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Editor: Hasan Fleyeh

Nr: 2015:09

Working papers in transport, tourism, information technology and microdata analysis
ISSN: 1650-5581

THE TRUE FIXED EFFECTS MODEL WITH NON-STATIONARY INEFFICIENCY DISTRIBUTION

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Abstract. About a decade ago William H. Greene introduced the so called 'True fixed effects estimator' (TFE), which is intended to separate between heterogeneity and efficiency in stochastic frontier analysis. We would say that it has had huge impact on applied stochastic frontier analysis. One problem with the original TFE estimator, is that the inefficiency distribution is assumed to be completely stationary over time. In this study we consider the TFE model with an inefficiency distribution that changes over time with respect to mean and variance as well as with higher moments. We consider a method of moments estimator that do not hinge on an explicit distribution of the random error. Only symmetry is necessary. The proposed estimators are consistent. We also derive basic asymptotic normality results and propose Wald-tests for testing stationarity.

Date: August 16, 2016 (Updated).
Key words and phrases. Stochastic frontier, Fixed effects, Panel data, Method of moments.
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1. Introduction

Greene introduced the so called 'True random effects estimator' (TRE) and the 'True fixed effects estimator' (TFE), which are intended to separate between heterogeneity and efficiency in stochastic frontier analysis (Greene 2005*a*,*b*). However, the proposed TFE estimator was inconsistent in cases with finite time observations. At least for the variance components of the composed error term.

Wang & Ho (2010) proposed a consistent estimator of a similar stochastic frontier model as Greene had considered. The difference is that the inefficiency term in Wang & Ho (2010) model is only allowed to vary over time with observable variables included by the researcher. The model proposed by Greene varies over time due unobserved factors.

Chen et al. (2014) proposes a consistent estimator based on Greene's model and they derive the distribution of the composed error for the within transformed model where the fixed effects have been removed. Based on the within transformed model Chen et al. (2014) derives a likelihood function which does not depend on the fixed effects and therefore argue their estimator does not suffer from the incidental parameters problem. Wikström (2015) suggested a class of consistent method of moment estimators that goes beyond the normal-half-normal TFE model proposed by (Greene 2005*a*,*b*). Wikström (2015) demonstrate this by deriving a consisitent normal-gamma TFE estimator.

All of these studies are limited to stationary inefficiency distributions, i.e. the population distribution is fixed over time. In this study we consider the TFE model with an inefficiency distribution that changes over time with respect to mean and dispersion and actually any characteristic of the distribution since the parameters of the composed error term are allowed to vary over time. We consider the normal-half-normal model. However, the results are general applicable to any one parameter inefficiency distribution that has an invertible third central moment with respect to the parameter. For example the exponential distribution.

2. Stochastic frontier Model

In this paper we consider the following stochastic frontier model:

$$y_{it} = \alpha_i + \delta_t + x_{it}\beta + \varepsilon_{it}, \quad \varepsilon_{it} = \tau_{it} - u_{it},\ u_{it} \geq 0,\ t = 1, 2\ldots, T\ \&\ i = 1, 2\ldots, N, \tag{1}$$

where y_t is a single output (possibly log-transformed), x_t is a $K \times 1$ of inputs, β is a $K \times 1$ coefficient vector and α_i is the firm effect of firm i, u_{it} is the measure of technical inefficiency of firm i at time t and ν_{it} is a random error term. The inefficiency terms are iid with $E(u_{it}) = \mu_{u,t}$ and $V(u_{it}) = \sigma^2_{u,t}$ that are not necessarily constant over time. The term ε_{it} is a 'composed error' term for which we assume $E(\varepsilon_{it}) = \mu_{\tau,t} - \mu_{u,t}$, i.e. the average may change over time. The τ_{it} are iid as well as the u_{it}'s and x, ν and u are mutually independent. No assumptions are made about α_i. We have included time effects δ_t. To find method of moments estimators of this type of model, it is helpful to have a composed error term that have average zero for each time point. Therefore, we rewrite the model as follows:

$$y_{it} = \alpha_i + \alpha_t + x_{it}\beta + \epsilon_{it},\ \epsilon_{it} = \nu_{it} - u_{it},\ t = 1, 2\ldots, T\ \&\ i = 1, 2\ldots, N, \tag{2}$$

where $\alpha_t = \delta_t + \mu_{\tau,t}$, $\nu_{it} = \tau_{it} - \mu_{\tau,t}$ and by construction the composed error term ϵ_{it} has average zero for all t. To simplify the notation even further, we write the model like:

$$y_{it} = \alpha_i + x_{it}\beta + \epsilon_{it},\ \epsilon_{it} = \nu_{it} - u_{it},\ t = 1, 2 \ldots, T \ \&\ i = 1, 2 \ldots, N, \tag{3}$$

where time-dummies are included in the x_{it} vector. However, we want to stress that it is very important to include these dummies when estimation of β is conducted. Otherwise, the results derived in this paper will not be generally applicable, since they are derived given $E(\epsilon_t) = 0$ for all $t = 1, \ldots, T$.

3. Identification of the parameters of the composed error term

In this section we propose how to obtain expressions of the parameters of the composed error term of the original (normal-) half-normal model proposed by Greene (2005*a*,*b*) extended to allow for a non-stationary composed error term and inefficiency. In the next section we present consistent moment estimators to consistently estimate the parameter of the model in this section.

