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Teaching Mathematics in English to Swedish Speaking Students

A systematic review of strategies for teaching mathematics to second language learners

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Abstract: The Swedish government has authorised the teaching of mathematics in English to Swedish speaking students. Much of that teaching is performed by foreign trained native English speaking teachers lacking training in second language learners. This systematic review summarises international studies from the last ten years that deal with the teaching of mathematics to second language learners. The review shows that second language students working in a bilingual environment achieve higher rates of content and language knowledge than learners in a monolingual environment. This study also summarises some of the teacher practices that are effective for teaching mathematics in English to second language learners.

Keywords: Second language learner • Mathematics teaching practices • CLIL • Bilingual classroom

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1 Introduction

My practical work experience during teacher training consisted of teaching mathematics at a large bilingual school in central Sweden. The school's mandate from the Swedish school authority, Skolverket, allowed for Mathematics, Natural Sciences and English to be taught in English by native English speakers, while Swedish and Social Sciences were taught in Swedish. Aesthetics subjects were taught in either English or Swedish depending on the abilities of the teachers hired at the time. In the student population of over 1320 students, there were only 44 students who were native English speakers themselves or had one native English speaking parent. The reality of this particular school environment is that the vast majority of students were participating in mathematics lessons that were being taught in English, a second language for most students and a third language for those who had immigrated to Sweden from a non-English speaking country.

During the 2008/2009 school year, there were 9,400 students, or about one percent of all school age children in Sweden, who were studying some subjects with English as the medium of instruction in 44 schools across Sweden (Skolverket 2010, p.24). A little over half of those, 4900, were students at the same school organisation where I completed my teacher training. As of 2015, my training school's organisation alone had 19,800 students registered across 29 schools. The growing popularity of students choosing schools with English as the medium of instruction for some subjects means that the number of students studying mathematics in a second language will probably increase throughout schools in Sweden in the near future.

Although the focus is on Swedish students learning mathematics in English, this paper will use the term second language learner to refer to the individual learning mathematics in a language other than their first language. The term second language environment will be used to refer to teaching with a language of instruction that is not the student's first language.

As the mathematics lessons at my training school were taught by teachers who were native speakers of English, consisting almost exclusively of teachers trained in countries other than Sweden, the teachers carried with them the experience of a cultural background different than the one that Swedish students were familiar with. In one of my earliest teaching experiences, I was introducing a series of probability lessons to a grade 7 class of Swedish speaking students. As I did not speak any Swedish at the time, I was very careful to speak English clearly and to define all mathematical terms, using both English and Swedish keywords that I had made a note of before the lesson. I was demonstrating dependent and independent probability events by discussing the selection of marbles from a bag and noting possible outcomes. After speaking to the class for 30 minutes, giving the students plenty of time to take down my notes from the whiteboard, I asked if any students had any questions. One student timidly raised their hand and asked "What's a marble?" I asked if any other student had the same question and more than half of the class raised their hands. This demonstrated a simple cultural assumption I had erroneously made, that I could simply teach mathematical concepts and terms to the students while ignoring common everyday terms in English.

Implicit assumptions about student cultural knowledge and language skills are made when preparing mathematical materials such as lesson plans and tests and these

assumptions, often incorrect in the case of students studying in a second language, increase the cognitive load for the students (Campbell, Davis & Adams, 2007, p.6). A second factor that must be considered for second language learners is their level of academic language proficiency. In his studies of second language learners, Cummins noted that it generally takes from 5 to 7 years for second language learners to acquire cognitive academic language proficiency, CALP (Ellis, 2008, p.307). It is reasonable to assume that Swedish students studying mathematics in English, taught by non-Swedish teachers, may experience many of the same difficulties that immigrant students experience, such as difficulties with language or difficulties regarding cultural expectations. Those cultural difficulties for Swedish students are usually eliminated when the students write the National Tests in Mathematics in grade 9 as they are prepared from a Swedish cultural perspective but it is the issue of teaching practices during regular lessons that is the focus of this study.

Skolverket has noted that few teachers working in English as the medium of instruction environments have been trained in content and language integrated learning (CLIL) or second language learning (Skolverket 2010, p.7). Yet the Swedish national school curriculum states that all students have the right to an equivalent education and that teaching must be adapted so that every student has the opportunity to develop according to their highest potential (Skolverket 2011, p.10). Although the majority of my students were native Swedish speakers, they had chosen to study at a school where mathematics and science were taught in English, a second language for them. How should I adapt my teaching methods for them? Many studies of second language learners in mathematics deal with immigrant students learning in a majority language environment. What does educational research into strategies for teaching mathematics in English to second language learners conclude?

This leads to my wondering what I, as a mathematics teacher working in a second language teaching environment, should be doing with my students to ensure that I, through my teaching, maximise the possibilities for them to develop their mathematical abilities to their fullest potential.

2 Study aim

The desired outcome of this systematic review is an inventory of recent knowledge about effective teaching practices for second language students of mathematics in early secondary school. Such an inventory can be a useful tool for other teachers of mathematics working with a similar population and context but can also be useful in suggesting methods that can be used or adapted by teachers working with other year groups in mathematics and sciences, two subjects with large cognitive loads when taught in a second language due to the large number of subject specific terminology (see Paas, Renkl & Sweller, 2003 for an overview of Cognitive Load Theory).

This systematic review will investigate international research published recently with the purpose of answering the following questions;

- What is known about teaching mathematics in a second language environment?
- What characterises effective teaching strategies for teaching mathematics in a second language environment?

3 Background

The first part of this chapter will define the term 'second language learner' and then describe some of the educational theories regarding language and learning in general and how they are reflected in the Swedish school curriculum. The focus will then narrow down to how the theories of language and learning apply to the teaching of mathematics and then finish with the focus on second language learners.

3.1 Definition of second language learner

In the context introduced at the beginning of this paper, many of the twenty-thousand students in the school organisation that I did my teacher training in have Swedish as their first language and are learning mathematics in English. Nonetheless, there are a number of students who do not regularly speak Swedish at home, students who have themselves immigrated to Sweden or were born in Sweden after their parents immigrated and who do not speak Swedish at home with their families. Swedish itself is their second or third language and English is now their third or fourth language. Ellis defines *second* (within the context of second language acquisition) as referring "to any language other than the first language" (Ellis, 2008, p.5). This paper will use Ellis' definition and all second, and third or more, language speakers will, in this context of being English language learners, be referred to as second language learners. Some researchers have offered separate definitions for second language acquisition and second language learning but Ellis argues that there is no need to make a distinction between the subconscious process of acquisition and the conscious process of learning because of the difficulty in determining whether the processes involved are conscious or not (Ellis, 2008, p.7). Ellis uses the terms *acquisition* and *learning* interchangeably but this paper will generally use the term *learning* for consistency unless paraphrasing another researcher's use of the term *acquisition*. The common notation for a learner's first language is L1 while L2 is used to refer to a second language.

