.5) includes constant, $c_{1,y} = c_{1,m} = c_{1,\epsilon} = 0$, $c_{i,y} = c_{i,m} = c_{i,\varepsilon} = 0$ for $i = 2, ..., S$.

(ii) When (3.5) includes constant and trend, $c_{i,y} = c_{i,m} = c_{i,\varepsilon} = 0$ for $i = 2, ..., S$.

(iii) When (3.5) includes constant and seasonal dummies, $c_{1,y} = c_{1,m} = c_{1,\epsilon} = 0$.

Due to the asymptotic orthogonality we get the coefficient of each term $c_{1,m} = \frac{\sum_{t=0}^{T}(x_{m,t}-\overline{x}_m)(t-\overline{t})}{\sum_{t=1}^{T}(t-\overline{t})^2} \approx \frac{12\sum x_{m,t}t - 6\overline{x}_mT}{T^3}$, $c_{1,\varepsilon} = \frac{\sum_{t=0}^{T}(\varepsilon_t-\overline{\varepsilon})(t-\overline{t})}{\sum_{t=1}^{T}(t-\overline{t})^2} \approx \frac{12\sum\varepsilon_t t - 6\overline{\varepsilon}T}{T^3}$. The approximation comes from $\sum_{t=1}^{T}(t-\overline{t})^2 \approx \frac{T^3}{12}$ and $T+1 \approx T$. The same expressions and approximations of these terms are also derived by Beaulieu and Miron (1992).

With (8.11), the t-statistics could be expressed as $t_m = \frac{\hat{\rho}_m}{\hat{\sigma}_{\rho_m}} = \frac{\sum_{t=0}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t}{s_T(\sum_{t=0}^{T}\widetilde{x}_{m,t}^2)^{1/2}}$.

For the numerator:

$$
\begin{aligned}
\sum_{t=1}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t &= \sum_{t=0}^{T}(x_{m,t}-c_{1,m}(t-\overline{t})-d_{m,t})(\varepsilon_t-c_{1,\varepsilon}(t-\overline{t})-d_{\varepsilon,t}) \\
&= \sum_{t=1}^{T}x_{m,t}\varepsilon_t-\sum_{t=1}^{T}x_m c_{1,\varepsilon}(t-\overline{t})-\sum_{t=1}^{T}x_{m,t}d_{\varepsilon,t}-\sum_{t=1}^{T}c_{1,m}(t-\overline{t})\varepsilon_t \\
&\quad+\sum_{t=1}^{T}c_{1,m}c_{1,\varepsilon}(t-\overline{t})^2+\sum_{t=1}^{T}d_{\varepsilon,t}c_{1,m}(t-\overline{t})-\sum_{t=0}^{T}d_{m,t}\varepsilon_t \\
&\quad+\sum_{t=0}^{T}d_{m,t}c_{1,\varepsilon}(t-\overline{t})-\sum_{t=0}^{T}d_{m,t}d_{\varepsilon,t}
\end{aligned}
$$

For the terms above, $\sum_{t=1}^{T} x_{m,t}d_{\varepsilon,t} = \sum_{t=0}^{T} d_{m,t}\varepsilon_t = \sum_{t=0}^{T} d_{m,t}d_{\varepsilon,t} = J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p$, $\sum_{t=1}^{T} c_{1,m}c_{1,\varepsilon}(t-\overline{t}.)^2 = c_{1,m}\sum_{t=1}^{T}(t-\overline{t})(\varepsilon_t-\overline{\varepsilon}) = c_{1,m}\sum_{t=1}^{T}\varepsilon_t(t-\overline{t})$

Next we show that $\sum_{t=1}^{T} d_{m,t}c_{1,\varepsilon}(t-\overline{t})$ and $\sum_{t=1}^{T} x_m c_{1,\varepsilon}(t-\overline{t})$ converge to 0 when T goes to infinity.