The normal-half-normal model consider the inefficiency term to be half-normally distributed and the random error to be normally distributed. To obtain a consistent estimator of the parameter of the half-normal distribution, it is enough to assume that the random error is symmetrically distributed, i.e. that the third central moment is zero.[1] The half-normal distribution is a one-parameter distribution and we define the parameter as follows:

$$\theta_t = \sqrt{\frac{\pi}{2}}\frac{1}{\sigma_t}$$

where $u_t \sim |N(0, \sigma_t^2)|$. Given that the random error is symmetric the third order central moment of the composed error term is equal to the negative of the third order central moment of the inefficiency term:

$$\mu_{3\epsilon,t} = \mu_{3\nu,t} - \mu_{3u,t} = -\mu_{3u,t}.$$

The third central moment of a half-normal distributed random variable is:

$$-\mu_{3\epsilon,t} = \frac{4-\pi}{2\theta_t^3},$$

which is strictly positive since θ_t is, and an expression of θ_t can then be shown to be:

$$\theta_t = \left(\frac{4-\pi}{-2\mu_{3\epsilon,t}}\right)^{1/3}. \tag{4}$$

Thus if we have a consistent estimator of $\mu_{3\epsilon,t}$ for time-point t, we can consistently estimate θ_t. Given consistent estimators of θ_t and of $\mu_{\epsilon 2,t}$ one can in turn obtain consistent estimators of the expected value and the variance of the random error ν_t from the following expressions:

[1] However, to obtain firm specific predictions of the inefficiencies one need to assume normality following Jondrow et al. (1982).

$$E(\epsilon_t) = E(\nu_t) - E(u_t) = 0, \tag{5}$$

$$\mu_{2\epsilon,t} = V(\nu_t) + V(u_t). \tag{6}$$

This enables us to obtain firm-specific predictions of the inefficiencies given a normal assumption on ν_t (see eq. (6) Greene 2005*b*).

In the next section we demonstrate how to obtain necessary consistent moment estimators for obtaining a consistent estimators of the parameters $\theta_t,\ 1,\dots,T$ presented in this section.

4. Consistent Moment estimators

In this section we present consistent moment estimators as $N \to \infty$. To obtain consistent estimators of half-normal model we need consistent estimators of the second and the third central moments of the composed error term.

For this purpose we work with the within-transformed model and the OLS estimators of this model. We use the following 'dot-dot' notation for the within-transformed variables: $\ddot{z}_{it} = z_{it} - \bar{z}_i$ where $\bar{z}_i = \sum_t z_{it}/T$.

The connection between the original model and the within-transformed model is as follows for the second central moment at time t:

$$\begin{aligned} E(\ddot{\epsilon}_t^2) = E(\epsilon_t - \bar{\epsilon})^2 = E(\epsilon_t^2 - 2\epsilon_t\bar{\epsilon} + \bar{\epsilon}^2) = \\ E(\epsilon_t^2) - 2E(\epsilon_t\bar{\epsilon}) + E(\bar{\epsilon}^2) \end{aligned} \tag{7}$$

where

$$E(\epsilon_t\bar{\epsilon}) = E\left(\frac{\epsilon_t^2 + \sum_{s\neq t}\epsilon_t\epsilon_s}{T}\right) = \frac{E(\epsilon_t^2)}{T},$$

$$E(\bar{\epsilon}^2) = E\left(\frac{\sum_t \epsilon_t^2 + \sum_t\sum_{s\neq t}\epsilon_t\epsilon_s}{T^2}\right) = \frac{E\left(\sum_t \epsilon_t^2\right)}{T^2}$$

and thus:

$$E(\ddot{\epsilon}_t^2) = E(\epsilon_t^2) - 2\frac{E(\epsilon_t^2)}{T} + \frac{\sum_t E\left(\epsilon_t^2\right)}{T^2},$$

that can be rearranged to:

$$E(\epsilon_t^2) = \frac{T^2}{(T-1)^2}\left[E(\ddot{\epsilon}_t^2) - \frac{\sum_{s\neq t} E(\epsilon_s^2)}{T^2}\right] \tag{8}$$

and for time period s we have

$$E(\epsilon_s^2) = \frac{T^2}{(T-1)^2}\left[E(\ddot{\epsilon}_s^2) - \frac{\sum_{j\neq s} E(\epsilon_j^2)}{T^2}\right]. \tag{9}$$

Insert (9) into (8) and obtain :

$$E(\epsilon_t^2) = \frac{T^2}{(T-1)^2}E(\ddot{\epsilon}_t^2) - \frac{T^2}{(T-1)^4}\sum_{s\neq t} E(\ddot{\epsilon}_s^2) + \frac{1}{(T-1)^4}\sum_{s\neq t}\sum_{j\neq s} E(\epsilon_j^2) \tag{10}$$

and given $\sum_{s\neq t}\sum_{j\neq s}E(\epsilon_j^2)=(T-1)E(\epsilon_t^2)+(T-2)\sum_{j\neq t}E(\epsilon_j^2)$, we can solve for $\sum_{j\neq t}E(\epsilon_j^2)$, such that

$$\sum_{j\neq t}E(\epsilon_j^2)=\frac{(T-1)\left[(T-1)^3-1\right]}{T-2}E(\epsilon_t^2)-\frac{T^2(T-1)^2}{T-2}E(\ddot{\epsilon}_t^2)+\frac{T^2}{(T-2)}\sum_{j\neq t}E(\ddot{\epsilon}_j^2) \tag{11}$$

and finally this sum is substituted into (8) to obtain :

$$E(\epsilon_t^2)=\frac{1}{(T-1)(T-2)}\left[\left[(T-2)+(T-1)^2\right]E(\ddot{\epsilon}_t^2)-\sum_{s\neq t}E(\ddot{\epsilon}_s^2)\right] \tag{12}$$

