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Does road network density matter in optimally locating facilities?

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Abstract

Optimal location on the transport infrastructure is the preferable requirement for many decision making processes. Most studies have focused on evaluating performances of optimally locate *p* facilities by minimizing their distances to a geographically distributed demand (n) when *p* and *n* vary. The optimal locations are also sensitive to geographical context such as road network, especially when they are asymmetrically distributed in the plane. The influence of alternating road network density is however not a very well-studied problem especially when it is applied in a real world context. This paper aims to investigate how the density level of the road network affects finding optimal location by solving the specific case of *p*-median location problem. A denser network is found needed when a higher number of facilities are to locate. The best solution will not always be obtained in the most detailed network but in a middle density level. The solutions do not further improve or improve insignificantly as the density exceeds 12,000 nodes, some solutions even deteriorate. The hierarchy of the different densities of network can be used according to location and transportation purposes and increase the efficiency of heuristic methods. The method in this study can be applied to other location-allocation problem in transportation analysis where the road network density can be differentiated.

Key Words: Road network; Density; *p* – median model; CPLEX; Heuristics

1. Introduction

Transportation is a requirement for the society, regardless of its industrial capacity, population size or technological development. Road network is a necessary component to facilitate the movement between transportation and social-economy activities. It plays a key role in the spatial structure of urban planning and fosters the development of regions. Whatever the purpose of journey, the mode of transport and the available route to take, location of the destination is a key factor to be considered. Optimal location on the transport infrastructure is the expected requirement for many decision making processes. Therefore the road network information is required to be easily accessible, reliable, organized and detailed.

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However, when the road network becomes more accurate and complex, finding the optimal location could be troublesome. The transportation surface tends to be asymmetric; the distance measure needs to be more sophisticated and accurate than a normal Euclidean distance. The efficiency of the algorithms could be affected and then cause the risk of increasing solution space without providing higher quality solutions.

Algorithms, spatial aggregation of demand points, and choice of distance measure have been studied extensively in finding optimal locations in the transportation scenario. It is considered that a better and more accurate representation of a road network is an important criterion to have better results (Peeters and Thomas, 1995). In other words, the denser a network is, the more information is represented by edges and nodes and the higher accuracy can be achieved. Therefore, better solutions for an optimal location problem can be expected. However, few studies have scrutinized the density level of the road network as an influential factor to find the optimal locations.

This paper aims to investigate how the density level of the road network affects finding optimal location by applying the specific case of p-median problem. We aim to locate a set of facilities on a road network in order to minimize the sum of travel distances between each single demand point and its closest facility on all demand points. The specific experiments are conducted by optimally locating 5 to 50 facilitates on a real complex road network in the region Dalarna, Sweden.

To avoid the one-sidedness of using one single algorithm, four different algorithms being greedy search (Kuehn and Hamburger, 1963), CPLEX, simulated annealing (Al-Khedhairi, 2008) and imp-GA (Rebreyend et al., 2015) algorithm are applied to solve the p-median problem to have a benchmark and validated performance. The CPLEX is commonly used software and is the only method that is able to find the optimal solution except for the volume algorithm (Barahona and Anbil, 2000). All other three methods can give suboptimal solutions.

The standard greedy algorithm for the p-median problem was studied back to Cornuejolset al.(1977). Resende and Werneck (2004) conducted a constructive greedy to perform the most profitable move. Rebreyend et al.(2015) conducted a detailed review on the other three algorithms, this paper mainly follows the desciptions and paprameter settings in their study.

The density levels synchronized with the hierarchy of the road network are classified by differentiating the 10 road classes in Sweden (0 for European highways and up to 9 for small local streets). The numbers of candidate points vary from 452 to 67,020 depending on the density of the road network. Network distance is chosen to be the distance measure (Carling et al., 2012). All network distances are calculated in the densest network in order to have a valid measure to make comparisons among different networks. This will also make sure that the variation of candidate points from different network densities does not affect the distance measure.

The remaining parts of this paper are organized as follows. In section 2 we discuss some related previous studies. Section 3 describes the p-median problem and the four algorithms. Section 4 shows the road network data used in this paper. The results and corresponding analysis are given in Section 5 while Section 6 presents the conclusions and discussions.