$$
\begin{aligned}
\sum_{t=1}^{T}d_{m,t}c_{1,\varepsilon}(t-\overline{t}) &= c_{1,\varepsilon}\sum_{p=1}^{S}\overline{x}_{m,p}\sum_{j=0}^{J-1}(Sj+p-\frac{T+1}{2}) = c_{1,\varepsilon}\sum_{p=1}^{S}\overline{x}_{m,p}J(p-\frac{S+1}{2}) \\
&= \frac{12}{T^3}(\sum_{t=1}^{T}\varepsilon_t t)(\frac{T}{S}\sum_{p=1}^{S}\overline{x}_{m,p}(p-\frac{S+1}{2}))-\frac{6}{T^3}\overline{\varepsilon}\sum_{p=1}^{S}\overline{x}_{m,p}J(p-\frac{S+1}{2})
\end{aligned}
$$

The last term comes from substituting $c_{1,\varepsilon}$ with its approximation term. For the first term, $\frac{\sum\varepsilon_t t}{T^{3/2}}$ converge according to Hamilton (See Hamilton, 1994, page 486), $\frac{1}{TS}\left|\sum_{p=1}^{S}\overline{x}_{m,p}(p-\frac{S+1}{2})\right| < \frac{\sum_{p=1}^{S}\overline{x}_{m,p}^2}{2TS}+\frac{\sum_{p=1}^{S}(2p-S-1)^2}{8TS}$ both converge to 0 when T goes to infinity (Lemma 4 (d)), so the first term converges to 0 when T goes to infinity. Similarly, the second term also converges to 0 as T goes to infinity.

With the same method, $\sum_{t=1}^{T} x_m c_{1,\varepsilon}(t-\overline{t})$ also converges to 0 as T goes to infinity.

Then for the numerator of the t-statistics becomes:

$$
\sum_{t=1}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t = \sum_{t=1}^{T}x_{m,t}\varepsilon_t - J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p - \sum_{t=1}^{T}x_m c_{1,\varepsilon}(t-\overline{t}) \tag{8.12}
$$

For the last term, again substitute $c_{1,\varepsilon}$ with its approximation term, we get:

$$
-\sum_{t=1}^{T}x_{m,t}c_{1,\varepsilon}(t-\overline{t}) = -3T\overline{x}_m\overline{\varepsilon}-12T^{-3}(\sum_{t=1}^{T}x_{m,t}t)(\sum_{t=1}^{T}t\varepsilon_t)+6T^{-1}\overline{x}_m\sum_{t=1}^{T}t\varepsilon_t+6T^{-1}\overline{\varepsilon}\sum_{t=1}^{T}x_{m,t}t
$$

Substitute the term above in (8.12), we get (8.13)

$$
\begin{aligned}
\sum_{t=1}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t = \sum_{t=1}^{T}x_{m,t}\varepsilon_t - J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p - 3T\overline{x}_m\overline{\varepsilon}-12T^{-3}(\sum_{t=1}^{T}x_{m,t}t)(\sum_{t=1}^{T}t\varepsilon_t) \\
+6T^{-1}\overline{x}_m\sum_{t=1}^{T}t\varepsilon_t+6T^{-1}\overline{\varepsilon}\sum_{t=1}^{T}x_{m,t}t
\end{aligned}
\tag{8.13}
$$

Divide each term by T, (8.13) would be reexpress as:

$$\sum_{t=1}^{T} x_{m,t}\varepsilon_t - \frac{1}{s}\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p - 3(T^{-3/2}\sum_{t=1}^{T} x_{m,t})(T^{-1/2}\sum_{t=1}^{T}\varepsilon_t) - 12(T^{-5/2}\sum_{t=1}^{T} x_{m,t}t)(T^{-3/2}\sum_{t=1}^{T} t\varepsilon_t)$$

$$+6(T^{-3/2}\sum_{t=1}^{T} x_{m,t})(T^{-3/2}\sum_{t=1}^{T} t\varepsilon_t) + 6(T^{-1/2}\sum_{t=1}^{T}\varepsilon_t)(T^{-5/2}\sum_{t=1}^{T} x_{m,t}t) \tag{8.14}$$

We directly give the covergence in Hamilton (See Hamilton 1994, Page 486):

$$\frac{1}{\sigma}T^{-1/2}\sum_{t=1}^{T}\varepsilon_t \xrightarrow{L} W_1(1) \tag{8.15}$$

$$\frac{1}{\sigma}T^{-3/2}\sum_{t=1}^{T} t\varepsilon_t \xrightarrow{L} W_1(1) - \int_0^1 W_1(r)dr \tag{8.16}$$

With (8.15), (8.16) and Lemma 4, we could get the asymptotic distributions for each term. Thus we could get the asymptotic distribution of the numerator of the t-statistics when constant, trend and seasonal dummies are included in regression (3.5).