3.2 Language and learning

> "Language, learning, and the development of a personal identity are all closely related. By providing a wealth of opportunities for discussion, reading and writing, all pupils should be able to develop their ability to communicate and thus enhance confidence in their own language abilities" (Skolverket, 2011, p.11).

In identifying the fundamental values and task of the school, Sweden's school curriculum identifies language as a vitally important aspect of learning. It is important to note that the relationship between language and learning changes over time. As a young child, one learns a verbal language as a form of communication, one of several means used to express oneself. In primary school, the process of codifying the verbal language into a written form begins as the student begins to learn how to read and write. Later on, by secondary school, language becomes a tool for learning. Gibbons (2009) writes that by grade 7, language proficiency and school language become more important in the process of knowledge acquisition and students are now "reading to learn" instead of "learning to read" (Gibbons, 2009, p.27). This distinction over the various stages of language in school can be clearly seen in the school curriculum when comparing the core content for the subject of Swedish for grades 1 – 3 and for grades

7 – 9. Under the heading Core content – *Reading and writing*, students in the earlier grades are expected to show knowledge of the "structure of language with large and small letters, full stops, question marks, exclamation marks as well as spelling rules for frequently occurring words in texts closely related to pupils' daily life" (Skolverket, 2011, p.212). Under the same heading for the later years of compulsory schooling students are now expected to demonstrate ability in "Identifying messages in texts, themes and motives, as well as their purpose, sender and context" (Skolverket, 2011, p214). As there is a clear distinction between the different age groups, it is important to note that this paper will be focusing on the older group of learners.

Many theories about learning have been proposed but I will mention two here briefly. Dewey's pragmatist theory 'learning by doing' can be seen as a move to student-centered learning (Säljö, 2012, p.176). The central theme of pragmatism is that people learn, through inquiry, what they need to learn in order to complete a task (Säljö, 2012, p.183). This theme is behind the development of problem based learning where students learn through the experience of solving open-ended problems by working and communicating together (Säljö, 2012, p.183 & 209).

The sociocultural perspective of learning and development is a group of theories that posit that verbal communication with other people is how we build the higher functions associated with thought, imagination and the ability to remember in an advanced way (Säljö, 2012, p.191). An important concept within the sociocultural perspective of learning is Vygotsky's Zone of Proximal Development, the zone where people are prepared for instruction and explanations, where the student is prepared to reach the next level of understanding. When given a task that is just beyond the learner's current abilities, if the learner is secure with the previous content, the learner is now prepared for the teacher, or a fellow student who is more knowledgeable, to provide guidance and support to the learner (Säljö, 2012, p.193). Vygotsky's theory is that this support through communication, later known as scaffolding, allows the learner to appropriate the knowledge from the more learned person (Säljö, 2012, p.194). According to the sociocultural perspective theories of learning, "interaction and communication are the keys to learning and development" (Säljö, 2012, p.195). The influence of sociocultural perspective theories on learning and their relation to language can be seen in the Swedish school curriculum with its repeated references to communication and interaction.

3.3 Language and the teaching of mathematics

As with learning in general, language is an important aspect of mathematics and the development of subject specific language is vital for a learner's success in mathematics as they advance through school. Moschkovich (2002) notes that "Students are now expected to communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments" (Moschkovich 2002, p.190). Mathematics is itself a language that should be used in all of its forms, both subtle and exact (Boaler, 2008, p.35). This emphasis on language and communication is reflected in one of the Swedish curriculum's five aims for mathematics, "Teaching in mathematics should essentially give pupils the opportunities to develop their ability to […] use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions"

(Skolverket 2011, p.60). This emphasis is also seen in the knowledge requirements for grade E at the end of grade 9,

> "Pupils can account for and discuss their approaches in a basically functional way and use symbols, algebraic expressions, formulae, graphs, functions and other mathematical forms of expression with some adaptation to purpose and context. In their accounts and discussions, pupils apply and follow mathematical reasoning by putting forward and responding to mathematical arguments in a way which to some extent takes the reasoning forward" (Skolverket 2011, p.67).

Mathematical language is exact and specific and lacks redundancy (Myndighet för skolutveckling, 2008, p.18). Redundancy, the repetition of an idea which supports understanding, is a strategy that can be used in communication to eliminate ambiguity. A person saying "Look at that house over there" while pointing at a house clarifies the ambiguous phrase 'that house' with the repetition in meaning by the gesture of pointing (gestures being a form of communication). The repetition in meaning allows the receiver to be more or less correct at inferring which house the speaker is referring to (unless the speaker is pointing to a group of houses some distance away!). With mathematical terms having a more specific meaning and no redundancy, it is more difficult for the learner to infer the meaning of an unknown word. Without knowledge of the meaning *isosceles*, there are no clues to allow a student to infer the meaning of the term, *isosceles*, in the very simple question, "How large are the angles in a right-angled isosceles triangle?" (Moschkovich, 2002, p.192)

Subject-specific language in mathematics refers to words that are particular to mathematics, words such as *hypotenuse* and *quadrilateral*, but also encompasses words that can be used in a general context, such as *mean* and *area* (Moschkovich, 2002, p.194). This latter aspect can be a point of difficulty for students as the same word can have different meanings in a mathematical context or in daily use (Moschkovich, 2002, p.194). When presented with the term "*one quarter*" (en kvart) Swedish students may associate it with the common expression "Vi ses om en kvart", or "We'll meet in fifteen minutes" which they are more likely to learn in conversational language before encountering the mathematical meaning of the fraction, *one quarter* in school. In daily life, the expression "*one quarter*" can be linked to the concept of fifteen minutes rather than the more exact mathematical proportional ratio of "one of four equal parts". Students with a weaker grasp of the mathematical concept of *one quarter* may also go on to confuse the decimal representation, 0.25 hours, as representing 25 minutes instead of one quarter of an hour. The challenges of subject specific language learning play a large role in learners' achievement in mathematics (Myndighet för skolutveckling, 2008, p.8).

Studies made for Skolverket indicate that somewhere between grades 4-6, the gap in understanding between the students who comprehend mathematical texts and the students who do not comprehend the texts widens due to the increasingly decontextualised nature and the specificity of the mathematical terms in the texts presented to the students (Myndighet för skolutveckling, 2008, p.9). This time frame coincides with the increasing emphasis on subject specific language noted earlier by Gibbons. Parszyk (1999) noted the same problem with second language students. By the time they reached secondary school, the students who thought that mathematics was boring were often the same students who did not understand the texts (Parszyk,

1999, p.191). By not comprehending the mathematical language used in a lesson or a textbook, students lose interest in mathematics.