2. The p-median problem

The p-median problem was first introduced by Hakimi (1964) which became the significant mark in the development of location science. The basic expression of the *p*-median objective function was given to be the sum of weighted distances between demand points and their respective nearest facilities. Minimize $f = \sum_{i=1}^{Q} \sum_{j=1}^{N} w_i * d_{ij} * X_{ij}$, subject to $\sum_{j=1}^{N} X_{ij} = 1$, $\forall\, i \in (1,2, \ldots \ldots Q)$ and $\sum_{j=1}^{N} Y_j = p$; f is the value of objective function, Q is the number of demand nodes, N is the number of location candidates, w_i is the weight of each demand location, d_{ij} is the distance between demand node i and potential facility j, X_{ij} is a dummy variable that having 1 if demand node i is allocated to facility at site j. The aim was to find p facility locations which would minimize f. Hakimi (1965) first investigated the p-median problem on a network and the Hakimi property stated that for the *p*-median problem on a network, at least one of the alternative optimal solutions would entirely consist of vertices of the network.

To find the optimal location for *p* supply points in relation to the demand using the *p*-median problem is NP-hard (Kariv and Hakimi, 1979), optimal solutions to large problems are difficult to obtain (Al-Khedhairi, 2008). The complexity depends both on the number of supply points to be located, the number of demand points, as well as on how distance is measured. Francis *et al*. (2009) conducted an explicit review of the *p*-median problem, in which about half of the 40 reviewed articles are studies based on real data, it showed that almost all distance measures are Euclidean distance and rectilinear distance.

Although Euclidean distance is most widely used, the network distance in most cases is more accurate in measuring the travel distance between two points. Further, a refined denser network should give the possibility to more accurate distance measures than a sparser network (Carling et al., 2012). There are a few studies which evaluate network effects on optimal locations. Peeters and Thomas (1995) examined the performance of the *p*-median model in different types of networks by changing the nature of the links. They found that there is a difference in optimal solutions when the links are changed but they registered no differences in computational efforts. Schilling *et al*. (2000) examined the Euclidean distance, network distance and a randomly generated network distance; they found that it is much easier and takes less computational effort to obtain the optimal solution by using Euclidean and network distance. The problem scale in their study is however small and they did not study the effect of network with different levels.

As for this NP-hard problem, if *p* is fixed, the p-median problem on a general network is solvable in polynomial time. This does not mean that the fixed *p* problem is computationally easy. This tough challenge urged different algorithms showing up, in which, enumeration and heuristics were the earliest techniques proposed to solve p-median problem. As enumeration is the method cannot work the NP-hard out, there was merely trial on using it and heuristics play a very important role in the method research.

Metaheuristics and approximation algorithms are those predominant techniques explored in the literature over the last few years (Farahani, et al., 2012). They provide a general framework to build heuristics for combinatorial and global optimization problems. Simulated Annealing, Genetic Algorithms are two of the most commonly used ones. Murray and Church (1996) proposed a basic SA algorithm for the p-median location problem. Levanova and Loresh (2004) studied the SA heuristic that used the 1-interchange neighborhood structure. Carling et al. (2012), Han et al. (2013) have used tuned SA to solve specific *p*-median problem in the real context of network. Thorough treatments of GAs can be found in (Davis, 1987, 1991; Reeves, 1993; Dowsland, 1996). Bozkaya et al. (2002) described GA which is able to produce solutions that are better than exchange algorithm. However, convergence is very slow. Alp et al. (2003) proposed a GA which is simpler and produces good solutions faster.

Tansel et al. (1983) reveiewd 100 studies of network structure influence on location problem. Reese (2006) conducted an annotated bibliography about the methods for solving the incapacitated and capacitated p-median problem on a network. Mladenović et al. (2007) did a thorough overview which is very helpful to illustrate the metaheuristics in solving the p-median problem. Varnamkhasti (2012) concentrated in summarizing the literatures on solution algorithms for the p-median problem.

However, when facing to the problem on complex networks with arbitrary *p* and various densities, the acceptable computation time, the solvable problem size, the capability of current computers and computer codes should all be taken in to account. The current heuristics must be modified to acquire good solutions for larger and tougher problems.