When different deterministic components are included in (3.5), certain changes are needed for (8.13):

When only constant is included in (3.5), $c_{1,y} = c_{1,m} = c_{1,\epsilon} = 0$, $c_{i,y} = c_{i,m} = c_{i,\varepsilon} = 0$ for $i = 2, ..., S$. Then we get $d_{m,t} = c_{0,m} = \overline{x}_m$, and $d_{\varepsilon,t} = c_{0,\varepsilon} = \overline{\varepsilon}$, thus we get $J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p = \sum_{t=0}^{T} d_{m,t}d_{\varepsilon,t} = T\overline{x}_m\overline{\varepsilon}$, (8.13) would change to $\sum_{t=1}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t = \sum_{t=1}^{T} x_{m,t}\varepsilon_t - T\overline{x}_m\overline{\varepsilon}$.

When constant and trend are included in (3.5), $c_{i,y} = c_{i,m} = c_{i,\varepsilon} = 0$ for $i = 2, ..., S$. Same as above we get $J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p = T\overline{x}_m\overline{\varepsilon}$, (8.13) would change to $\sum_{t=1}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t = \sum_{t=1}^{T} x_{m,t}\varepsilon_t - T\overline{x}_m\overline{\varepsilon} - \sum_{t=1}^{T} x_m c_{1,\varepsilon}(t-\overline{t})$.

When constant and dummies are included in (3.5), $c_{1,y} = c_{1,m} = c_{1,\epsilon} = 0$, (8.13) would change to $\sum_{t=1}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t = \sum_{t=1}^{T} x_{m,t}\varepsilon_t - J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p$.

So we could express (8.13) as

$$\sum_{t=1}^{T}\widetilde{x}_{m,t}\widetilde{\varepsilon}_t = \sum_{t=1}^{T} x_{m,t}\varepsilon_t - T\overline{x}_m\overline{\varepsilon} - 1^{\mu}[J\sum_{p=1}^{S}\overline{x}_{m,p}\overline{\varepsilon}_p - T\overline{x}_m\overline{\varepsilon}] - 1^{t}[3T\overline{x}_m\overline{\varepsilon} - 12T^{-3}(\sum_{t=1}^{T} x_{m,t}t)(\sum_{t=1}^{T} t\varepsilon_t)$$

$$+6T^{-1}\overline{x}_m\sum_{t=1}^{T} t\varepsilon_t + 6T^{-1}\overline{\varepsilon}\sum_{t=1}^{T} x_{m,t}t] \tag{8.17}$$

With Lemma 4, (8.15) and (8.16), the asymptotic distributions of the numerators for t-statistics are derived.

For the denominator, we apply the same procedure above and substitute ε_t with $x_{m,t}$, we could derive the asymptotic distributions of the denominator of the statistics with Lemma 4. Then with the continuous mapping theorem we could derive the asymptotic distributions of the t-statistics. Theorem 2 is proofed.

Appendix IV

A4.1 Critical value for hourly data when different deterministic components included.

Table 14: Critical values for the t_1 of hourly data. Only intercept included

Sample size T	Probability that t_1 is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-3.21	-2.85	-2.55	-2.26	-0.29	0.03	0.34	0.72
T=240	-3.22	-2.92	-2.70	-2.43	-0.41	-0.04	0.25	0.65
T=360	-3.28	-3.00	-2.89	-2.49	-0.44	-0.10	0.24	0.65
T=480	-3.35	-3.08	-2.82	-2.52	-0.44	-0.10	0.23	0.66
$T=\infty$	-3.41	-3.14	-2.89	-2.57	-0.44	-0.07	0.26	0.61