To sum up, sociocultural perspectives on learning posits that language is a vital tool used in learning to develop knowledge and understanding in mathematics and its relevant subject specific language for all learners. What considerations need to be made for second language learners?

3.4 Second language learners and cultural experience

Referring to the National Education Act's stipulation that every student in Sweden must receive an equivalent education (Sweden, 2010:800, 1 kapitel, 9§), the national school curriculum in Sweden, points out that this does not mean that education should be the same for all students nor should the school's resources be divided equally. The curriculum clearly states that schools have the responsibility to adapt to each student's individual needs to ensure that they reach the established goals (Skolverket 2011, p. 10). How does this affect Swedish students who have chosen to learn mathematics in English?

As noted before, as students make their way through higher grade levels, each subject begins to place a greater emphasis on subject specific language which is more abstract, has greater lexical density and is more structured than the student's own daily language (Gibbons, 2009, p.31). This places more demands on second language learners who are learning both a subject, mathematics, and a language, English, at the same time. One of the added difficulties to the second language learner is the fact that some words from their first and second languages may sound similar but have different meanings. Two simple examples from Swedish and English are the terms *mil/mile* and *biljon/billion*. Although they sound the same, a Swedish mile (mil) represents ten kilometers while an imperial or American mile is about 1.6 kilometers. Potentially more confusing is the Swedish term *biljon* which is pronounced in the same way as the English *billion*. Although *miljon* and *million* represent the same order of magnitude, *biljon* and *billion* do not (*biljon* = 10^{12} while *billion* = 10^{9}) and I have seen many second language learners struggle with those terms.

This potential for interference between the learners' first and second languages can add to the cognitive load for a second language learner. When solving problems in mathematics, the learner must keep in mind the different parts of the problem, the different strategies that may be needed to solve the problem and how they all connect together (Campbell, Davis & Adams, 2007, p6). To this already substantial load on working memory, the second language learner must add their level of understanding of mathematical language and any cultural differences that may arise.

Language develops within a cultural context (Säljö, 2012, p.188) and second language learners often grow up in a different cultural context with different experiences than their teachers. This is the case with first-generation immigrants to a country but can also be the case for native students who are taught in a second language by teachers coming from another culture. Although modern-language teachers may be aware of and be mindful in using those cultural differences as learning opportunities for students, teachers of mathematics need also be cognisant of the importance of the multicultural classroom setting. Teachers must integrate students' cultures and experiences within the learning process and allow the student to build knowledge

from their own cultural perspective and experiences (Dornoo, 2015, p.84). If the teachers themselves are from another culture, the teachers must be aware of any cultural assumptions made while teaching but that does not mean the teacher should avoid cultural differences. Those cultural differences can be used as learning opportunities. Students with different cultural experiences from those of the teacher may struggle to understand if the teacher uses only his or her own personal culture as a framework for teaching. If cultural experiences are treated equitably, both being equally valid, if different, ways of approaching a problem or a concept, students will gain greater confidence in their abilities, in their contributions and their understanding of the topic (Dornoo, 2015, p.84).

As noted by Skolverket in 2010, few subject teachers, working with English as a medium of instruction when teaching Swedish students, have training in second language learning.

3.5 Second language learning research

Moschkovich (2002) identified three perspectives that have been used in the study of second language mathematics learners. The first perspective focuses on students acquiring vocabulary in order to be able to "solve traditional word problems" (Moschkovich, 2002, p.192). The second perspective deals with the difficulty caused for second language learners by multiple meanings across registers. Namely, words can have a different meaning in a mathematics lesson than the meaning intended in everyday use. An example in Swedish is the word for *fraction* (*bråk*) which in everyday language has the more common meaning of brawl or disturbance.

The third perspective identified by Moschkovich is based on sociocultural perspective theories of learning and development and focuses on how students participate in mathematical discussions. Moschkovich posits that "participants bring multiple views to a situation; that representations have multiple meanings for participants; and that these multiple meanings for representations and inscriptions are negotiated through conversations" (Moschkovich, 2002, p.197). Although these three perspectives focus on how the students develop their knowledge, the choice of perspective by the teacher, conscious or not, implicitly defines which methods the teacher will use in their lessons, whether they focus on acquiring vocabulary, working out multiple meanings, or provide classroom situations where students can talk through terms, definitions and problems. One of the analytical criteria used in this review will be how the found articles can be categorised in one of the three perspectives suggested by Moschkovich.

4 Methodology

A study by the Swedish government in 2008 pointed out that research in education and learning was a prerequisite to ensure good quality education (Sweden, 2008, p.384). This was then written into the most recent version of the Swedish school law, issued in 2010, which states that all work within the educational system should be based on scientific research and proven experience (Sweden, 2010:800, 1 kapitel, 5§). Scientific research is necessary in order for us as a society to properly understand and explain events so that we may develop or even change them (Larsen, 2009, p.11).

Proven experience is systematically evaluated experience that can be of use to other colleagues in the same field (Kornhall, 2014, p.77) but proven experience is not the same as personal experience. For experiences to qualify as proven experiences, they must be documented and then analysed to ascertain what aspect was effective. Improvements are then proposed and tested again. Collegial learning, through for example, *Learning study*, where teaching colleagues observe and reflect on each others' lessons, can be an effective way to improve teaching practices. Besides an improvement in the effectiveness of teaching practices, another positive effect of peer-to-peer classroom observation and reflection is that teachers begin to understand how much of an impact they actually have on the learning taking place in the classroom (Kornhall, 2014, p.50). Unfortunately, not all school administrations offer the time needed for collegial learning and teachers are left to rely on the research done by social science researchers in the field of education and psychology.

There is a mountain of research available to teachers and prospective teachers. A quick search on Högskolan Dalarna's library search page with the search term *Teaching* yielded 1,945,219 results. Much of the research is highly specialised and focused, with a lot of detail given over to method and statistical calculations. It can be challenging for teachers to find general guidelines or strategies within the specific conclusions of all the scientific reports published. This underlines the importance of systematic reviews and meta-analysis.

A systematic review is a synthesis of data from previously published empirical studies (Barajas et al., 2013, p.31). It attempts to identify all the relevant data available for a specific topic of inquiry and in expanding the database of results available to analyse, reduces the probability for selection bias that may occur with a general review (Barajas et al., 2014, p.28). Focusing on recently published research within a specified field of study, the conclusions of a high-quality systematic review can be used as grounds for decisions by educational institutions, businesses and government as well as a basis for further research (Larsen, 2009; Barajas et al, 2013; Kornhall, 2014).