For the third order central moments we have the following relations:

$$\begin{aligned}E(\ddot{\epsilon}_t^3)&=E(\epsilon_t-\bar{\epsilon})^3=E(\epsilon_t^3-3\epsilon_t^2\bar{\epsilon}+3\epsilon_t\bar{\epsilon}^2-\bar{\epsilon}^3)=\\&E(\epsilon_t^3)-3E(\epsilon_t^2\bar{\epsilon})+3E(\epsilon_t\bar{\epsilon}^2)-E(\bar{\epsilon}^3)\end{aligned} \tag{13}$$

where

$$\begin{aligned}
&E(\epsilon_t^2\bar{\epsilon})=E\left(\epsilon_t^2\frac{\sum_s\epsilon_s}{T}\right)=E\left(\frac{\epsilon_t^3+\sum_{s\neq t}\epsilon_s\epsilon_t^2}{T}\right)=\frac{E\left(\epsilon_t^3\right)}{T}\\
&E(\epsilon_t\bar{\epsilon}^2)=E\left(\epsilon_t\frac{\sum_s\epsilon_s^2+\sum_s\sum_{j\neq s}\epsilon_s\epsilon_j}{T^2}\right)=\\
&E\left(\frac{\epsilon_t^3+\sum_{s\neq t}\epsilon_s^2\epsilon_t+\epsilon_t\sum_s\sum_{j\neq s}\epsilon_s\epsilon_j}{T^2}\right)=\frac{E\left(\epsilon_t^3\right)}{T^2}\\
&E(\bar{\epsilon}^3)=E\left(\frac{\sum_t\epsilon_t}{T}\frac{\sum_s\epsilon_s^2+\sum_s\sum_{j\neq s}\epsilon_s\epsilon_j}{T^2}\right)=\\
&E\left(\frac{\sum_t\epsilon_t\sum_s\epsilon_s^2+\sum_t\epsilon_t\sum_s\sum_{j\neq s}\epsilon_s\epsilon_j}{T^3}\right)=\\
&E\left(\frac{\sum_t\epsilon_t^3+\sum_t\sum_{s\neq t}\epsilon_t\epsilon_s^2+\sum_t\sum_{s\neq t}\epsilon_t^2\epsilon_s+\sum_t\sum_{s\neq t}\epsilon_t\sum_{j\neq s}\epsilon_s\epsilon_j}{T^3}\right)=\\
&E\left(\frac{\sum_t\epsilon_t^3+3\sum_t\sum_{s\neq t}\epsilon_t\epsilon_s^2+\sum_t\sum_{s\neq t}\sum_{j\neq s,t}\epsilon_t\epsilon_s\epsilon_j}{T^3}\right)=\frac{\sum_t E(\epsilon_t^3)}{T^3}
\end{aligned}$$

For the last equality, independence or actually mean-independence assumptions like: $E(\epsilon_t|\epsilon_s)=E(\epsilon_t)$ and $E(\epsilon_t^2|\epsilon_s)=E(\epsilon_t^2)$ for all $t\neq s$ are sufficient. Given (13), we can now obtain:

$$E(\epsilon_t^3)=\frac{T^3}{(T-1)^3}\left[E(\ddot{\epsilon}_t^3)+\frac{\sum_{s\neq t}E(\epsilon_s^3)}{T^3}\right] \tag{14}$$

and furthermore we can obtain the following sum, the same way we did it for (11):

$$\frac{\sum_{s\neq t}E(\epsilon_s^3)}{T^3}=\frac{(T-1)\left[(T-1)^5-1\right]}{(T-2)T^3}E(\epsilon_t^3)-\frac{(T-1)^3}{T-2}E(\ddot{\epsilon}_t^3)-\frac{1}{T-2}\sum_{s\neq t}E(\ddot{\epsilon}_s^3) \tag{15}$$

and if we substitute the right-hand side into (13), we finally obtain:

(16)

$$E(\epsilon_t^3) = \frac{T^3}{(T-1)\left[(T-1)^2(T-2)-(T-1)^5+1\right]}\left[\left[(T-2)-(T-1)^3\right]E(\ddot{\epsilon}_t^3) - \sum_{s\neq t} E(\ddot{\epsilon}_s^3)\right]$$

For the third raw moment $E(\ddot{\epsilon_t}^3)$ we propose the following consistent estimator:

$$\begin{aligned}(17)\qquad \frac{\sum_i \hat{\ddot{\epsilon}}_t^3}{N} &= \frac{\sum_i(\ddot{y}_t - \ddot{x}_t\hat{\beta})^3}{N} = \frac{\sum_i(\ddot{\epsilon}_t + \ddot{x}_t(\beta-\hat{\beta}))^3}{N} \\ &= \frac{\sum_i \ddot{\epsilon}_t^3 + 3\ddot{\epsilon}_t^2\ddot{x}_t(\beta-\hat{\beta}) + 3\ddot{\epsilon}_t\left[\ddot{x}_t(\beta-\hat{\beta})\right]^2 + \left[\ddot{x}_t(\beta-\hat{\beta})\right]^3}{N} \\ &= \frac{\sum_i \ddot{\epsilon}_t^3}{N} + O_{a.s.}(N^{-1/2}) + O_{a.s.}(N^{-1}) + O_{a.s.}(N^{-3/2}) \xrightarrow{a.s.} E(\ddot{\epsilon}_t^3)\end{aligned}$$

where $a.s.$ denotes 'almost sure' (or 'strong') convergence. For the forth equality Lemma 1 in the appendix is applied. Sufficient moment restrictions are $E(|\epsilon_t|^3) < \infty$ and $E(|x_t|^3) < \infty$. This can be verified with help of Lemma 1 in the supplementary materials of Wikström (2016).

