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How do different densities in a network affect the optimal location of service centers?

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Abstract: The p -median problem is often used to locate p service centers by minimizing their distances to a geographically distributed demand (n). The optimal locations are sensitive to geographical context such as road network and demand points especially when they are asymmetrically distributed in the plane. Most studies focus on evaluating performances of the p -median model when p and n vary. To our knowledge this is not a very well-studied problem when the road network is alternated especially when it is applied in a real world context. The aim in this study is to analyze how the optimal location solutions vary, using the p -median model, when the density in the road network is alternated. The investigation is conducted by the means of a case study in a region in Sweden with an asymmetrically distributed population (15,000 weighted demand points), Dalecarlia. To locate 5 to 50 service centers we use the national transport administrations official road network (NVDB). The road network consists of 1.5 million nodes. To find the optimal location we start with 500 candidate nodes in the network and increase the number of candidate nodes in steps up to 67,000. To find the optimal solution we use a simulated annealing algorithm with adaptive tuning of the temperature. The results show that there is a limited improvement in the optimal solutions when nodes in the road network increase and p is low. When p is high the improvements are larger. The results also show that choice of the best network depends on p . The larger p the larger density of the network is needed.

Key words: location-allocation problem, inter-urban location, intra-urban location, p -median model, network distance, simulated annealing heuristics.

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1. Introduction

To have an as accurate representation of a road network as possible is important for many researchers and planners using the network for transportation and planning optimization. The focus is mainly on how the roads are used and maintained. Less focus is given to using the road network to locate facilities in order to minimize transportation. In this study we focus on the inter-urban and the intra-urban location allocation problem in relation to the density of the road network. To do so we turn to the p -median problem.

The p -median location problem is well-studied (Farahani *et al.*, 2012). However, most studies are not based on real road distances. Francis *et al.* (2009) made an explicit review of the p -median location problem. Among the 40 published articles, about half of them are studies based on real data. From that survey it is also obvious that almost all of the distance measures are Euclidean distance and rectilinear distance. In a recent study by Carling *et al.* (2012) the performance of the p -median model was evaluated when the distance measure was alternated between Euclidian, network and travel time. It was shown that for region with an asymmetrical distributed population and road network due to natural barriers the choice of distance measure has affected the optimal locations, and that the use of Euclidian distance leads to sub optimal solutions.

The work in this study follows the work of Carling *et al.* (2012). In Carling *et al.* (2012) the road network was limited to 1579 nodes and there was no analysis done of the effects on the suggested solutions by varying the number of nodes in the road network. However, differences in accuracy of the road networks could also influence the optimal location of service centers.

In a discrete location allocation problem complexity varies due to the number of demand points, number of service centers to locate and/or number of nodes in a network. However the p -median model is NP-hard (Kariv and Hakimi, 1979) and so aggregation has often been used to reduce the size of the problem. In our study we use a real world road network which consists of about 1.5 million nodes. To our knowledge there is no study which has used such a large real world network density applied on a discrete p -median problem. Based on that, the aim of this paper is to analyze

how the optimal location solutions vary, using the p -median model, when both the number of service centers and the density of the road network are alternated. The investigation is conducted by the means of a case study in a region of Sweden, Dalecarlia. The population is distributed at 15,000 weighted demand points. The road network we elaborate is from the Swedish digital road system: NVDB (The National Road Database) and it is administrated by the Swedish Traffic administration. We start with 500 candidate nodes to locate on and increase them in steps up to 67,000.—

To evaluate the effects of different road networks on the optimal location solutions in different situations we compare the results from the experiments in which we have alternated both the density in the road network and the number of service centers that are located within Dalecarlia. In this study we simulate an inter-urban location problem and both inter-urban and intra-urban location problems. The location of emergency hospitals and courts is typical inter-urban location problems in a region like Dalecarlia. To locate for instance high schools and post offices could be seen as typical inter-urban as well as intra-urban location problems. In this study we therefore systematically alternate P between 5 and 50.

To do this, several computer experiments using the p -median model were implemented. Since the exact optimal solution is difficult to obtain, the experiments are conducted by use of a simulated annealing algorithm.