Many of the research articles published are investigations with a limited number of subjects and conducted under specific circumstances but the conclusions, while giving us new knowledge about that particular circumstance, cannot be extended or generalised to a larger group in society. Meta-analyses allow a researcher to combine the data of many studies thereby expanding the total number of research subjects and the circumstances of the individual investigations to form a much broader collection of data (Barajas et al., 2014 p.28). This large quantity of data is then processed using analytical statistical methods to arrive at a conclusion that can be generalised. For this reason, meta-analysis ranks very highly in terms of the reliability of evidence with high quality meta-analysis at the top of the list (Kornhall, 2014, p.79).

4.1 Study design

The method used for this thesis is a systematic review which will answer the questions posed in the study aims by reviewing recently published scientific research that have been judged as relevant to the study's aims and purpose. A systematic review is an examination of studies with a clearly defined method that itself can be reviewed and replicated (Barajas et al., 2013, p.28). That the results of a study can be replicated by another researcher is one sign of its validity (Gilje & Grimen, 1992, p.22).

Barajas et al. (2013) list the following steps that must be included in a systematic review:

- Justification for the review (study aim).
- Formulation of questions that can be answered.
- Formulation of a plan of action for the literature review.
- Selection of search words and a search strategy.
- Identification and selection of literature in the form of scientific articles or reports.
- Critical evaluation and quality grading of the selected literature.
- Analysis and discussion of the results.
- Final review and conclusion.

The rest of this chapter will describe the methods used in detail for this systematic review.

4.2 Ethical Consideration

The Swedish Research Council (Vetenskapsrådet) succinctly summarises ethical considerations in research as "largely a matter of finding a reasonable balance between various interests that are all legitimate" (Vetenskapsrådet, 2011, p.8). Two significant interests that can cause conflict are the quest for knowledge and respect for individual privacy.

Scientific research in Sweden should follow the guidelines established by the Swedish Research Council. The 'criteria of quality' for scientific research means that research should provide new knowledge, reveal never previously known conditions or throw new light on previously known phenomena and relationships, giving us more reliable maps of knowledge to navigate (Vetenskapsrådet, 2011, p.24).

Although this thesis is a systematic review of existing literature, there are still some ethical aspects to be taken into consideration. The Swedish Science Council, have published their research guidelines, Good Research Practice (*God forskningssed,* Vetenskapsrådet 2011) which is the primary source used for ethical consideration in this paper. Some of the general rules for ethical research do not directly apply to this literature review as detailed below but should be considered for the research performed in the selected literature in some cases.

The council summarises their advice in eight general rules for ethical research as follows:
1) Researchers should tell the truth about their research. This paper will include all search results that apply to the theme being investigated and not exclude papers that disagree with the intent of the review. The exclusion of any articles will be justified within the scope of this paper so that other researchers have the chance to question the legitimacy of the exclusion.

2) Researchers must consciously examine and present the starting points for their studies. This paper's introduction clearly presents the review's starting point and a PICOC-analysis (Barajas et al., 2013, p.71) examines and refines the focus for the review. PICOC is a strategy for identifying the key components when formulating a topic of research

where Population, Intervention, Control, Outcome and Context help to define and further refine the problem in focus.

With this in mind, the focus population for this review consists of students attending lower secondary or middle school, specifically grade 9 in Sweden or the equivalent of 15 years of age internationally. The students in focus are studying mathematics in a second language environment, specifically students studying mathematics in a second language which is not the majority language. Recent studies of immigrant students studying mathematics in a second language which is the language of the majority will also be included if they fulfill all the other defined criteria.

Intervention refers to what specific action serves as the basis for the research. This systematic review is looking at the methods or adaptations used by teachers in second language mathematics instruction. John Hattie (2003) argues that after student ability, which accounts for a 50% variance in student achievement, the second-largest factor influencing student achievement is the work of the teacher, accounting for 30% of the variance and much higher than other factors such as home, school, principals and peer-effects. For this reason, this review will focus on strategies used by teachers teaching mathematics in a second language learning environment.

Control indicates whether the data search will also include a control group on whom the effects of, in this case, no targeted method for second language mathematics instruction is used. Much of the selected literature was of an observational nature where existing classroom situations where observed and analysed. In many cases there was no control group that could be analysed for comparison. The few articles that did include a control group will be noted in the results section of this paper.

The intended outcome for this thesis is a summary of existing knowledge of the teaching of mathematics in a second language that can be of relevance to the teaching of mathematics in English to Swedish speaking students.

3) Researchers should disclose the methods and results used. The methods used are detailed in full in this chapter. The result of the database searches are included in table format and discussed. The systematic review of the selected literature is documented in the Result section of the paper and all selected literature is noted in brief in tables and with complete details in the reference list at the of this paper.

4) Researchers must openly declare commercial interests and other interests. There are no commercial interests involved in the preparation of this systematic review but the review is prepared as part of a Master's thesis required for the completion of a Masters degree in Education at Högskolan Dalarna. There is also my personal interest in the subject as a teacher of mathematics in English to Swedish speaking students.

5) Researchers must not steal the research of others. All research, in the form of database searches, is that of the author of this paper. The literature analysis is the author's interpretation of the reading of the selected literature and the conclusion is the author's interpretation and summation of the results. All other work and ideas, belonging to other writers and researchers, are cited with reference to the author(s) last name, year of publication and page number when necessary. The titles of the cited articles, along with year of publication and the name of the publisher or

publication are listed in alphabetical order by the author's name in the reference list at the end of the paper.

6) Researchers should keep good order in their research including thorough documentation and archiving. The result of all database searches are complied into a spreadsheet document (and included as a table in this paper) and all selected and/ or referenced literature will be archived for ten years from the publication date of this paper according to established norms (Barajas, Forsberg & Wengström, 2013, p.70).

7) Researchers should strive to conduct their research without harming people, animals or the environment. This ethical consideration does not apply to a systematic review of literature unless any of the selected literature caused harm to humans, animals or the environment. As the selected studies have gone through the process of peer-review as well as an examiner process before being selected for publishing, it can be assumed that each study has undergone the ethical analysis as required by the country of origin. All of the publishing journals of the selected articles are on the list of reputable publications administered by the Norwegian Social Science Database (NSD) as detailed in section *4.3.4* of this paper. The use of research and databases from other countries brings up the issue that the question of ethics has become more challenging with research becoming a more international effort with multicenter studies and major international projects (Vetenskapsrådet, 2011, p.8).