Table 15: Critical values for the t_{24} of hourly data. Only intercept included

Sample size T	Probability that t_2 is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-2.26	-1.92	-1.66	-1.36	0.91	1.28	1.65	2.01
T=240	-2.52	-2.11	-1.81	-1.49	0.90	1.26	1.63	1.99
T=360	-2.52	-2.16	-1.84	-1.52	0.89	1.25	1.63	1.98
T=480	-2.53	-2.19	-1.90	-1.57	0.86	1.23	1.60	1.96
$T=\infty$	-2.60	-2.24	-1.96	-1.63	0.90	1.28	1.61	2.03

Table 16: Critical values for the F-statistics of hourly data. Only intercept included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.01	0.03	0.05	0.11	2.02	2.62	3.24	4.08
T=240	0.01	0.03	0.05	0.11	2.22	2.86	3.52	4.41
T=360	0.01	0.03	0.05	0.11	2.30	2.95	3.60	4.51
T=480	0.01	0.03	0.05	0.11	2.33	3.00	3.68	4.58
$T=\infty$	0.01	0.03	0.05	0.11	2.41	3.13	3.81	4.60

Table 17: Critical values for the F-statistics of hourly data. Only intercept included

Sample size T	Probability that $Fall = \frac{1}{24}\sum_1^{24} t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.43	0.49	0.54	0.61	1.32	1.48	1.62	1.79
T=240	0.48	0.55	0.61	0.69	1.43	1.58	1.71	1.91
T=360	0.49	0.55	0.62	0.70	1.50	1.62	1.75	1.95
T=480	0.51	0.57	0.64	0.72	1.52	1.66	1.81	2.00
$T=\infty$	0.54	0.61	0.68	0.77	1.57	1.72	1.85	2.00

Table 18: Critical values for the t_1 of hourly data. Intercept and trend included

Sample size T	Probability that t_1 is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-3.63	-3.33	-3.06	-2.79	-0.99	-0.68	-0.41	-0.10
T=240	-3.82	-3.51	-3.25	-2.96	-1.14	-0.84	-0.61	-0.28
T=360	-3.78	-3.51	-3.27	-2.99	-1.16	-0.88	-0.60	-0.27
T=480	-3.91	-3.57	-3.35	-3.05	-1.19	-0.92	-0.64	-0.34
$T=\infty$	-3.98	-3.66	-3.40	-3.12	-1.25	-0.95	-0.69	-0.30

Table 19: Critical values for the t_{24} of hourly data. Intercept and trend included

Sample size T	Probability that t_{24} is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-2.28	-1.94	-1.67	-1.37	0.91	1.29	1.59	1.95
T=240	-2.40	-2.08	-1.80	-1.49	0.97	1.34	1.66	2.03
T=360	-2.48	-2.14	-1.83	-1.52	0.88	1.26	1.59	2.02
T=480	-2.52	-2.20	-1.92	-1.57	0.90	1.27	1.58	2.00
T= ∞	-2.58	-2.27	-1.97	-1.64	0.91	1.31	1.60	2.02

Table 20: Critical values for the F-statistics of hourly data. Intercept and trend included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.01	0.03	0.05	0.10	2.01	2.63	3.25	4.09
T=240	0.01	0.03	0.05	0.11	2.21	2.85	3.51	4.38
T=360	0.01	0.03	0.05	0.11	2.29	2.97	3.62	4.50
T=480	0.01	0.03	0.05	0.11	2.32	3.00	3.69	4.58
T= ∞	0.01	0.03	0.06	0.11	2.46	3.16	3.88	4.64

Table 21: Critical values for the F-statistics of hourly data. Intercept and trend included

Sample size T	Probability that $Fall = \frac{1}{24}\sum_1^{24} t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.54	0.61	0.67	0.76	1.53	1.67	1.86	2.27
T=240	0.54	0.61	0.67	0.75	1.54	1.68	1.81	2.01
T=360	0.56	0.65	0.69	0.78	1.54	1.69	1.85	2.06
T=480	0.57	0.64	0.71	0.80	1.59	1.74	1.89	2.06
T= ∞	0.61	0.69	0.76	0.85	1.66	1.82	1.94	2.11