The remaining parts of this paper are organized as follows. In section 2 we discuss some relevant literature. In section 3 we present the data used. In the fourth section we present the simulated annealing methods used. In section five we present and comment results and in section six we have a concluding discussion.

2. Literature Review

The discrete p -median model was first introduced by Hakimi (1964). The goal with the model is to find p service centers which minimize the summed distances between demands and their nearest centers. This problem can be formulated as follows.

Minimize $f = \sum_{i=1}^n \sum_{j=1}^n w_i d_{ij} x_{ij}$, subject to $\sum_{j=1}^n x_{ij} = 1$ and $\sum_{j=1}^n x_{jj} = p$, where f is the value of objective function. n is the number demand locations. w_i is the weight of each demand location. d_{ij} is the distance from demand location i to the center j . x_{ij} is a dummy variable: taking 1 if location i is allocated to center j .

Since we model our problem as a p -median problem, our objective function will be to minimize the value f which is the sum of all network distances between a person and the closest service center. (d_{ij} is one for the closest location in our case). By dividing this value by the total population, we obtain the average distance between a person and its closest service center.

To find the optimal location for p -service centers in relation to the demand using the p -median model is NP-hard, Kariv and Hakimi (1979). The complexity depends both on the number of service centers to be located, the number of demand points, as well as on how distance is measured.

Although Euclidean distance is most widely used, the network distance is in most cases more accurate in measuring the travel distance between two points (e.g. Carling *et al.* 2012). Further, a refined network should give the possibility to more accurate distance measures between two points compared to a sparser network. There are a few studies which evaluate network effects on optimal locations. Peeters and Thomas (1995) examined the p -median problem for different types of networks by changing the nature of the links. They found that there was a difference in optimal solutions when the links were changed but they registered no differences in computational effort.

Morris (1978) tested the linear programming algorithm for 600 random generated data sets. He generated a benchmark to simulate the effect of a road network by adding a random noise to the Euclidian distance. His conclusion was that regardless whether he was using the pure Euclidian distance or the simulated networks he was able to solve the problem, implying that the choice of distance measure is not significant. However, the data set were very small and it was only a simulated network with values close to the Euclidian distance. Further he did not really evaluate the effect of the choice of the distance measure to the quality of the solutions.

Schilling *et al.* (2000) examined the Euclidean distance, network distance and a randomly generated network distance. Their conclusion is that it is much easier for the Euclidean and

network to obtain the optimal solution and with less computational effort. However, the problem is small scale and they did not provide the effect of network with different numbers of nodes in the networks. In our study we are dealing with large networks and we systematically alternate the number of nodes in it to evaluate the quality in the optimal solutions. None of the previous studies provided any analysis of network aggregation.

In a recent study Avella *et al.* (2012) tested a large size p -median problem using a new heuristic based on Lagrangean relaxation. The number of nodes varies from 3,038 to 89,600. They compared their computational results to the results found by Hansen *et al.* (2009) under 4 instance sets (from Birch and TSP library). The largest data set is Birch 1. The Birch data set are synthetically generated, designed to test clustering algorithms. Birch 1 and 3 differ in two significant ways. Birch 1 is the largest data set used (89,600 nodes) and it consists of symmetrical distributed demand points and nodes in the network which are also organized in tight clusters. Birch 3 consists of up to 20,000 nodes and the demand points and the nodes in the network are more asymmetrically distributed and the clusters also vary more in their characteristics. They found that the new heuristic is fast and efficient. They also showed that the quality of the optimal solutions was quite different when Birch 1 was used compared to when Birch 3 was used. Instances of type Birch 3 also took longer computing time to be solved. Larger instances exhibit worse results. However, they did not consider a real world network, when the number of nodes in the network are alternated systematically.

3. Data

3.1 Demand Points and Service Centers

The demand points represent the distribution of the population's residence in Dalecarlia. In this study we use the population in 2002. The figures are public produced and controlled data from Statistics Sweden (www.scb.se). The populations' residents are registered on 250 meter by 250 meter squares. We generalize each square is to its central point. Each point is then weighted by the number of people living in each square. The populations' residence location is represented by 15,729 weighted points. In total 277,725 lived in Dalecarlia during the study year. The distribution of the residents is shown in Figure 1a. The figure illustrate that the population in the region is

asymmetrically distributed. The majority of residents live in the southeast corner, while the remaining residents are primarily located along the county's major rivers and lakes. Overall, the region is not only non-symmetrical distributed, but it is also sparsely populated with an average of nine residents per square kilometer (the average for Sweden overall is 21).