Apart from Han et al.(2013), very few study has examined the effects on the optimal solutions by varying the densities of the road network. However, different densities of the road networks could have influences on the performances of algorithms in deriving preferable solutions of optimal locations, especially when the road network is complex. Our study therefore focuses on examining how large and complicated networks affect the quality of optimal solutions by systematically alternating the densities of the network.

3. Methods and Data

3.1 Methods

Greedy algorithm follows the problem solving heuristic of finding the locally optimal choice at each stage with the hope to get a global optimum. It always choose the optimal choice at current stage rather than consider all other conditions to make sure to find a global optimal. This can be characterized as being 'short sighted', but it is easy to implement and can get acceptable results within a short time.

The *p*-median problem can be formulated as a 0-1 binary programming problem (BP) and then can be solved by a Mixed Integer Problem (MIP) solver through using a branch and cut approach. CPLEX software from IBM (version 12.6, Linux 64 bits) is applied to solve the *p*-median problem. Following Rebreyend et al.(2015), some parameters of the solver have been tuned in order to adapt CPLEX to work on large problem instances, specifically, removing default computation time limits, allowing intermediate data storage, and tuning branch & cut search tree strategies according to the manual[1].

Simulated annealing (SA) is one sub-class of metaheuristics that has been widespread used in the last few years. The basic idea of SA is not only accepting all the better results on the search process, but also accepting some worse results based on certain probabilities. It is capable of solving complicated nonlinear optimization problems. It is simple to implement and it can provide high quality solutions to many problems. The performances of SA are sensitive to the values of control parameters. In this study we employ the same parameters of SA to the same real world network data as Han et al. (2013) did. The specific parameter settings and the implementation of SA follow Zhao et al. (2013). After testing with other tricks and parameter settings, we found that the settings used to increase the temperature works well to avoid the search being trapped in the local optimal for a long time and finally provides appealing results.

Genetic algorithms (GA) are heuristic search methods that are designed to mimic the evolution process. New solutions are produced from old solutions in ways that are reminiscent of the interaction of genes. GA has been applied with success to problems with very complex objective functions (Alp et al., 2003). Most previous studies used a classical string representation, in which, each chromosome is represented as a single string of length *p* embedding the index of the selected facilities or nodes. Followed Correa et al. (2001), Rebreyend et al. (2015) proposed an improved genetic algorithm called imp-GA to efficiently solve large scale *p*-median problems. The imp-GA used in this paper follows the description and settings as in Rebreyend et al. (2015).

[1] http://www-01.ibm.com/support/knowledgecenter/SSSA5P_12.6.1/ilog.odms.studio.help/pdf/gscplex.pdf

We run CPLEX and Greedy once due to the deterministic property of the methods. We randomly select 3 different initial configurations to conduct the SA and record the solutions with the minimum objective function value, each run contains 20,000 iterations. As for imp-GA, considering the time and computer memory, we set 1 run with 100 iterations. In this paper, all the programs are coded with C and compiled using GCC version 4.8.2; they are launched under a system Linux (Kernel 3.11-2-amd 64). The computer has the memory of 32 G, CPU of Intel Core i7-3770.

3.2 Data

The data in this study is the complete digitalized representation of the road network and geo-coding of the population of Dalarna in Sweden. The population data comes from Statistics Sweden[2]. The population number of Dalarna is 277,725 by December 2010. The data of road network comes from the Swedish digital road system: National Road Data Base (NVDB)[3].

Figure 1 depicts the distribution of the population and the road network. The population distribution in Figure 1a shows people that are registered on 250 meters by 250 meters squares apart in the four directions of north, west, south and east. The center point of each registry square represents a demand point. Each demand point has a weight being the number of people in that square. There are 15,729 weighted demand points to represent the population in this region. The population in the studied region is asymmetrically distributed with the majority live in the southeast part.