Similarly one can find a consistent estimator of the second raw moment, such that:

$$(18)\qquad \hat{E}(\ddot{\epsilon}_t) = 0,$$

$$(19)\qquad \hat{E}(\ddot{\epsilon}_t^2) = \frac{\sum_i \hat{\ddot{\epsilon}}_{it}^2}{N} \xrightarrow{a.s.} E(\ddot{\epsilon}_t^2),$$

$$(20)\qquad \hat{E}(\ddot{\epsilon}_t^3) = \frac{\sum_i \hat{\ddot{\epsilon}}_{it}^3}{N} \xrightarrow{a.s.} E(\ddot{\epsilon}_t^3).$$

Now it is straightforward to construct consistent estimators of the third central moments of ϵ_t for all t. We first substitute the estimators (18)-(20) into (16) to obtain a consistent estimator of $E(\epsilon_t^3)$ and this is also the estimator of the third central moment:

$$\mu_{3\epsilon,t} = E(\epsilon_t^3) - 3E(\epsilon_t)E(\epsilon_t^2) + 2E(\epsilon_t)^3 = E(\epsilon_t^3).$$

5. Asymptotic normality

In this section we derive key asymptotic normality results for general method of moment estimators including raw moments up to order three. For the third raw moment estimator we have

$$\begin{aligned}(21)\qquad \sqrt{N}\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^3}{N} &= \sqrt{N}\frac{\sum_i(\ddot{\epsilon}_{it} + \ddot{x}_{it}(\beta-\hat{\beta}))^3}{N} \\ &= \sqrt{N}\frac{\sum_i \ddot{\epsilon}_{it}^3 + 3\ddot{\epsilon}_{it}^2\ddot{x}_{it}(\beta-\hat{\beta}) + 3\ddot{\epsilon}_{it}\left[\ddot{x}_{it}(\beta-\hat{\beta})\right]^2 + \left[\ddot{x}_{it}(\beta-\hat{\beta})\right]^3}{N} \\ &= \sqrt{N}\frac{\sum_i \ddot{\epsilon}_t^3 + 3\ddot{\epsilon}_t^2\ddot{x}_{it}(\beta-\hat{\beta})}{N} + O_{a.s.}(N^{-1/2}) + O_{a.s.}(N^{-1}) \xrightarrow{a.s.}\end{aligned}$$

$$\sqrt{N}\frac{\sum_i \ddot{\epsilon}_{it}^3}{N} - 3E(\ddot{x}_t\ddot{\epsilon}_t^2)\left(E(\ddot{X}'\ddot{X})\right)^{-1}\sqrt{N}\frac{\sum_i \ddot{X}_i'\ddot{\boldsymbol{\epsilon}}_i}{N},$$

where $\ddot{\boldsymbol{\epsilon}}' = [\ddot{\epsilon}_1 \dots \ddot{\epsilon}_T]$ is an $1 \times T$ vector and $\ddot{X}' = [\ddot{x}_1 \dots \ddot{x}_T]$ is a $K \times T$ matrix. For the third equality, we apply Lemma 1 given in the appendix.

In the same way we can show that:

$$\sqrt{N}\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^2}{N} \xrightarrow{a.s.} \sqrt{N}\frac{\sum_i \ddot{\epsilon}_{it}^2}{N}.$$

We can summon the random variables in these expressions into the following multivariate asymptotic normal result:

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) = \sqrt{N}\begin{pmatrix} \hat{\theta}_{21} - \theta_{21} \\ \vdots \\ \hat{\theta}_{2T} - \theta_{2T} \\ \hat{\theta}_{31} - \theta_{31} \\ \vdots \\ \hat{\theta}_{3T} - \theta_{3T} \\ \hat{\theta}_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}} - \theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}} \end{pmatrix} \xrightarrow{d} N(\mathbf{0}, \Sigma). \tag{22}$$

where $\hat{\theta}_{2t} = \frac{\sum_i \ddot{\epsilon}_{it}^2}{N}$, $\theta_{2t} = E(\ddot{\epsilon}_t^2)$, $\hat{\theta}_{3t} = \frac{\sum_i \ddot{\epsilon}_{it}^3}{N}$, $\theta_{3t} = E(\ddot{\epsilon}_t^3)$, $\hat{\theta}_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}} = \frac{\sum_i \ddot{X}_i'\ddot{\boldsymbol{\epsilon}}_i}{N}$ and $\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}} = \mathbf{0}$. The covariance matrix is a $(2T+K) \times (2T+K)$ matrix that can be written as follows:

$$\Sigma = \begin{pmatrix} \sigma^2_{\theta_{21}} & \cdots & \sigma_{\theta_{21},\theta_{2T}} & \sigma_{\theta_{21},\theta_{31}} & \cdots & \sigma_{\theta_{21},\theta_{3T}} & \sigma_{\theta_{21},\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}}} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{\theta_{2T},\theta_{21}} & \cdots & \sigma^2_{\theta_{2T}} & \sigma_{\theta_{2T},\theta_{31}} & \cdots & \cdots & \sigma_{\theta_{2T},\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}}} \\ \sigma_{\theta_{31},\theta_{21}} & \cdots & \sigma_{\theta_{31},\theta_{2T}} & \sigma^2_{\theta_{31}} & \cdots & \cdots & \sigma_{\theta_{31},\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{\theta_{3T},\theta_{21}} & \cdots & \sigma_{\theta_{3T},\theta_{2T}} & \sigma_{\theta_{3T},\theta_{31}} & \cdots & \sigma^2_{\theta_{3T}} & \sigma_{\theta_{3T},\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}}} \\ \sigma_{\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}},\theta_{21}} & \cdots & \sigma_{\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}},\theta_{2T}} & \sigma_{\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}},\theta_{31}} & \cdots & \sigma_{\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}},\theta_{3T}} & \sigma^2_{\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}}} \end{pmatrix}, \tag{23}$$

where $\sigma^2_{\theta_{2t}} = \left[E(\ddot{\epsilon}_t^4) - E(\ddot{\epsilon}_t^2)E(\ddot{\epsilon}_t^2)\right]$, $\sigma^2_{\theta_{3t}} = \left[E(\ddot{\epsilon}_t^6) - E(\ddot{\epsilon}_t^3)E(\ddot{\epsilon}_t^3)\right]$, $\sigma_{\theta_{2t},\theta_{3t}} = \left[E(\ddot{\epsilon}_t^5) - E(\ddot{\epsilon}_t^2)E(\ddot{\epsilon}_t^3)\right]$, $\sigma_{\theta_{2t},\theta_{3s}} = \left[E(\ddot{\epsilon}_t^2\ddot{\epsilon}_s^3) - E(\ddot{\epsilon}_t^2)E(\ddot{\epsilon}_s^3)\right]$, $\sigma_{\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}},\theta_{pt}} = E(\ddot{X}'\ddot{\boldsymbol{\epsilon}}\ddot{\epsilon}_t^p)$ and $\sigma^2_{\theta_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}}} = E(\ddot{X}'\ddot{\boldsymbol{\epsilon}}\ddot{\boldsymbol{\epsilon}}'\ddot{X})$, and with the delta-method we can derive asymptotic normality results for the method of moment estimators at each t. However, to have practical use of this we need an estimator of the covariance matrix.