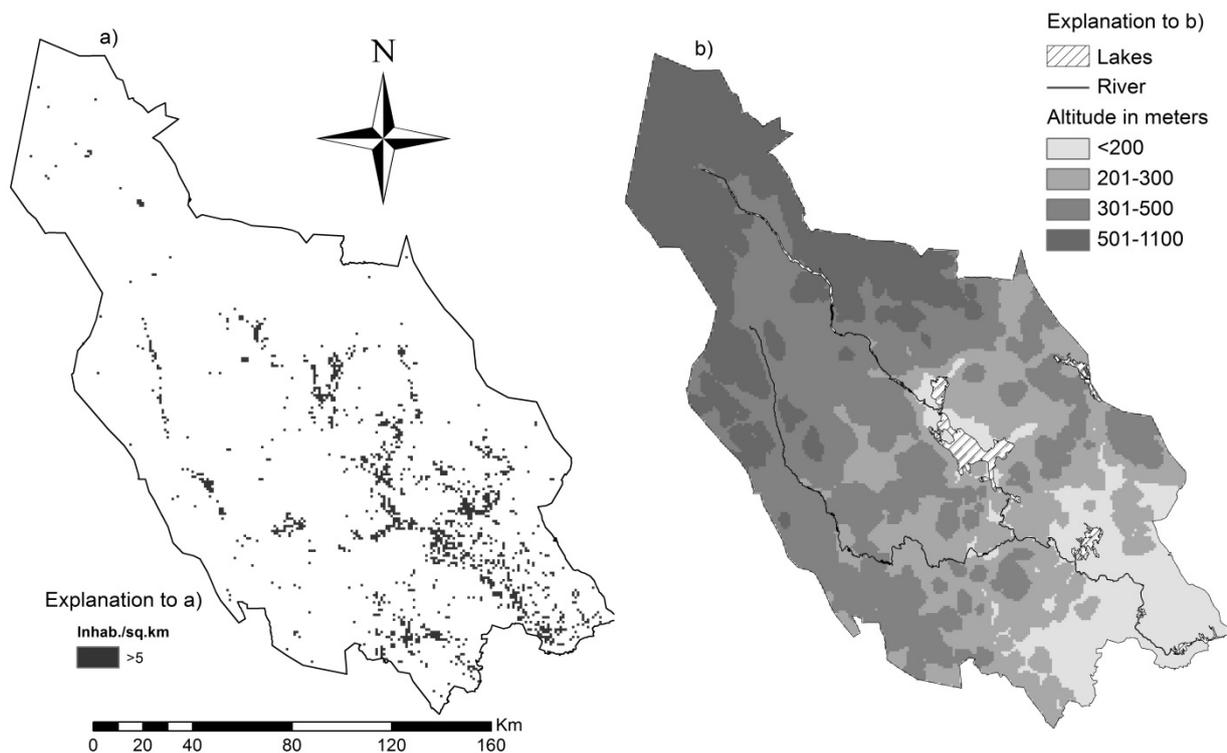


Figure 1. The distribution of the population on 1 by 1 km squares (a) and natural landscape (b) in Dalecarlia.

Figure 1b shows some important features of the natural landscape in Dalecarlia. Firstly it is shown that the altitude in Dalecarlia vary a lot. From the south east corner with altitude below 200 the altitude increase in general towards north east. Secondly it is shown that a major river (Dalecarlia River) and some large lakes also act as natural barriers. Clearly, when comparing the distribution of the population (figure 1a) with the natural barriers (figure 1b) there is a correlation.

Concerning the service centers, in this study, we search for optimal locations for p equal 5, 10, 15, 20, 25 30, 35, 40, 45 and 50.