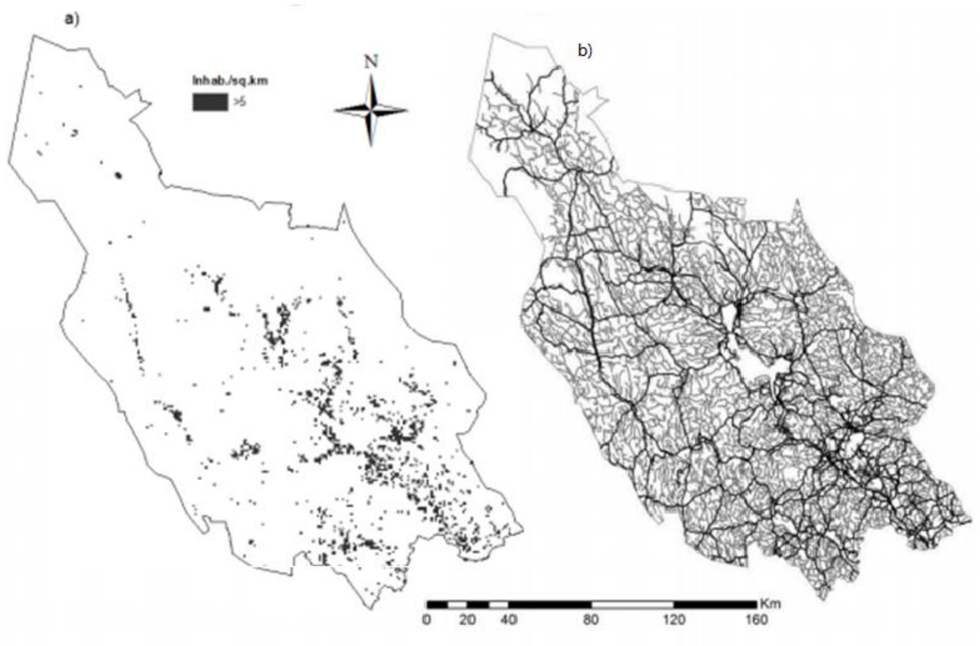

Figure 1. Map of the Dalarna region showing (a) one-by-one kilometer cells where the population exceeds 5 inhabitants, (b) road network of 10 road classes

[2] www.scb.se
[3] www.nvdb.se

Figure 1b shows the road network with all the 10 road classes. There are European highway, national roads, local roads and private streets in the Swedish road system. The total length of the road network is 39,454 kilometers. The density levels are corresponding to the 10 accumulated road classes (0, 0-1, 0-2 … 0-9).

This paper follows Han et al. (2013) to represent the population by grid aggregation in 500m by 500m instead of the original registration of 250m by 250m. The whole road network of Dalarna is composed by 10 road classes. The road classes are corresponding to the densities of the road network, 0 represents the sparsest network with the least nodes and 0-9 represents the densest network with the most nodes.

There are 1,797,939 nodes and 1,964,801 road segments on the whole road network. The network is quite a large one for applying the *p*-median model to find relatively small *p* optimal locations. Besides, it is not necessary to set all the nodes to be the candidates for optimal locations due to the non-symmetrical sparse distribution of the population. We therefore use grid aggregation to select the candidates.

The basic idea of grid aggregation is to change the density of road network by dividing it into smaller grids with the same size. In each grid at most one node is kept as a potential facility point. Three criteria are applied to select potential facilities. First, choose the node connected with most road segments as the potential facility. Second, choose the node with the highest level as the potential facility. Third, choose the node which is the nearest one to the center of the grid as the potential facility. Table 1 summarizes the number of nodes according to the density level of the road segment that the node connects. The nodes range from 452 to 67020.

Table 1. Number of nodes in the hierarchy of different road classes under the grid aggregation level of 500m by 500m on the network in Dalarna

Road Classes	Number of Nodes	Length (km)
0	452	167
0 – 1	1994	883
0– 2	2909	1299
0 – 3	3926	1725
0– 4	6735	2923
0 – 5	12417	5479
0 – 6	12552	5631
0 – 7	20718	10964
0 – 8	45336	23086
0 – 9	67020	39454

As the coordinates of the residents do not perfectly coincide with the road nodes, we use the nearest node in the network to represent the location of corresponding residents. This approximation partially introduces some errors in the computation. However, people usually reside at the locations that are easy to get access to the transportation; some even live within the walk distance. Comparing to the distance that residents traveling to the nearest service center, the distance of reaching to the nearest available road node can be ignored. In this study the average distance between the residents node and the nearest network node is only 62m, it does not produce much difference on the final results.