Table 22: Critical values for the t_1 of hourly data. Intercept and dummies included

Sample size T	Probability that $t_1$is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-2.89	-2.56	-2.32	-2.03	-0.20	0.07	0.33	0.68
T=240	-3.14	-2.89	-2.61	-2.32	-0.38	-0.04	0.23	0.60
T=360	-3.27	-2.94	-2.67	-2.39	-0.40	-0.07	0.22	0.57
T=480	-3.27	-2.98	-2.70	-2.42	-0.39	-0.05	0.23	0.62
T= ∞	-3.45	-3.13	-2.87	-2.57	-0.47	-0.10	0.21	0.57

Table 23: Critical values for the t_{24} of hourly data. Intercept and dummies included

Sample size T	Probability that t_{24} is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-2.99	-2.73	-2.45	-2.07	-0.23	-0.06	0.37	0.75
T=240	-3.24	-2.91	-2.64	-2.36	-0.34	-0.04	0.25	0.64
T=360	-3.30	-2.98	-2.74	-2.45	-0.39	-0.04	0.26	0.63
T=480	-3.30	-3.00	-2.76	-2.47	-0.39	-0.05	0.23	0.63
T= ∞	-3.46	-3.15	-2.87	-2.58	-0.44	-0.12	0.20	0.55

Table 24: Critical values for the F-statistics of hourly data. Intercept and dummies included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.05	0.12	0.23	0.42	3.68	4.45	5.23	6.20
T=240	0.09	0.20	0.35	0.62	4.71	5.57	6.42	7.48
T=360	0.09	0.21	0.39	0.68	5.02	5.95	6.83	7.97
T=480	0.10	0.23	0.41	0.71	5.15	6.11	7.11	8.10
T= ∞	0.09	0.23	0.44	0.77	5.60	6.60	7.57	8.54

Table 25: Critical values for the F-statistics of hourly data. Intercept and dummies included

Sample size T	Probability that $Fall = \frac{1}{24}\sum_1^{24} t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	1.02	1.12	1.23	1.35	2.50	2.72	2.93	3.20
T=240	1.45	1.58	1.70	1.87	3.15	3.37	3.57	3.85
T=360	1.60	1.74	1.88	2.02	3.38	3.60	3.81	4.04
T=480	1.66	1.80	1.93	2.09	3.46	3.69	3.89	4.16
T= ∞	1.89	2.03	2.17	2.33	3.76	4.00	4.21	4.42

Table 26: Critical values for the t_1 of hourly data. Intercept, trend and dummies included

Sample size T	Probability that $t_1$is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-3.34	-3.05	-2.80	-2.53	-0.79	-0.50	-0.33	-0.12
T=240	-3.59	-3.34	-3.09	-2.82	-1.08	-0.80	-0.56	-0.27
T=360	-3.77	-3.45	-3.18	-2.91	-1.14	-0.82	-0.54	-0.24
T=480	-3.85	-3.51	-3.25	-2.99	-1.16	-0.87	-0.59	-0.26
T= ∞	-3.98	-3.68	-3.41	-3.11	-1.22	-0.92	-0.65	-0.30

Table 27: Critical values for the t_{24} of hourly data. Intercept, trend and dummies included

Sample size T	Probability that t_{24} is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	-2.89	-2.54	-2.31	-2.05	-0.17	0.15	0.45	0.83
T=240	-3.13	-2.86	-2.62	-2.33	-0.34	0.01	0.29	0.66
T=360	-3.34	-3.00	-2.71	-2.41	-0.34	0.00	0.27	0.63
T=480	-3.29	-2.99	-2.72	-2.45	-0.40	-0.07	0.24	0.63
T= ∞	-3.38	-3.08	-2.84	-2.55	-0.44	-0.07	0.29	0.70

Table 28: Critical values for the F-statistics of hourly data. Intercept, trend and dummies included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.05	0.12	0.23	0.42	3.68	4.46	5.21	6.22
T=240	0.09	0.20	0.37	0.63	4.65	5.53	6.35	7.41
T=360	0.10	0.22	0.40	0.69	4.98	5.90	6.78	7.88
T=480	0.10	0.23	0.41	0.71	5.17	6.11	7.02	8.11
T= ∞	0.10	0.25	0.45	0.82	5.55	6.56	7.61	8.87