3.2 The Road Network

The road network used is the 2011 national road database (NVDB) for Dalecarlia. NVDB was formed in 1996 on behalf of the government. It is organized and updated by the National Transport Administration (Trafikverket) in Sweden. In total the road network for Dalecarlia contains about 1.5 million nodes and 1,964,801 segments. The total length is 39,452 kilometers. The average distance between the nodes in NVDB is about 40 meters. The minority of the nodes is nodes in intersections or at points where roads starts or begin. Most nodes describe the geographical shape of the road and by that they give a precise description of the length of the road. We use this network to calculate the distance between the demand points and the closest service center. To do so we use the Euclidian distance to identify the closest node on the road network. Then we add the shortest network distance. To find the shortest network distance the Dijkstra algorithm has been used (Dijkstra 1959).

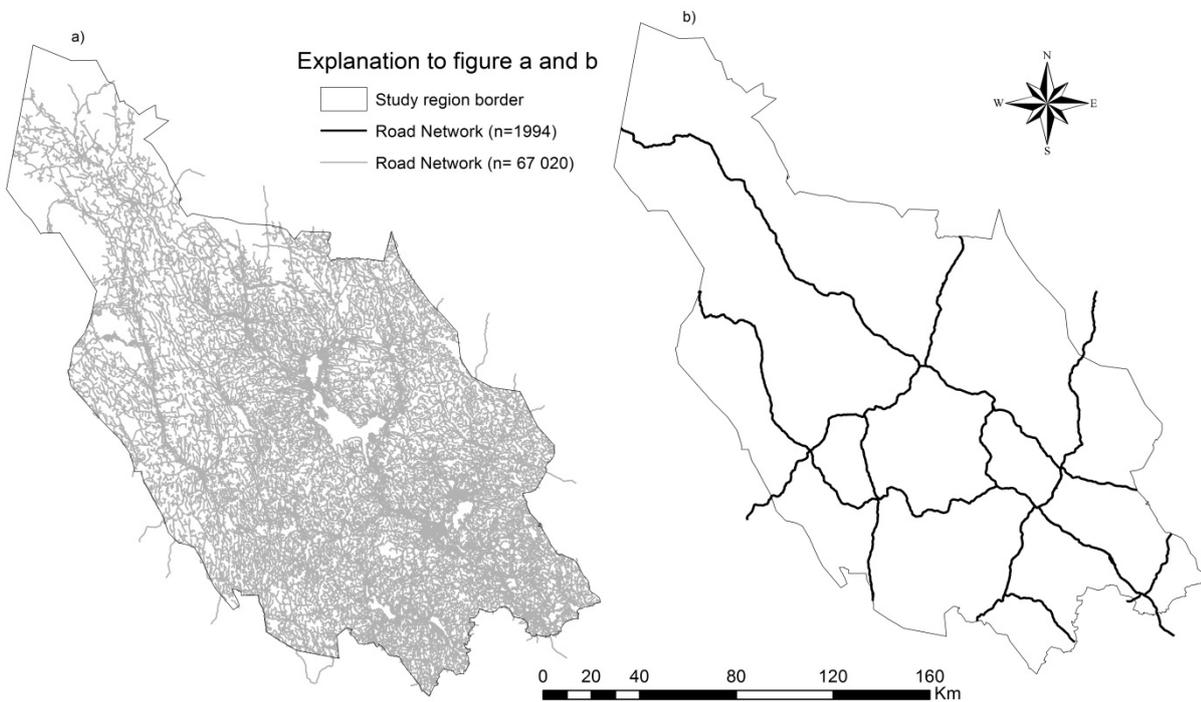


Figure 2. All roads in a dense network with 67,020 candidate nodes (a) and all roads in a sparse network with 1,994 candidate nodes (b) in Dalecarlia

To identify the candidate nodes to locate on we select one node in each 500 by 500 meter square in which the roads pass through. By reducing the number of nodes within a square an in-built location error occurs. However by selecting the center of the square as the representative node the maximum location error due to this could be 354 meters in Euclidian metric. Finally we used at the most 67,020 nodes in the road network as candidate nodes to locate on. (see Figure 2a).