When dealing with the objective function of the *p*-median problem, distance measure is one crucial aspect that should be taken into consideration. Carling et al. (2012) empirically investigated the consequences of different distance measures for the optimal location of multiple service centers in rural areas. They stated the shortest travel time or minimal cost along an existing network intuitively seems to be the most accurate measure for most settings, yet it is infrequently employed. One is the difficulty and cost associated with collecting data on travel time. Another is the complication which arises in modeling the inherent variation in travel time. This paper employs network distance as the distance measure.

The Dijkstra algorithm (Dijkstra, 1959) is used to calculate the shortest distance between each potential location node to all the population nodes. It randomly starts from a node and then calculates the distance between it and all the other nodes. After the computation, there are 9,020 nodes that do not connect with the main part of the network. Those nodes and the road segments between them are deleted. The matching between the residents' locations and the network is based on the network after deletion.

3 Results and Analysis

It is preferable to apply CPLEX due to its potential to find the optimal solution while all other three chosen methods can give suboptimal solutions. However, Rebreyend et al. (2015) found that, due to the current memory limitation of computer, the CPLEX approach is not able to provide results for netwrok densities larger than 1938 nodes. As expected, CPLEX fails to derive results when the desitiy goes up to level 1 and beyond.

Figure 2- 4 show that, the optimal solutions are always not found in the sparsest density level. As the density level goes highr, the optimal solutions are not gurannteed to be always found in a denser network . The improvement of optimal solutions are huge when the density level goes higher from the sparsest one, the improvement of best solutions become smaller and insignificant along the density's increase, after a density level, the improvement stops even turns to deteriorate. In Fiture 2-4, the road classes are employed in the x-axis, which are corresponding to the nodes numbers as well as the densities of the network. The figures show the differences in percentage in comparison

to the best solutions respectively from Greedy search, SA and imp-GA when different densities of the road network are used.

SA and imp-GA are more sensitive to the variation of network density than Greedy search when the density goes up to level 5. The best solutions from Greedy search stay the same after the density level 5 for all the *p*; the best solutions from SA deteriorate when the density level is higher than 6 for all the *p*. Optimal solutions from GA have the tendency of improving when the density level goes up to 7. The improvment is however insignificant comparing to the results found in the density level 5, the biggest improvement being 2.78% (149 meters) is within people's walk distance, which is resonable to regard it as no difference. The time cost and computation effort on the other hand are getting tremendously large so that results cannot be derived for higher density level 8 and 9 within the time threshold of 3 days.

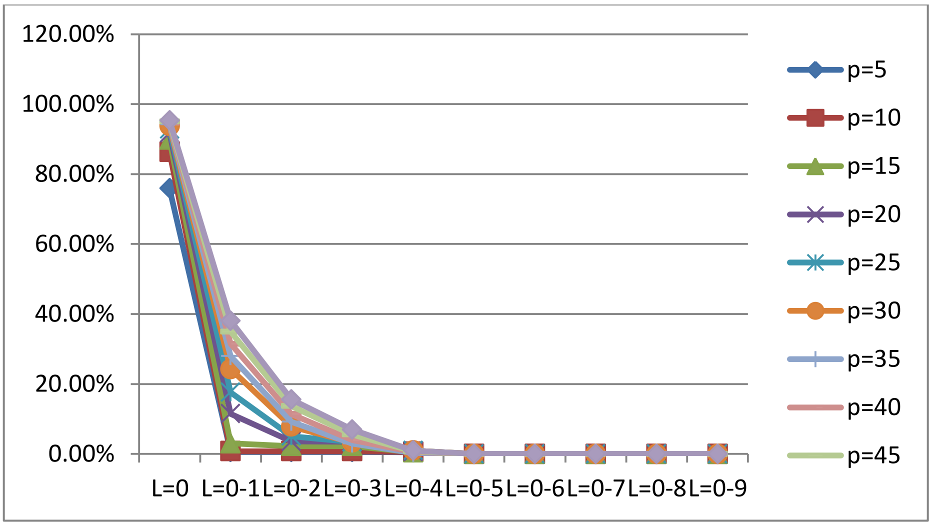

Figure 2. Variations from Greedy in excess distances (in percent) compared to the best solutions for an increased density level. The x-axis shows the density level. The y-axis shows the difference in percentage between the best solution and the current solution in accordance to ($\frac{|\text{current solution}-\text{best solution}|}{\text{current solution}} * 100\%$).