8) Researchers must be fair in the assessment of other's research. This paper will objectively and systematically review the selected literature in order to produce an inventory of teaching methods for teachers teaching mathematics to second language learners, based on the questions raised in the paper's aims.

4.3 Search strategies

The following section will describe the search process in detail so that the search is reproducible in the future. The section begins with a description of the databases used and the search strategies used. The selection criteria will then be detailed and the selection process described. The section will end with a description of the search process results and the list of articles included for the systematic review.

4.3.1 Databases

In order to find the largest possible number of articles relevant to the theme of this review, several databases were used. The databases, described in more detail below, were selected due to their primary focus on research within a pedagogical context. This ensured that the search results would be relevant for the purposes of this review.

ERIC (via PROQUEST) - The Education Resources Information Center was first opened in 1964 by the Office of Education (USA) to "embrace all education research and research in other disciplines that have implications for educational theory and practice" (ERIC, 2014). It is now the world's largest database of literature in the field of education. The search was limited to peer-reviewed articles published between 2001 and 2015. To further limit the search results to articles that would be relevant to the aim of this paper, the Education Level option was selected for Junior high schools, Middle schools and Secondary education.

LIBRIS - A Swedish national database of articles collected in Swedish libraries (LIBRIS 2015), LIBRIS is the most complete database of doctoral theses published in Swedish according to the Högskolan Dalarna search guide. This search was limited to doctoral theses relevant to the study's aim but there was no limitation to the year of publication because the search engine did not offer that option. The search was manually limited to the years 2005-2015. No relevant articles were found which could indicate an incorrect choice of search terms or that no research has been published regarding this topic.

PROQUEST (LLBA) - The Linguistics and Language Behaviour Abstracts database was searched via PROQUEST. This database covers all aspects of language and "indexes international literature in linguistics and related disciplines in the language sciences" (PROQUEST, 2015). The searches were limited to peer-reviewed articles published between 2005 and 2015. 161 articles were found but the titles became less relevant as they were perused and only the first 80 results were looked at.

SUMMON@DALARNA - The SUMMON service at Högskolan Dalarna is the university library's search engine which contains references for most the library's printed and electronically stored material. The search engine also searches other databases that the library has a subscription for, including ERIC and LIBRIS (Högskolan Dalarna, 2015). It was not easily possible to limit the search results with the SUMMON database resulting in a very large number of search results. As it was not possible to limit the search to secondary education and middle school studies, the publication dates were limited to 2010-2015. This still returned over 4000 results but after reading the titles of the first 120 results, it was noted that the titles became less relevant and the search was stopped.

4.3.2 Selection Criteria

The choice of selection criteria was delineated by this paper's question; what are effective teaching strategies for the teaching of mathematics to second language learners in the context of Grade 9 Swedish students studying mathematics in English? Therefore, the contents of the literature searched for should be empirical research within the field of educational sciences of teaching strategies, that is to say, articles with a teacher's perspective that defined the teaching strategies used. The focus of the research should be the teaching of mathematics to the equivalent of Grade 9 in Sweden, or fifteen year-olds. As teaching ages vary slightly around the world, research that included Junior high schools, Middle schools and Secondary education were deemed relevant to this paper.

The following criteria were also used in the actual database searches. As this systematic review is interested in recent research in the field of education relating to language, the choice of databases was limited to those within the field of education and language. In order to access current research, the search results were first limited to articles published in the last fifteen years, 2001 – 2015. Later searches were further limited to the last ten years and lastly, to the most recent five years as detailed in the previous paragraph. The search was limited to peer-reviewed material and doctoral theses and the search terms used were used first in English and then in Swedish. The

result was that most of the search results were limited to English and Swedish language publications though there were some exceptions written in other languages that appeared due to their English abstract. These few foreign language articles were not relevant with regard to teaching mathematics or teaching strategies and were therefore not selected for closer investigation.

4.3.3 Search strategies

Based on this paper's aim, key search words could be defined for use in searching the different databases that were described earlier in section *4.3.1*. The keywords selected for the first search were *language, second language*, *mathematics* and *teaching mathematics*. The search was conducted using different pairings of the terms using the Boolean operator AND. As the first search was in the American database, ERIC, only English search words were used in the initial search. The ERIC database offered the option to limit search results by 'Educational level' which was used to limit the results to research dealing with Junior high schools, Middle schools and Secondary education. Publication dates were limited to the range of 2001 to 2015. An analysis of the results from the first search revealed that no relevant articles were found using *language* and *mathematics* as search terms and so they were dropped from later searches.

The second search in the LIBRIS database used *teaching mathematics, second language, matematikundervisning, andraspråk,* and *Engelska* as search terms. The variation *mathematik undervisning* was also used to ensure that no relevant articles were missed. Again, the terms were used in pairs using the Boolean operator AND. This search was limited to doctoral theses to see if there existed any research dealing with the situation of Swedish students choosing to learn mathematics in English. LIBRIS did not have the option for limiting the publication date and the search gave results dating from 1952 to 2015.

The third search was conducted using the Linguistics and Language Behavior Abstracts database accessed via PROQUEST. Here the search term was *teaching mathematics in English*. No Boolean operators were used and the publication dates were limited to between 2005 and 2015. This gave a significantly larger number of results but when sorted for relevance it was noted that the relevance to this paper's questions and criteria dropped off rather quickly. Only the results from the first half of the returned literature were considered. The search terms did open up the possible results considerably compared to the first two searches and a variation was used for the fourth search a few days later.

The fourth database search was performed using Högskolan Dalarna's SUMMON database. The search term this time was *English mathematics teaching methods* AND *secondary school* which would yield a large number of results. Limiting the search to peer-reviewed journal articles published since 2001 gave almost 12,000 results which was an unrealistic number of results to investigate. The publication dates were then narrowed to articles published between 2010 and 2015. This narrowed the results returned to 4134. A sort by relevance revealed that the articles again quickly became no longer relevant to the teaching of mathematics to second language students and the decision was made to stop reviewing the titles after the first 120 titles.

As a supplement to the database searches, the reference lists in the articles and books used for background information were also reviewed manually to see if any of the

articles referenced could be relevant to this paper's question. The reference lists in the selected articles were also perused to expand the search where applicable. The manual searches were subject to the same criteria as the database searches. Articles had to be about research in the last ten years into teaching strategies for teaching mathematics to second language learners.