Table 29: Critical values for the F-statistics of hourly data. Intercept, trend and dummies included

Sample size T	Probability that $Fall = \frac{1}{24}\sum_1^{24} t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	1.03	1.16	1.26	1.38	2.59	2.81	3.02	3.34
T=240	1.52	1.65	1.77	1.93	3.23	3.46	3.67	3.90
T=360	1.66	1.82	1.94	2.11	3.45	3.65	3.86	4.09
T=480	1.74	1.89	2.03	2.20	3.54	3.77	3.99	4.22
T= ∞	1.88	2.06	2.21	2.40	3.84	4.07	4.28	4.54

A4.2 Critical value for daily data when different deterministic components included.

Table 30: Critical values for the t_1 of daily data. Only intercept included

Sample size T	Probability that t_1 is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	-3.40	-3.05	-2.78	-2.51	-0.38	-0.02	0.29	0.60
T=280	-3.45	-3.10	-2.83	-2.52	-0.40	-0.06	0.23	0.55
T=420	-3.40	-3.12	-2.88	-2.56	-0.43	-0.06	0.26	0.60
T=560	-3.41	-3.12	-2.89	-2.55	-0.44	-0.08	0.25	0.61
T= ∞	-3.41	-3.13	-2.91	-2.57	-0.45	-0.09	0.26	0.61

Table 31: Critical values for the F-statistics of daily data. Only intercept included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.01	0.03	0.05	0.11	2.29	3.01	3.73	4.64
T=280	0.01	0.03	0.06	0.11	2.34	3.03	3.70	4.55
T=420	0.01	0.03	0.05	0.11	2.37	3.08	3.75	4.59
T=560	0.01	0.03	0.06	0.12	2.40	3.10	3.82	4.70
T= ∞	0.01	0.03	0.05	0.12	2.40	3.13	3.83	4.70

Table 32: Critical values for the F-statistics of daily data. Only intercept included

Sample size T	Probability that $Fall = \frac{1}{7}\sum_1^{24} t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.27	0.35	0.45	0.58	2.09	2.42	2.76	3.19
T=280	0.28	0.37	0.46	0.59	2.13	2.43	2.75	3.13
T=420	0.27	0.36	0.45	0.59	2.18	2.50	2.81	3.23
T=560	0.27	0.37	0.47	0.61	2.19	2.50	2.80	3.17
T= ∞	0.28	0.39	0.48	0.63	2.19	2.55	2.83	3.21

Table 33: Critical values for the $t_1$of daily data. Intercept and trend included

Sample size T	Probability that $t_1$is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	-3.96	-3.64	-3.35	-3.07	-1.18	-0.88	-0.58	-0.26
T=280	-3.96	-3.66	-3.40	-3.10	-1.20	-0.89	-0.61	-0.27
T=420	-3.93	-3.62	-3.37	-3.11	-1.24	-0.93	-0.66	-0.30
T=560	-3.94	-3.66	-3.40	-3.11	-1.24	-0.93	-0.66	-0.29
T= ∞	-3.96	-3.66	-3.40	-3.12	-1.25	-0.95	-0.69	-0.30

Table 34: Critical values for the F-statistics of daily data. Intercept and trend included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=120	0.01	0.03	0.05	0.11	2.30	2.98	3.67	4.62
T=240	0.01	0.03	0.05	0.11	2.35	3.05	3.76	4.68
T=360	0.01	0.03	0.05	0.12	2.39	3.08	3.78	4.60
T=480	0.01	0.03	0.05	0.12	2.42	3.10	3.80	4.60
T= ∞	0.01	0.03	0.06	0.12	2.47	3.15	3.81	4.61

Table 35: Critical values for the F-statistics of daily data. Intercept and trend included