NVDB is divided into 10 different categories according to the quality of the roads (see Table 1). To alternate the density in the road network we used those road classes. In Dalecarlia there is just one road (class 0) which is a European highway. For this reason, class 0 roads are merged into class 1 in this study. By just taking into account the largest roads (class 0 and 1) the set of candidates to find an optimal location of a service center are as many as about 2000 nodes distributed in a rather sparse network (see Figure 2b). This is still quite large; so to decrease the density in the road network further we add two new classes which consist of 500 and 1000 candidates to locate service centers in. We select these candidates randomly from candidates in class 0 and 1. From Table 1 we can see that average distance between the candidate nodes varies rather little when the road classes 0 to 9 are concerned. However, the average distances between candidate nodes become significant longer when the density in the road network is decreased further.

Table 1. Number of candidate nodes, road length and average road distance between candidate nodes with different road classes on the road network in Dalecarlia.

Road classes	Number of nodes	Length (km)	Meters between Candidate nodes
0 to 9	67020	39454	588
0 to 8	45336	23086	509
0 to 7	20718	10964	529
0 to 6	12552	5631	449
0 to 5	12417	5479	441
0 to 4	6735	2923	434
0 to 3	3926	1725	439
0 to 2	2909	1299	446
0 to 1	1994	883	443
0 to 1 (randomized)	1000	883	883
0 to 1 (randomized)	500	883	1766

Figures 2a and 2b illustrate that the road network becomes denser and more homogenous in areas in the region's southeast corner. In the southeast and in the center of the region, a sparse network of larger roads supplements the smaller roads. From Figure 2a it is obvious that the smaller local roads and streets are oriented to the larger roads. It is also evident that the smaller roads make the road network more homogenous when it comes to its distribution in the region.

4. Simulated Annealing

4.1 Algorithm

Since the p -median problem is NP-hard, for large number problems, the exact optimal solution is difficult to obtain. That is why there are only a few studies examining the exact solutions (Hakimi, 1965; Marsten, 1972; Galvão, 1980; Christofides and Beasley, 1982). Instead most studies regarding p -median problem use heuristics and meta-heuristics (e.g. Kuehn and Hamburger, 1963; Maranzana, 1964; Rahman and Smith, 1991; Rolland *et al.*, 1996 Crainic, 2003; and Ashayeri, 2005). In our case, the cost of evaluating a solution is rather high therefore we focus on an algorithm which tries to keep the needed number of evaluated solutions low. This excludes, for example, algorithms such as the Genetic and to some extent Branch and Bound algorithms.

Another sub-class in meta-heuristics is simulating the annealing method, which we will use in this paper (e.g. Kirkpatrick 1983, Chiyoshi and Galvão, 2000; Al-khedhairi, 2008; and Murray and Church, 1996). This randomized algorithm has been chosen due to its flexibility, its ease of implementation and the quality of results in the case of complex problems. Al-khedhairi (2008) gave the general SA heuristic procedures.

SA starts with a random initial solution s , a choice of a control parameter named the initial temperature T_0 , and the corresponding temperature counter $t = 0$. The next step is to improve the initial solution. The counter of the number of iterations is initially set as $n = 0$ and the procedure is repeated until $n = L$, where L is the pre-specified number of iterations of the algorithm. A neighborhood solution s' is evaluated by randomly exchanging one facility in the

current solution to the one not in the current solution. The difference, Δ , of the two values of the objective function is evaluated. We replace s by s' if $\Delta < 0$, otherwise a random variable $X \sim U(0,1)$ is generated. If $X < e^{\frac{\Delta}{T}}$, s still replaces s' . The counter is updated as $n = n + 1$ whenever the replacement does not occur. Once n reaches L , the temperature counter is updated as $t = t + 1$ and T is a decreasing function of t . The procedure stops when the stopping condition for t is reached.

Given p we start the simulated annealing by randomly selecting points to locate the service centers. We then randomly select one of the suggested service center location sites and define a neighborhood around it. As the neighborhood we apply a square of 25 km centered on the selected site. If we have less than 50 candidates for a service center location we increase the neighborhood by steps of 2.5 km until this criterion is satisfied. This was necessary in just a few cases.