4.3.4 Selection process

After each round of searches, an evaluation of the results and the search strategy itself was performed (the evaluation of each stage of the search strategy is discussed in the previous section). Search results were first subjected to an appraisal of the article title to see if the article had any significance for this systematic review. All titles deemed to have potential for relevance to the review were then selected for further evaluation. The abstracts of each selected article were then read and a second selection was made using the criteria established for this review. The third round of selections involved reading the article in its entirety. All articles that dealt with research into teaching strategies for the teaching of mathematics in a second language to Grade 9 students (or equivalent) were saved for use in this systematic review.

The results of the searches and selection phases were documented in a spreadsheet during each step in the search and selection. All literature selected from a reading of the abstract were printed out and saved when possible. A summary of the keywords used, search results and selection results are provided in table 1.

Table 1 - Database Search results and selection

Search in ERIC PROQUEST	**Limits: Peer reviewed, 2001-2015, Education level; Secondary education, Junior high schools, Middle schools**				
Search terms	Number of results	Selection from title	Selection from abstract	Selection after reading	Notes
Language AND Mathematics	61	4	0	0	
Second language AND Mathematics	84	14	12	2	
teaching mathematics AND second language	88	13	6	4	
Search in LIBRIS	**Limits: Doctoral thesis**				
Search terms					
matematikundervisning AND Engelska	53	-	-	-	
teaching mathematics AND second language	47	-	-	-	
matematikundervisning AND andraspråk	2	-	-	-	

Search in PROQUEST Linguistics and Language Behavior Abstracts	**Limits: Peer reviewed, 2005-2015**				
Search terms					
teaching mathematics in English	161	4	3	2	less relevant after 80
Search in SUMMONS	**Limits: Peer-reviewed, 2001-2015**				
Search terms					
English mathematics teaching methods AND secondary school	11,931*	-	-	-	
					*Results not included in totals
Search in SUMMONS	**New limit: Peer-reviewed, 2010-2015**				
English mathematics teaching methods AND secondary school	4134	10	8	3	less relevant after 120
Totals	4630	45	29	11	

The remaining 28 articles were read thoroughly and an evaluation was made to determine if the articles fulfilled all of the criteria for this review as listed in section *4.3.2*. A number of articles were immediately excluded and the reason for exclusion was noted on the copy of the article and then entered into the summary spreadsheet. Reasons for the exclusion of an article can be summed up in the following comments:

- Several articles were not about empirical research but a review of educational theories and other empirical research. As such, the articles did not have any results to be reviewed. All articles were excluded from the results of this review but one article was used as a source for some of the background information in chapter two of this paper.

- Two of the articles dealt with teacher training and not actual classroom experiences. Although some of the strategies suggested in the articles are similar to the strategies in the other selected articles, there was a lack of empirical data that could be used in this review.

- Several articles dealt with an age group outside the scope of this review. As discussed in the introduction and background chapters, the cognitive loads and language demands are not the same between elementary and secondary school students and so any study with subjects below Grade 7 were excluded.

- Three articles dealt with research into summer school or supplementary programs scheduled outside of the regular school lessons. The aim of this review is to investigate teaching strategies that can be used within a normal lesson so these three articles were excluded.

- Two other articles did not deal with second language learners at all but general mathematics teaching strategies and were also excluded.

After the exclusions listed above, eleven articles were initially selected for a comprehensive reading which determined their justification for inclusion in this review. After selection of the articles, the names of the publishing journals were entered into the Norwegian Social Science Data Service (NSD) which publishes a list of publications related to higher education that the NSD have determined have passed their criteria and can be used as relevant data for use in research. Their criteria can be found at the NSD website listed in the reference section of this essay. Of the eleven articles, only eight had been published in journals receiving a Level 1 or Level 2 grade from NSD. One article was published in the Asian Social Science journal which had a Level 1 grading until 2012 but the article was published in 2014, after the journal had lost its rating. These three articles, in general, supported the results in eight other articles and their inclusion or exclusion would not change the results of this review so it was decided to exclude them from this review. At the end of the selection process, eight articles were chosen for systematic review.

4.4 Search results

The result of the database search and subsequent selection process was a total of eight articles that were determined to be relevant for the purposes of this systematic review. As further searches with the key search terms continued to produce the same results, the selection process was deemed to be complete. The next stage was a critical analysis of the quality and summary of the content of the selected research articles. This next section presents the selected articles with a very brief summary of their relevance to this review as well as a summary of the quality analysis.

4.4.1 Presentation of selected literature

The eight articles selected for this systematic review are presented in table 2. Every article selected has been published in a recognised journal listed in the NSD database and has undergone the process of peer-review. To be published in a recognised journal, an article must meet the following conditions:

- The research results are published for the first time and the journal retains exclusive rights for publication. The article cannot be reproduced without permission and acknowledgement of original publication.
- The article is accessible. The article is published and available to other researchers and interested parties.
- The article is written in an established form and structure with an introduction, background, purpose, method, results, analysis and discussion.
- A critical examination of the article was performed before publication, a peer-review.

(Barajas et al., 2013, p.61)

All the articles selected were published in English and, as can be seen in the table below, come from a variety of countries. Two studies were produced in the United States of America, the greatest number from any individual country, but they are only a small fraction of the selected studies. The research covers most of the continents with a total of two from North America, two from Europe, two from Asia and the remaining two from Africa and Australia. A simple conclusion is that the issue of second language learners is one that is being investigated in most regions of the world. The only region for which recent research on the teaching of mathematics in English to second language learners was not found was South America. There are no

conclusions to be drawn from such a limited sampling other than there was no research from South America that fulfilled all of this review's criteria.

Table 2 - Presentation of selected literature

	Author	Year	Title	Publication	Country	Type of study	NSD
1	Jäppinen, A.K.	2005	Thinking and Content Learning of Mathematics and Science as Cognitional Development in Content and Language Integrated Learning (CLIL): Teaching Through a Foreign Language in Finland	Language and Education	Finland	Quantitative (Testing with control group)	1
2	Planas, N.	2014	One speaker, two languages: Learning opportunities in the mathematics classroom	Educational Studies in Mathematics	Spain	Qualitative - Phenomenology (Videotaped observations)	2
3	Riconscente, M.M.	2014	Effects of Perceived Teacher Practices on Latino High School Students' Interest, Self-Efficacy, and Achievement in Mathematics	The Journal of Experimental Education	USA	Quantitative (Questionaire)	1
4	Tan, M. & Lan, O.S.	2010	Teaching mathematics and science in English in Malaysian classrooms: The impact of teacher beliefs on classroom practices and student learning	Journal of English for Academic Purposes	Malaysia	Mixed Quantitative & qualitative (Observation & questionaire)	2
5	Tavares, N.J.	2015	How strategic use of L1 in an L2-medium mathematics classroom facilitates L2 interaction and comprehension	International Journal of Bilingual Education and Bilingualism	Hong Kong	Qualitative (Observation & interviews)	1
6	Uribe-Florez, L., Araujo, B., Franzak, M., & Haynes Writer, J.	2014	Mathematics, Power, and Language: Implications from Lived Experiences to Empower English Learners	Action in Teacher Education	USA	Qualitative (Observation)	1
7	Wildsmith-Cromarty, R. & Gordon, M.	2009	Policy versus practice: the role of the home language in learning mathematics and science in English-medium classrooms	The Language Learning Journal	South Africa	Qualitative (Observation & interviews)	1
8	Zevenbergen, R., Mousley, J., & Sulliva, P.	2004	Making the pedagogical relay inclusive for indigenous Australian students in mathematics classrooms	International Journal of Inclusive Education	Australia	Qualitative (Case study)	2

The publishing date for all selected articles range from 2004 to 2015 which ensures the criterion for current research has been met by the selection process.