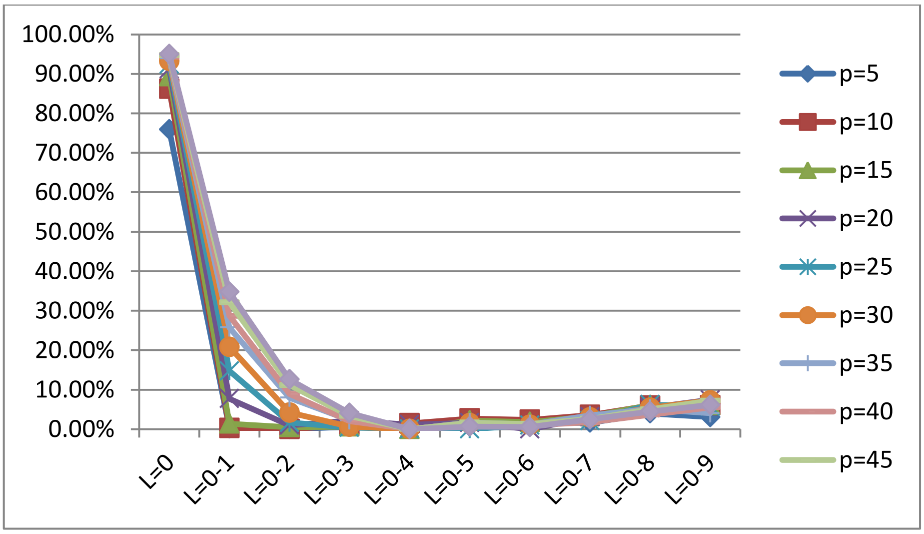

Figure 3. Variations from SA in excess distances (in percent) compared to the best solutions for an increased density level. The x-axis shows the density level. The y-axis shows the difference in percentage between the best solution and the current solution in accordance to ($\frac{|\text{current solution}-\text{best solution}|}{\text{current solution}} * 100\%$).

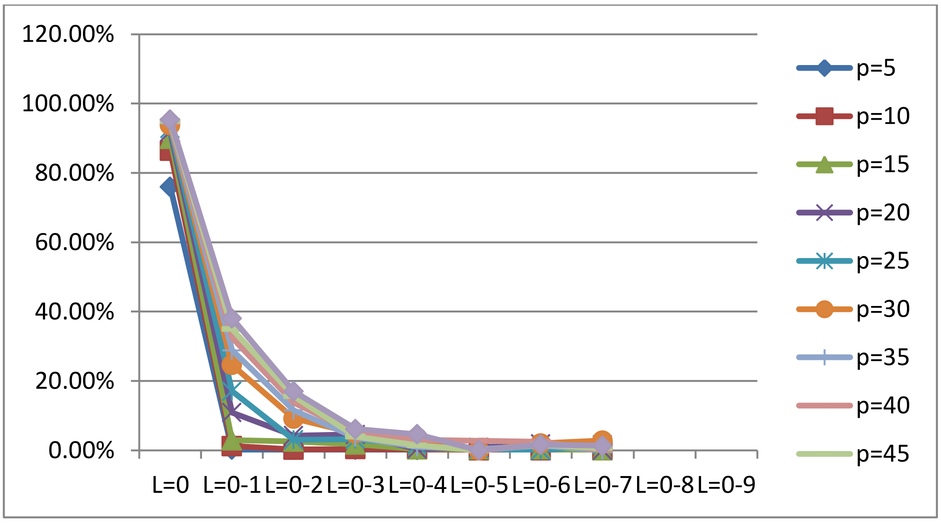

Figure 4. Variations from imp-GA in excess distances (in percent) compared to the best solutions for an increased density level. The x-axis shows the density level. The y-axis shows the difference in percentage between the best solution and the current solution in accordance to ($\frac{|\text{current solution} - \text{best solution}|}{\text{current solution}} * 100\%$).