An estimator is obtained by replacing each expected value with its consistent counterpart among the following estimators:

(24)

$$\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^{p}}{N} = \frac{\sum_i (\ddot{\epsilon}_{it} + \ddot{x}_{it}(\beta - \hat{\beta}))^p}{N} = \frac{\sum_i \ddot{\epsilon}_{it}^{p}}{N} + O_{a.s.}(N^{-1/2}) \xrightarrow{a.s.} E(\ddot{\epsilon}_t^{p}),$$

(25)

$$\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^{p} \hat{\ddot{\epsilon}}_{is}^{q}}{N} = \frac{\sum_i \ddot{\epsilon}_{it}^{p} \ddot{\epsilon}_{is}^{q}}{N} + O_{a.s.}(N^{-1/2}) \xrightarrow{a.s.} E(\ddot{\epsilon}_t^{p} \ddot{\epsilon}_s^{q})$$

(26)

$$\frac{\sum_i \ddot{X}_i' \hat{\ddot{\boldsymbol{\epsilon}}}_i \hat{\ddot{\epsilon}}_{it}^{p}}{N} = \frac{\sum_i}{N} \begin{pmatrix} \sum_t \ddot{x}_{1it} \hat{\ddot{\epsilon}}_{it}^{p+1} + \sum_t \sum_{s \neq t} \hat{\ddot{\epsilon}}_{it} \hat{\ddot{\epsilon}}_{is}^{p} \\ \vdots \\ \sum_t \ddot{x}_{Kit} \hat{\ddot{\epsilon}}_{it}^{p+1} + \sum_t \sum_{s \neq t} \hat{\ddot{\epsilon}}_{it} \hat{\ddot{\epsilon}}_{is}^{p} \end{pmatrix} =$$

$$\frac{\sum_i}{N} \begin{pmatrix} \sum_t \ddot{x}_{1it} \ddot{\epsilon}_{it}^{p+1} + \sum_t \sum_{s \neq t} \ddot{\epsilon}_{it} \ddot{\epsilon}_{is}^{p} \\ \vdots \\ \sum_t \ddot{x}_{Kit} \ddot{\epsilon}_{it}^{p+1} + \sum_t \sum_{s \neq t} \hat{\ddot{\epsilon}}_{it} \hat{\ddot{\epsilon}}_{is}^{p} \end{pmatrix} + O_{a.s.}(N^{-1/2}) \xrightarrow{a.s.} E(\ddot{X}' \ddot{\boldsymbol{\epsilon}} \ddot{\epsilon}_t^{p}),$$

(27)

$$\frac{\sum_i \ddot{X}_i' \hat{\ddot{\boldsymbol{\epsilon}}}_i \hat{\ddot{\boldsymbol{\epsilon}}}_i \ddot{X}_i}{N} =$$

$$\frac{\sum_i}{N} \begin{pmatrix} \left(\sum_t \ddot{x}_{1it} \hat{\ddot{\epsilon}}_{it}\right)^2 & \cdots & \sum_t \ddot{x}_{1it} \hat{\ddot{\epsilon}}_{it} \sum_t \ddot{x}_{Kit} \hat{\ddot{\epsilon}}_{it} \\ \vdots & \ddots & \vdots \\ \sum_t \ddot{x}_{Kit} \hat{\ddot{\epsilon}}_{it} \sum_t \ddot{x}_{1it} \hat{\ddot{\epsilon}}_{it} & \cdots & \left(\sum_t \ddot{x}_{Kit} \hat{\ddot{\epsilon}}_{it}\right)^2 \end{pmatrix} =$$

$$\frac{\sum_i}{N} \begin{pmatrix} \sum_t \ddot{x}_{1it}^2 \hat{\ddot{\epsilon}}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{1it} \ddot{x}_{1is} \hat{\ddot{\epsilon}}_{it} \hat{\ddot{\epsilon}}_{is} & \cdots & \sum_t \ddot{x}_{1it} \ddot{x}_{Kit} \hat{\ddot{\epsilon}}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{1it} \ddot{x}_{Kis} \hat{\ddot{\epsilon}}_{it} \hat{\ddot{\epsilon}}_{is} \\ \vdots & \ddots & \vdots \\ \sum_t \ddot{x}_{Kit} \ddot{x}_{1it} \hat{\ddot{\epsilon}}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{Kit} \ddot{x}_{1is} \hat{\ddot{\epsilon}}_{it} \hat{\ddot{\epsilon}}_{is} & \cdots & \sum_t \ddot{x}_{Kit}^2 \hat{\ddot{\epsilon}}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{Kit} \ddot{x}_{Kis} \hat{\ddot{\epsilon}}_{it} \hat{\ddot{\epsilon}}_{is} \end{pmatrix} =$$

$$\frac{\sum_i}{N} \begin{pmatrix} \sum_t \ddot{x}_{1it}^2 \ddot{\epsilon}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{1it} \ddot{x}_{1is} \ddot{\epsilon}_{it} \ddot{\epsilon}_{is} & \cdots & \sum_t \ddot{x}_{1it} \ddot{x}_{Kit} \ddot{\epsilon}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{1it} \ddot{x}_{Kis} \ddot{\epsilon}_{it} \ddot{\epsilon}_{is} \\ \vdots & \ddots & \vdots \\ \sum_t \ddot{x}_{Kit} \ddot{x}_{1it} \ddot{\epsilon}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{Kit} \ddot{x}_{1is} \ddot{\epsilon}_{it} \ddot{\epsilon}_{is} & \cdots & \sum_t \ddot{x}_{Kit}^2 \ddot{\epsilon}_{it}^2 + \sum_t \sum_{s \neq t} \ddot{x}_{Kit} \ddot{x}_{Kis} \ddot{\epsilon}_{it} \ddot{\epsilon}_{is} \end{pmatrix} +$$

$$O_{a.s.}(N^{-1/2}) \xrightarrow{a.s.} E(\ddot{X}' \ddot{\boldsymbol{\epsilon}} \ddot{\boldsymbol{\epsilon}}' \ddot{X}).$$