Although all of the selected articles deal with research performed on the subject of the teaching of mathematics to second language learners, four of the articles were published in journals whose main focus is on language and learning while three

articles were published in journals with a focus on education. Only one article was published in a journal dealing with issues of mathematics and education. Mathematics teachers looking to find more information about strategies for teaching second language learners would be advised not to limit their searches to journals with a focus on mathematics and education.

All of the selected literature involved empirical research of an either qualitative or quantitative nature. Five of the articles reported on research performed using qualitative methods, two were quantitative and one was mixed. No reports used experimental intervention methods which involves a specific strategy used with a select group of students. One report did have a control group as well which was used for comparative purposes but the intervention was not one created by the researchers but rather a policy enacted by the government. Several of the studies were of a phenomenological nature where the researchers' efforts were focused on describing a phenomenon, often by recording, through questionnaire surveys and interviews, the feelings and experiences of the subjects involved (Szklarski, 2015, p.136).

4.4.2 Quality of the selected literature

It is vital that any systematic review determine the quality of the studies included in the review. Such a quality review consists of an evaluation of each individual study's treatment of the different elements that make up the study, including the study's purpose/aim, design, population selection, data collection method, analysis and discussion or interpretation of the collected data (Barajas et al., 2013, p.115).

As noted earlier, *4.3*.4, all of the articles were published in research journals that appear on the Norwegian Social Science Data Service (NSD) database which is another indication of the quality of the article. Three of the journals, *Educational Studies in Mathematics*, *Journal of English for Academic Purposes* and *International Journal of Inclusive Education* were given a ranking of 2 which indicates that the journal is a leader in its field; responsible for publishing the most significant articles available (NSD, 2015). On the basis of the evaluation of the quality of each report, it was determined that these eight reports were of sufficient quality to be included in this systematic review.

4.4.3 Content analysis of the selected literature

> *"Analysis means breaking up into smaller parts"*
> (Barajas et al., 2013, p.163,) [*my translation*]

After the evaluation of the quality of each article, an analysis of the content of the selected literature was then performed. Content analysis is a systematic and ordered method for classifying data so that patterns and themes can be identified with the goal of describing specific phenomena (Barajas et al., 2013, p.147). As the majority of the selected articles involve qualitative methods of research, a qualitative meta-synthesis, a compilation of the data to identify consensus or differences, was completed. The comparison and analysis of the results of different studies can lead to a new or broader interpretation of the results than is possible from each individual study alone (Barajas et al., 2013, p. 154). In other words, although the conclusions of qualitative research are often limited to specific situations and populations, a qualitative meta-

synthesis can lead to conclusions that apply to a wider population, thereby adding to general scientific knowledge.

5 Results

This chapter will present a thorough review of the results of the content analysis performed on all of the selected literature. The first part will examine what is known about teaching mathematics in a second language environment. Research has revealed situations with positive experiences and situations with negative results. The second part of this chapter will explore the different teaching strategies that have been effective for learning mathematics in a second language environment.

Moschkovich's identification of three different perspectives used when looking at second language mathematics learners can be useful in situating the different views on learning and language that are presented in the selected literature. This is not to imply that the researchers themselves are proposing one perspective over another but rather that the cultural (social and political) situation presented in each study can relate to one of the three perspectives identified by Moschkovich.

Second language mathematics learning can be seen as, (1) the challenge of learning vocabulary, as (2) the challenge of negotiating multiple meanings, or as (3) the opportunity to participate in mathematical discussions. Moschkovich argues that while the first two perspectives, vocabulary and multiple meanings, focus on obstacles and difficulties for the second language learner in mathematics, the third perspective, based on sociocultural theories of learning and development, defines the learner's native language and existing mathematical abilities as resources to be actively used in a social learning environment to develop the learner's knowledge and understanding of the content (Moschkovich, 2002, p.197). Broadly categorised, the research that revealed the difficulties in teaching mathematics to second language learners operated from the first two of Moschkovich's perspectives, the challenges of learning vocabulary and multiple perspectives. The research that looked at successful programs for teaching mathematics in English to second language learners overcame those difficulties by using language and discussion as an opportunity for learning both subject and language.

In the 'Background' chapter of this paper, it was noted that the Swedish curriculum defines one of the tasks of the school as to provide "a wealth of opportunities for discussion, reading and writing" with the outcome that "all pupils should be able to develop their ability to communicate and thus enhance confidence in their own language abilities" (Skolverket, 2011, p.11). The Swedish school curriculum also shows influences of sociocultural perspective theories of learning where "interaction and communication are the keys to learning and development" (Säljö 2012, p.195).

With this framework in mind, we look back to this review's two central aims:
(1) What is known about teaching mathematics in a second language environment?
(2) What characterises effective teaching strategies for teaching mathematics in a second language environment?

The content analysis identified the key points in each of the selected studies' results and conclusions and labelled them as relevant to the first question concerning

knowledge, or the second question concerning teaching strategies. When dealing with a large amount of data, much larger than the scope of this limited review, a content analysis allows the researcher to easily see the similarities and differences in the collected data. The results of this content analysis are briefly summarised in table 3.