Sample size T	Probability that $Fall = \frac{1}{7}\sum_1^7 t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.42	0.54	0.66	0.81	2.50	2.85	3.18	3.60
T=280	0.41	0.53	0.66	0.83	2.54	2.89	3.22	3.66
T=420	0.44	0.55	0.68	0.82	2.57	2.91	3.24	3.62
T=560	0.42	0.55	0.66	0.83	2.56	2.92	3.25	3.69
T= ∞	0.45	0.56	0.67	0.84	2.55	2.92	3.26	3.67

Table 36: Critical values for the t_1 of daily data. Intercept and dummies included

Sample size T	Probability that $t_1$is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	-3.33	-3.04	-2.76	-2.47	-0.38	-0.01	0.29	0.63
T=280	-3.36	-3.07	-2.76	-2.49	-0.39	-0.03	0.25	0.56
T=420	-3.39	-3.09	-2.83	-2.52	-0.40	-0.05	0.21	0.63
T=560	-3.41	-3.08	-2.83	-2.54	-0.44	-0.09	0.23	0.60
T= ∞	-3.45	-3.13	-2.87	-2.57	-0.47	-0.10	0.22	0.61

Table 37: Critical values for the F-statistics of daily data. Intercept and dummies included

Sample size T	Probability that $F_{m,m+1}$, $m = m_{even}$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.11	0.23	0.41	0.72	5.23	6.20	7.18	8.27
T=280	0.10	0.23	0.43	0.75	5.33	6.38	7.36	8.51
T=420	0.11	0.25	0.45	0.77	5.51	6.53	7.50	8.61
T=560	0.11	0.25	0.44	0.77	5.53	6.55	7.51	8.63
T= ∞	0.12	0.26	0.44	0.77	5.60	6.58	7.57	8.65

Table 38: Critical values for the F-statistics of daily data. Intercept and dummies included

Sample size T	Probability that $Fall = \frac{1}{7}\sum_1^7 t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.92	1.15	1.36	1.62	4.11	4.57	4.98	5.45
T=280	0.99	1.22	1.44	1.70	4.23	4.69	5.14	5.64
T=420	0.98	1.24	1.44	1.70	4.29	4.79	5.17	5.64
T=560	1.04	1.26	1.47	1.73	4.33	4.78	5.19	5.70
T= ∞	1.06	1.25	1.49	1.76	4.32	4.82	5.21	5.72

Table 39: Critical values for the t_1 of daily data Intercept, trend and dummies included

Sample size T	Probability that $t_1$is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	-3.91	-3.60	-3.32	-3.03	-1.17	-0.88	-0.58	-0.29
T=280	-3.93	-3.62	-3.36	-3.10	-1.22	-0.92	-0.63	-0.27
T=420	-3.94	-3.65	-3.39	-3.07	-1.22	-0.92	-0.66	-0.32
T=560	-3.93	-3.64	-3.39	-3.10	-1.24	-0.92	-0.65	-0.33
T= ∞	-3.99	-3.66	-3.41	-3.11	-1.24	-0.92	-0.66	-0.33

Table 40: Critical values for the F-statistics of daily data. Intercept, trend and dummies included

Sample size T	Probability that $F_{m,m+1}$, $m = cosi$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	0.10	0.22	0.40	0.70	5.22	6.17	7.11	8.23
T=280	0.11	0.24	0.42	0.74	5.41	6.40	7.35	8.60
T=420	0.11	0.26	0.45	0.77	5.48	6.47	7.47	8.62
T=560	0.12	0.26	0.45	0.78	5.53	6.53	7.47	8.63
T= ∞	0.10	0.26	0.48	0.85	5.55	6.56	7.59	8.80

Table 41: Critical values for the F-statistics of daily data. Intercept, trend and dummies included

Sample size T	Probability that $Fall = \frac{1}{7}\sum_1^7 t_k^2$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
T=140	1.08	1.34	1.56	1.84	4.43	4.92	5.34	5.91
T=280	1.21	1.44	1.66	1.93	4.60	5.06	5.49	6.08
T=420	1.23	1.46	1.69	2.00	4.65	5.14	5.59	6.05
T=560	1.23	1.47	1.69	1.99	4.68	5.16	5.60	6.16
T= ∞	1.25	1.48	1.75	2.02	4.74	5.21	5.71	6.18

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