Table 2 further shows the optimal values derived by the four algorithms as well as the corresponding facility number *p* and network density. As for Greedy search and SA, the results are based on computations of all 10 density levels, while the imp-GA was not able to provide the results for density levels 8 and 9 due to the time complexity and computer memory limitations. Greedy search found all the optimal solutions on density level 5; SA found the optimal solutions on desity level 2,4 and 5; imp-GA found the optimal solutions on desity level 5, 6 and 7. The SA used in this paper does not apply any heuristics inside and it is outperformed by the Greedy, imp-GA performs best as expected to provide more preferable results than Greedy and SA, however, the differences are smaller than 445 meters.

Table2. The optimal resluts found on different network diensities by tested algorithms for various *p*

p	Greedy			SA			imp-GA			CPLEX		
	Optimal (m)	L	N	Optimal(m)	L	N	Optimal(m)	L	N	Optimal(m)	L	N
5	19685.1	5	12417	19715.02	2	2909	**19624.1**	5	12417	81681	0	452
10	11134.18	5	12417	11259.28	2	2909	**11080.03**	7	20718			
15	8316.884	5	12417	8625.458	4	6735	**8304.478**	7	20718			
20	6698.967	5	12417	7094.882	6	12552	**6652.11**	7	20718	Can only provide results on density level 0 and fails on upper levels due to the limitation of computer memory (the software aborts before completion on our computer with 32 Gb of memory)		
25	**5804.399**	5	12417	6155.251	5	12417	5845.335	6	12552			
30	5122.226	5	12417	5465.033	4	6735	**5082.8**	5	12417			
35	4708.024	5	12417	5013.738	6	12552	**4686.263**	7	20718			
40	4349.791	5	12417	4611.287	4	6735	**4298.012**	7	20718			

45	**4049.786**	5	12417	4306.7	4	6735	4052.92	5	12417	
50	3796.301	5	12417	4066.063	4	6735	**3794.33**	5	12417	
	Results from all 10 density levels			Results from all 10 density levels			Results from 8 density levels due to time complexity			

4. Conclusion and Discussion

This paper aims to investigate how the density level of the road network affects finding optimal location by solving the specific case of p-median problem. This paper used a detailed network of Dalarna Province in Sweden which has 10 density levels and up to 67, 020 nodes. As a remark, the network with 67, 020 nodes is the densest one we ever encountered in our literature review for this problem.

A denser network is found needed when a higher number of facilities are to locate. Results show that the best solutions can generally be obtained in the middle level of the network density but not always be found in the most detailed network. The solutions do not further improve or improve insignificantly as the density exceeds 12,000 nodes, some solutions even deteriorate. The choice of proper density of the network is crucial to the final results. Yet given an efficient algorithm and unlimited computing time, the solutions could improve monotonically along with the density increase of the network. The hierarchy of the different densities of network can be used to fit transportation purposes and to increase the efficiency of heuristic methods. The method in this study can be generalized and applied to other location-allocation problem in transportation analysis where the road network density can be differentiated.

In this study we choose 10 different *p* that varies from 5 to 50 as the facility numbers to investigate the influence of network densities on the performances of the *p*-median solutions. The road network is aggregated to the gird of 500m by 500m. More tests with larger *p* and closer aggregated approximations are needed to generalize our conclusions.

Various algorithms perform differently in solving the p-median problem and they perform different sensitivity to the change of the network density. There are many other algorithms that have been used to get optimal locations for the *p*-median problem. In this study the CPLEX being the only exact method is not able to give the solution due to the memory limitation, imp-GA failed for densities higher than 20,718 within an acceptable time. Other algorithm such as volume and hybrid heuristics can be tested. The real world data we have used in this study is quite detailed. However, it is still a small geographical rural area compared to the whole region of Sweden and those beyond. More case studies with different kinds of service centers are needed to modify the investigation at a national level.

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