5.1. **Joint testing for stationarity.** Given the asymptotic normality result in (22) it is possible to construct test-statistics for making joint hypotheses about the parameters of the composed error term and also about the inefficiency distribution. In this section we discuss how to construct a joint tests for stationarity. We first consider a stationarity test for the whole composed error term ϵ and then discuss possibilities for conduct testing about the inefficiency distribution.

Now we turn to a test for stationarity of the whole composed error term. In all versions of the true fixed effects model we know of, except for the one proposed here, stationarity is assumed for the composed error term. The test we propose here is completely nonparametric in the sense that no knowledge of the distribution of the composed error is necessary (except for basic central limit theorem assumptions). This test is based on the second order raw moments of the composed error.[2] We state the null hypothesis as follows:

$$H_0: \; E(\epsilon_1^2) = \ldots = E(\epsilon_T^2) \Leftrightarrow$$
$$H_0: \; E(\ddot{\epsilon}_1^2) = \ldots = E(\ddot{\epsilon}_T^2) \Leftrightarrow$$
$$H_0: \; E(\ddot{\epsilon}_1^2) = E(\ddot{\epsilon}_2^2) \;\&\; E(\ddot{\epsilon}_2^2) = E(\ddot{\epsilon}_1^2) \;\&\; \ldots \;\&\; E(\ddot{\epsilon}_{T-1}^2) = E(\ddot{\epsilon}_T^2),$$

where the first equivalence is verified by (7) and (12). Given

$$\sqrt{N}\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^2}{N} \xrightarrow{a.s.} \sqrt{N}\frac{\sum_i \ddot{\epsilon}_{it}^2}{N}, \; 1, \ldots, T,$$

it is more straightforward to testing this compared to the previous test. The following Wald-statistic can be used:

$$W_2 = N(\mathbf{R_1}\hat{\boldsymbol{\theta}})'(\mathbf{R_1}\Sigma\mathbf{R_1}')^{-1}\mathbf{R_1}\hat{\boldsymbol{\theta}}, \tag{28}$$

where $\mathbf{R_1}$ is a is a $(T-1)\times(2T+K)$ matrix. On row t the matrix has one on positions t and minus one on position $(t+1)$, where $t = 1, \ldots, T-1$. The statistic W_2 is asymptotically χ^2-distributed with $(T-1)$ degrees of freedom. This statistic can be made feasible by replacing $\frac{\sum_i \ddot{\epsilon}_{it}^2}{N}$ with $\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^2}{N}$ in $\hat{\boldsymbol{\theta}}$ and Σ is replaced with $\hat{\Sigma}$ including the consistent elements given by (24)-(27).

Thus, it is possible to make joint testing with a consistent estimator of the covariance matrix Σ. Nevertheless, this estimator is quite complicated and an alternative is to use a bootstrap estimator instead. We consider a simple non-parametric block bootstrap, where one block is the T observations of one firm, like the following bootstrap scheme:

(1) Given the data $\{x_{it}, y_{it}\}_{t=1}^T$, $i = 1, \ldots, N$, draw, with replacement, N firm blocks of data $\{x_{it}^*, y_{it}^*\}_{t=1}^T$, $i = 1, \ldots, N$.
(2) Calculate the bootstrap estimate of the vector:

$$\hat{\boldsymbol{\gamma}}^* = \sqrt{N}\widehat{(\mathbf{R_1}\hat{\boldsymbol{\theta}})}^* = \sqrt{N}\begin{pmatrix} \frac{\sum_i \hat{\ddot{\epsilon}}_{i1}^{2*}}{N} - \frac{\sum_i \hat{\ddot{\epsilon}}_{i2}^{2*}}{N} \\ \frac{\sum_i \hat{\ddot{\epsilon}}_{i2}^{2*}}{N} - \frac{\sum_i \hat{\ddot{\epsilon}}_{i3}^{2*}}{N} \\ \vdots \\ \frac{\sum_i \hat{\ddot{\epsilon}}_{i,T-1}^{2*}}{N} - \frac{\sum_i \hat{\ddot{\epsilon}}_{iT}^{2*}}{N} \end{pmatrix}$$

with help of the within residulas $\hat{\ddot{\epsilon}}_{it}$.
(3) Repeat 1. and 2. B times to obtain B estimated vectors $\hat{\boldsymbol{\gamma}}_1^*, \ldots, \hat{\boldsymbol{\gamma}}_B^*$.

[2]We realize there may be special cases of non-stationarity for which H_0 is still true. To be precise, this test is for second order raw-moment stationarity.

(4) Finally, obtain an estimate of the covariance matrix $\mathbf{R_1}\Sigma\mathbf{R_1}'$ as

$$\hat{\Omega}^* = \frac{1}{B-1}\sum_{b=1}^{B}(\hat{\boldsymbol{\gamma}}_b^* - \bar{\hat{\boldsymbol{\gamma}}}^*)(\hat{\boldsymbol{\gamma}}_b^* - \bar{\hat{\boldsymbol{\gamma}}}^*)'$$

where $\bar{\hat{\boldsymbol{\gamma}}}^* = \frac{\sum_{b=1}^{B}\hat{\boldsymbol{\gamma}}_b^*}{B}$.