4.2 Adaptive Tuning and Parameters

The parameters used here have been tuned after prior testing. In our study we start with the initial temperature T_0 of 400. We multiply the temperature by 0.95 at each new iteration. To avoid having our algorithm blocked in a local minimum, we have an adaptive scheme to reheat the system. If 10 times in a row we refuse a solution, we increase the temperature multiplying the temperature by 3^β . A suitable value of β is 0.5. Therefore, the initial value of β is 0.5 and if no solution is accepted between two updates of the temperature we increase beta β by 0.5. β will be reset to 0.5 as soon as we accept a solution. Experiments done with 2000 and 20,000 iterations have shown that for our cases 20,000 leads to significantly better results. The number of iterations has been fixed at 20,000. Our experiments have been conducted on an Intel Core2 duo E8200 cpu working at 2.66 GHz. The operating system used is Linux and programming has been done in C and compiled with gcc. It took us about 24 hours to compute 20,000 iterations.

5. Results

Table 2 shows some results from the computer experiments when different density in the Dalecarlia road network for the location of a different number of service centers is alternated.

The table gives information on the mean travel distance in the road network from their residence to the closest service center for the inhabitants in Dalecarlia. Highlighted figures in the table indicate the best solution found for a given number of service centers (p). When p is set to 15 the solutions computed continue to be better until road class 3 is added to road classes 0, 1 and 2. The best solution gives an average travel distance in the complete road network from the inhabitants' homes to the closest service center of 8.53 kilometers.

The main result which can be drawn from Table 2 is that a more complex location problem can take advantage of a more complex network. This is shown by the fact that when the number of service centers is below 20 the best solutions are found already with the density given by the road classes up to two while when the number of service centers is above 20 the best solutions are found with a higher density of road network.

Table 2. The mean network distance in kilometers to the closest service center given different p and densities of the road network to locate on.

p	Road classes in the road network										
	500pt	1000pt	0-1	0-2	0-3	0-4	0-5	0-6	0-7	0-8	0-9
5	22.71	20.34	19.74	19.73	20.20	20.17	20.25	19.99	20.07	20.33	20.52
10	11.20	11.23	11.18	11.17	11.20	11.22	11.26	11.38	11.44	11.74	11.80
15	8.63	8.64	8.63	8.53	8.58	8.58	8.66	8.63	8.75	8.92	9.21
20	7.67	7.61	7.11	7.11	7.03	7.08	7.18	6.99	7.24	7.54	7.82
25	7.15	7.31	7.19	6.19	6.24	6.12	6.16	6.15	6.30	6.61	6.94
30	6.90	6.93	6.94	5.78	5.56	5.34	5.58	5.55	5.68	5.88	6.05
35	6.72	6.67	6.67	5.33	5.10	4.96	5.15	5.13	5.16	5.29	5.43
40	6.52	6.47	6.54	5.09	4.71	4.70	4.71	4.72	4.69	4.89	5.29
45	6.34	6.31	6.33	4.90	4.45	4.29	4.40	4.42	4.49	4.69	4.84
50	6.27	6.24	6.25	4.69	4.24	4.14	4.09	4.12	4.24	4.40	4.46