Now we turn the attention to stationary testing for inefficiency.[3] We consider the straightforward hypothesis that the parameters of the inefficiency distribution are equal. Thus, we consider the following type of null hypothesis:

$$\begin{aligned}
&H_0:\ \theta_1 = \ldots = \theta_T \Leftrightarrow\\
&H_0:\ E(\epsilon_1^3) = \ldots = E(\epsilon_T^3) \Leftrightarrow\\
&H_0:\ E(\ddot{\epsilon}_1^3) = \ldots = E(\ddot{\epsilon}_T^3) \Leftrightarrow\\
&H_0:\ E(\ddot{\epsilon}_1^3) = E(\ddot{\epsilon}_2^3)\ \&\ E(\ddot{\epsilon}_2^3) = E(\ddot{\epsilon}_1^3)\ \&\ \ldots\ \&\ E(\ddot{\epsilon}_{T-1}^3) = E(\ddot{\epsilon}_T^3),
\end{aligned}$$

where the second equivalence is given by (13) and (16). This hypothesis can be tested by using the following asymptotic result:

$$\sqrt{N}\left(\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^3}{N}\right) \xrightarrow{a.s.} \sqrt{N}\left(\frac{\sum_i \ddot{\epsilon}_{it}^3}{N}\right) - 3E(\ddot{x}_t\ddot{\epsilon}_t^2)\left(E(\ddot{X}'\ddot{X})\right)^{-1}\sqrt{N}\frac{\sum_i \ddot{X}_i'\ddot{\boldsymbol{\epsilon}}_i}{N} = \tag{29}$$

$$\sqrt{N}\hat{\theta}_{3t} - 3E(\ddot{x}_t\ddot{\epsilon}_t^2)\left(E(\ddot{X}'\ddot{X})\right)^{-1}\sqrt{N}\hat{\theta}_{\ddot{X}'\ddot{\boldsymbol{\epsilon}}},\ 1,\ldots,T.$$

The null hypothesis can be further rewritten in matrix form as $\mathbf{R_2}\boldsymbol{\theta} = \mathbf{0}$, where $\mathbf{R_2}$ is a $(T-1)\times(2T+K)$ matrix and $\boldsymbol{\theta}$ is the parameter vector in (22). Row t of matrix $\mathbf{R_2}$ have one on positions $(T+t)$ and minus one on position $[T+(t+1)]$ and the vector $3E(\ddot{x}_{t+1}\ddot{\epsilon}_{t+1}^2)\left(E(\ddot{X}'\ddot{X})\right)^{-1} - 3E(\ddot{x}_t\ddot{\epsilon}_t^2)\left(E(\ddot{X}'\ddot{X})\right)^{-1}$ on the K final positions. This gives $\mathbf{R_{2t\bullet}}\boldsymbol{\theta} = E(\ddot{\epsilon}_t^3) - E(\ddot{\epsilon}_{t+1}^3)$, $t = 1\ldots,T-1$. Given this and the asymptotic normality result in (22), the following Wald statistic is asymptotically χ^2-distributed with $(T-1)$ degrees of freedom:

$$W_2 = N(\mathbf{R_2}\hat{\boldsymbol{\theta}})'(\mathbf{R_2}\Sigma\mathbf{R_2}')^{-1}\mathbf{R_2}\hat{\boldsymbol{\theta}}. \tag{30}$$

This statistic is not feasible but if each row t of $\mathbf{R_2}\hat{\boldsymbol{\theta}}$ is replaced by $\frac{\sum_i \hat{\ddot{\epsilon}}_{it}^3}{N} - \frac{\sum_i \hat{\ddot{\epsilon}}_{i,t+1}^3}{N}$, the terms $E(\ddot{x}_t\ddot{\epsilon}_t^2)\left(E(\ddot{X}'\ddot{X})\right)^{-1}$, $1,\ldots,T$ are replaced by consistent counterparts in $\mathbf{R_2}$ and if Σ is replaced with $\hat{\Sigma}$ including the consistent elements given by (24)-(27), then we obtain a feasible test-statistic with the same asymptotic distribution as W_1. For estimating $E(\ddot{X}'\ddot{X})$ one can use the sample moment and for $E(\ddot{x}_t\ddot{\epsilon}_t^2)$ one can use the sample moment where $\ddot{\epsilon}_i$ is

[3]Given a symmetric random error and a skewed one-parameter inefficiency distribution, we conjecture there are no special cases for which H_0 is true when the inefficiency distribution is not stationary.

replace with $\hat{\ddot{\epsilon}}_i$. That is the following estimator:

$$\frac{\sum_i \ddot{x}_{it}\hat{\ddot{\epsilon}}^2_{it}}{N} = \frac{\sum_i \ddot{x}_{it}\left(\ddot{\epsilon}_{it} + \ddot{x}_{it}(\beta - \hat{\beta})\right)^2}{N} = \frac{\sum_i \ddot{x}_{it}\ddot{\epsilon}^2_{it}}{N} + O_{a.s.}(N^{-1}) \xrightarrow{a.s.} E(\ddot{x}_t\ddot{\epsilon}^2_t). \tag{31}$$

Nevertheless, an estimator of the test-statistic W_2 is quite complicated and, therefore, we consider the following bootstrap scheme for obtaining the asymptotic covariance matrix:

(1) Given the data $\{x_{it}, y_{it}\}_{t=1}^{T}$, $i = 1, \ldots, N$, draw, with replacement, N firm blocks of data $\{x^*_{it}, y^*_{it}\}_{t=1}^{T}$, $i = 1, \ldots, N$.
(2) Calculate the bootstrap estimate of the vector:

$$\hat{\boldsymbol{\gamma}}^* = \sqrt{N}\widehat{(\mathbf{R_2}\hat{\boldsymbol{\theta}})}^* = \sqrt{N}\begin{pmatrix} \frac{\sum_i \hat{\ddot{\epsilon}}^{3*}_{i1}}{N} - \frac{\sum_i \hat{\ddot{\epsilon}}^{3*}_{i2}}{N} \\ \frac{\sum_i \hat{\ddot{\epsilon}}^{3*}_{i2}}{N} - \frac{\sum_i \hat{\ddot{\epsilon}}^{3*}_{i3}}{N} \\ \vdots \\ \frac{\sum_i \hat{\ddot{\epsilon}}^{3*}_{i,T-1}}{N} - \frac{\sum_i \hat{\ddot{\epsilon}}^{3*}_{iT}}{N} \end{pmatrix}$$

with help of the within residulas $\hat{\ddot{\epsilon}}_{it}$.
(3) Repeat 1. and 2. B times to obtain B estimated vectors $\hat{\boldsymbol{\gamma}}^*_1, \ldots, \hat{\boldsymbol{\gamma}}^*_B$.
(4) Finally, obtain an estimate of the covariance matrix $\mathbf{R_2}\Sigma\mathbf{R_2}'$ as

$$\hat{\Omega}^* = \frac{1}{B-1}\sum_{b=1}^{B}(\hat{\boldsymbol{\gamma}}^*_b - \bar{\hat{\boldsymbol{\gamma}}}^*)(\hat{\boldsymbol{\gamma}}^*_b - \bar{\hat{\boldsymbol{\gamma}}}^*)'$$

where $\bar{\hat{\boldsymbol{\gamma}}}^* = \frac{\sum_{b=1}^{B}\hat{\boldsymbol{\gamma}}^*_b}{B}$.

6. Conclusion

In this paper we have demonstrated how to derive consistent estimators of the so called ”true fixed effects” model based on method of moments. We have extended the modelling of Wikström (2015), to incorporate a non-stationary inefficiency distribution (and random error distribution). The focus has been on the half-normal model but the results are general and can be applied on other models with an inefficiency term that has a single-parameter distribution, for example, the exponential model.

We also derive Wald-tests for testing stationarity of both the composed error term as well as for the inefficiency distribution.

A natural extension of the work presented here would be to allow for dependence of the inefficiency distribution over time.

7. Appendix

Lemma 1. *If* $(\beta - \hat{\beta}) = O_{a.s.}(N^{-1/2})$, $\{(b_i, a_{ik})\}_{i=1}^N$ *is an iid sequence,* $E(|b_i^q|) < \infty$ *and* $E(|a_{ik}^q|) < \infty$ *for all* k*, then*

$$\frac{\sum_{i=1}^N b_i \left(a_i'(\beta - \hat{\beta})\right)^p}{N} = O_{a.s.}\left(N^{-p/2}\right)$$

for any finite $p \geq 2$ *and* $q = p + 1$*.*

Proof.

$$\frac{\sum_{i=1}^N b_i \left(a_i'(\beta - \hat{\beta})\right)^p}{N} = \frac{\sum_{i=1}^N b_i \left(\sum_{k=1}^K a_{ik}(\beta_k - \hat{\beta}_k)\right)^p}{N} \leq$$

$$\frac{\sum_{i=1}^N (b_i^2)^{1/2} \left[\left(\sum_{k=1}^K a_{ik}(\beta_k - \hat{\beta}_k)\right)^2\right]^{p/2}}{N} = \frac{\sum_{i=1}^N |b_i| \left[\left|\sum_{k=1}^K a_{ik}(\beta_k - \hat{\beta}_k)\right|\right]^p}{N} \leq$$

$$\frac{\sum_{i=1}^N |b_i| \left[\sum_{k=1}^K |a_{ik}(\beta_k - \hat{\beta}_k)|\right]^p}{N} \leq \frac{\sum_{i=1}^N |b_i| \left[\sum_{k=1}^K |a_{ik}|^2 \sum_{k=1}^K |(\beta_k - \hat{\beta}_k)|^2\right]^{p/2}}{N} \leq$$

$$\left[\sum_{k=1}^K (\beta_k - \hat{\beta}_k)^2\right]^{p/2} \frac{K^{p/2}}{K} \sum_{k=1}^K \frac{\sum_{i=1}^N |b_i||a_{ik}|^p}{N}$$

$$= O_{a.s.}(N^{-p/2}) O_{a.s.}(1) = O_{a.s.}(N^{-p/2}).$$

The first inequality arises from the fact that $X \leq (X^2)^{1/2}$, with equality iff $X \geq 0$. The second inequality is given by the triangle inequality. The third inequality is obtained by applying the Cauchy-Schwarz inequality and the fourth inequality is obtained using Jensen's inequality (if $p \geq 2$). If $E|b_i a_{ik}^p| < \infty$, then by the strong law of large numbers:

$$\frac{\sum_{i=1}^N |b_i||a_{ik}|^p}{N} = O_{a.s.}(1),$$

and

$$E|b_i a_{ik}^p| \leq \left[E|b_i|^q\right]^{1/q} \left[E|a_{ik}^p|^{q/(q-1)}\right]^{(q-1)/q} = \left[E|b_i|^q\right]^{1/q} \left[E|a_{ik}|^q\right]^{(q-1)/q} < \infty$$

by Hölder's inequality. This completes the proof. □

Note that for the consistency results we only apply the lemma when $p \leq 2$. In this case, $E|a_{ik}^3|$ is required, i.e. $q = 3$. When $p = 3$, then $b_i = 1$ and we only consider $\frac{\sum_{i=1}^N \left(a_i'(\beta - \hat{\beta})\right)^p}{N}$. If we then follow the proof of the lemma, then $\frac{\sum_{i=1}^N \left(a_i'(\beta - \hat{\beta})\right)^3}{N} = O_{a.s.}(N^{-3/2})$ under $E|a_{ik}^3| < \infty$. Thus, finite third-order moments are required to demonstrate the consistency of the estimators presented here, but no restrictions are applied on higher orders.

For the asymptotic normality results we apply the lemma up to $p = 5$ and for $p = 6$ the same reasoning as given for consistency applies.

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