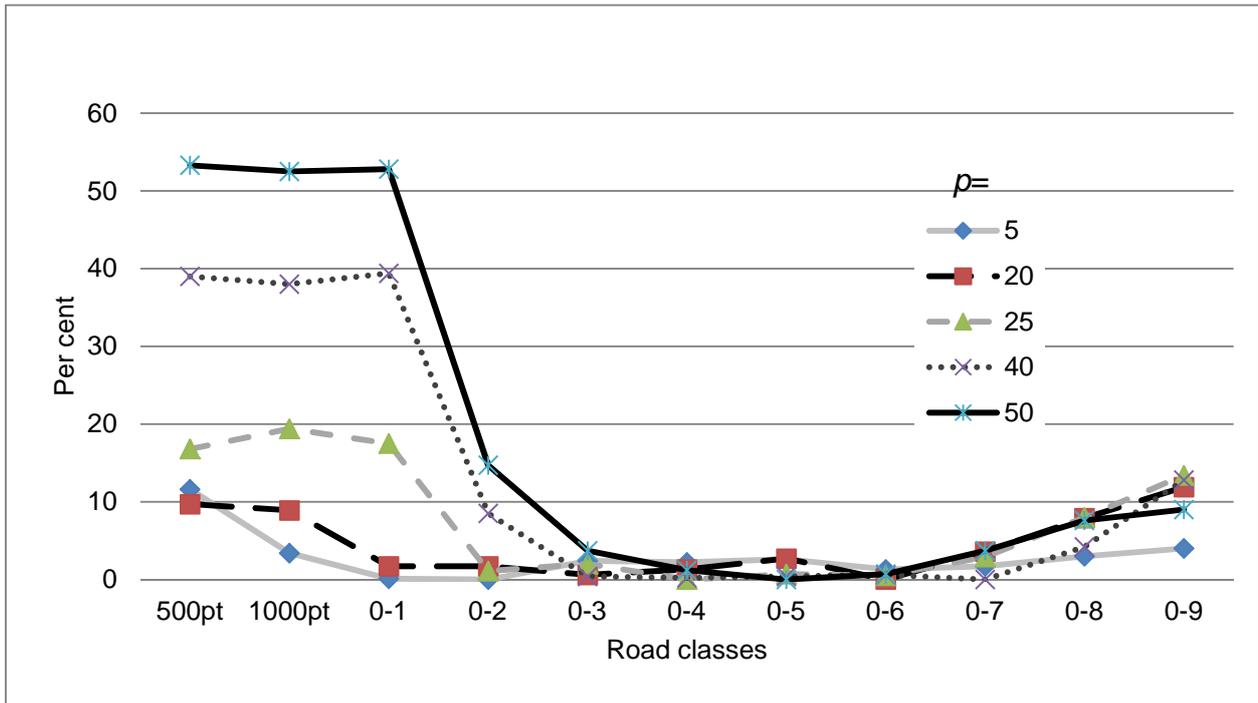


Figure 2. Variations in excess distances (in per cent) compared to the best solutions when different density in the network has been used to find an optimal location on.

Figure 2 illustrates how much worse (in per cent) solutions are in relation to the best solution for different densities in the network. In the figure this is illustrated with a selection of different p . The conclusion is that there is more to gain in choosing the right density level on the network when p is higher. This is clearly shown since when the number of service centers is 20 or less the worse solution found is not less than 12 per cent longer than the best one. On the other hand for location problems with more than 25 service centers the worst solution is at least 30 per cent longer than the best one.

6. Conclusions and Discussions

The paper aims to examine the effect of alternating the density in a road network when service center location problem is studied. To do so, we use a large scale real world road network with 1.5 million nodes in the region of Dalecarlia in Sweden and we alternate the density of the road network used to locate on from 500 to 67,000 candidate nodes. As demand points we use the population in the region registered on squares of 250 by 250 meters. The population and the network are asymmetrical distributed in the region due to natural barriers. To scrutinize the

problem we also alternate the number of p between 5 and 50. In doing so, we cover inter-urban location problems as well as intra-urban location problems. We use the p -median model and meta-heuristics to find the optimal solutions.

It has earlier been shown that it is important to use the network distances when optimal locations are sought. In this study we add the result that an increased density of the road network is only necessary up to a certain level. We also show that when the number of service centers increases the density needed in the network tends to be higher. This implies that for inter-urban location problems (like for instance locating emergency hospitals or courts) with lower p in a region of the size used here it is sufficient to use fairly simple networks, while dealing with inter-urban as well as intra-urban location problems (like for instance locating high schools or post offices) simultaneously with higher p the need for a more refined network is larger.

The road network used here was not constructed for the purpose of service centers location. The structure of it is probably suitable for a lot of issues related to what happens on the road. However, in organizing this network to be suitable for the purpose of being used in location problem we turn out to have between 500 candidate nodes up to 67,000 candidate nodes which are the extremes in our case. It turns out that these two extreme densities of the road network were not suitable for solving the location problem here. One possible future research question could be how the road network should be arranged to be suitable for location allocation problems.

In this study we use simulated annealing. It has obvious drawbacks. It would however be interesting to evaluate how other algorithms would perform in this kind of setting.

Further, the case here is quite a small geographical rural area, Dalecarlia. As such more case studies are needed. In addition, the important roads are first and foremost designed to be efficient in a national transportation system. Further, many public activities but also private businesses are taken conducted at a national level. There is a need to better evaluate the efficiency in present situations of where these activities are carried out. One suggestion for future research is therefore to scale up the present case study to national level. Advanced methods (e.g. more aggressive heuristics, distributed computing) will be needed to keep the computing time acceptable and still reach excellent solutions.

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