Table 3 – Summary of content analysis

Article	**Study conclusions relevant for the study (review aims within brackets)**
Jäppinen, A.K. (2005). Thinking and Content Learning of Mathematics and Science as Cognitional Development in Content and Language Integrated Learning (CLIL): Teaching Through a Foreign Language in Finland.	- CLIL environments have succeeded in offering favorable conditions for thinking and content learning in mathematics (1) - CLIL environments support thinking and content learning, in particular, in situations where the learner has to compare different concepts and meaning schemes with each other.(1)
Planas, N. (2014). "One speaker, two languages: Learning opportunities in the mathematics classroom"	- Student-student interaction creates opportunities for learning (2). - Tension between meeting the language needs of the classroom and promoting the mathematical goals of the learning (1) - Opportunities for participation in flexible language practices. (2)
Riconscente, M.M. (2014). "Effects of Perceived Teacher Practices on Latino High School Students' Interest, Self-Efficacy, and Achievement in Mathematics"	- Teacher practices contribute to students' mathematics interest at the end of the year. (1) - Negative trends associated with grade level may be mitigated by teacher practices to promote interest and explain content well.(1/ 2) - Teacher practices can play a powerful role in contributing to students' motivation and achievement. (1)
Tan, M. & Lan, O.S. (2010). Teaching mathematics and science in English in Malaysian classrooms: The impact of teacher beliefs on classroom practices and student learning.	- Focusing only on [L2] vocabulary may obscure the ability of the students to make connections between mathematical operations. (1,2) - Streaming widens the differences between high-performing and average to low-performing students (1/ 2) - Teachers reduce and simplify both language and content (2)
Tavares, N.J. (2015). "How strategic use of L1 in an L2-medium mathematics classroom facilitates L2 interaction and comprehension"	- Use of L1 will have a positive impact on student learning (2) - Welcome the use of L1 in student-student interaction (2) - Comprehension of the L2 academic language can be facilitated via effective use of L1 (2) and a range of other strategies such as questioning and think-pair-share (2) - Legitimise the use of L1 to support L2 content learning (1)
Uribe-Florez, L., Araujo, B., Franzak, M., & Haynes Writer, J. (2014). Mathematics, Power, and Language: Implications from Lived Experiences to Empower English Learners.	- Students should have been encouraged to read, write, watch movies and videos, and use all the resources in both languages.(2) - Value and support all languages, with the lived experiences of ELs driving teachers' decisions.(2) - Empower students (2)
Wildsmith-Cromarty, R. & Gordon, M. (2009). "Policy versus practice: the role of the home language in learning mathematics and science in English-medium classrooms".	- Instruction carried out in the home language mainly through the use of code-switching (2) - Use of the L1 book stimulated discussion amongst the learners, especially when using their L2 (2)
Zevenbergen, R., Mousley, J., & Sullivan, P. (2004). Making the pedagogic relay inclusive for indigenous Australian students in mathematics classrooms.	-Use of open ended tasks (2) - Include cultural knowledge.(2)

5.1 Teaching mathematics in a second language environment

This section will try to resolve the first question posed by this study's aims.

- What does scientific research reveal about teaching mathematics in a second language environment?

The combined literature presents research conducted in seven different countries with varying results in terms of the effectiveness of the teaching strategies studied. In half of the studies the language of instruction for mathematics was English even though English was not the majority language of the country. The other four studies discuss second language learning in the context of the language of instruction being the majority language of the country, that is to say, in the context of immigrant or indigenous students.

The first difference to note in the case of four studies is the circumstance that that led to second language learners learning mathematics in English. The success of second language learners appears to correlate with whether English as a medium of instruction (MoI) was mandated for all students by the government or offered as a choice. The following section will look at how all the selected studies present different attitudes to teaching mathematics to second language learners – whether teachers ignore or embrace the students' first language. The last section will present the effective teaching practices that appeared across repeated studies.

5.1.1 Mandate or choice?

In two of the countries studied by researchers, the national government mandated that English become the medium of instruction for mathematics and science. To increase competitiveness in the global market, Malaysia and South Africa decided that students in secondary school would study mathematics and science in English (Wildsmith-Cromarty & Gordon, 2009; Tan & Lan, 2010). The Finnish and Hong Kong governments decided that individual schools were given the flexibility to offer content and language integrated learning classes (CLIL) for the students who chose to study in a second language. Schools in Finland were free to offer CLIL classes in a number of languages while the English was the only language choice for CLIL in Hong Kong. The Education Board in Hong Kong had mandated that subjects taught in English must be taught in English only, with no interference from the learners' L1 (Jäppinen, 2005; Tavares, 2015).

Although four studies is much too small a sample size to draw any definitive conclusions, the results of second language learning in mathematics in countries that mandated language of instruction and those that offer a choice is markedly different. According to government data reported in the studies, in Malaysia and South Africa, student achievement in mathematics has fallen over the years since the introduction of English as the language of instruction for mathematics (Wildsmith-Cromarty & Gordon, 2009). There was no distinction made in the South African report between high and low ability students (Wildsmith-Cromarty & Gordon, 2009) but the Malaysian report noted that teaching practices such as streaming of students into different ability groups resulted in an increasing gap in both mathematical knowledge and English proficiency between high-achieving and low-achieving students (Tan & Lan, 2010).

Researchers proposed one possible reason for the decline in results of the weaker Malaysian students being that mathematics teachers have the option of teaching a restricted curriculum, which they did with the weaker classes, and that the teachers placed more emphasis on learning keywords than the learning of concepts (Tan & Lan, 2010). By focusing on limited content and on the vocabulary, the weaker students received less language and content instruction and performed poorly on assessments.

The situation in Hong Kong is a combination of the Finnish experience of choice, and the South African and Malaysian experiences of mandated language. Taking mathematics classes in English in Hong Kong is a choice that students can make but the Education Board has mandated that the lessons must be taught in English only. Teachers find it challenging to avoid using L1, especially with the weaker students. Similar to the Malaysian experience, weaker students in Hong Kong focus on the mathematical vocabulary rather than subject content. Although the Hong Kong administration boasts of stronger English results, research has shown that content knowledge of other subjects, including mathematics, has declined in recent years (Tavares, 2015).

One problem noted among Hong Kong, Malaysia and South Africa students was that students couldn't answer many problems in mathematics because, although they may know the words, they didn't understand the question (Wildsmith-Cromarty & Gordon, 2009; Tan & Lan, 2010; Tavares, 2015). Vocabulary was emphasised over content understanding. Another problem was that weaker students could not answer open-ended/essay questions (Wildsmith-Cromarty & Gordon, 2009; Tan & Lan, 2010; Tavares, 2015). Tavares (2015) noted that with proper support using second language learner teaching strategies, students in Hong Kong were achieving good results in both language and mathematics subject content (see section 5.2).

Content and language integrated learning (CLIL) is offered as a choice in Finland with no government restrictions on the use of L1 in the classrooms (Jäppinen, 2005). Comparing the results of content and language achievement, the study found that students' content knowledge did not suffer in the CLIL environment. "Finnish CLIL environments support thinking and content learning, in particular, in situations where the learner has to compare different concepts and meaning schemes with each other. This is assumed to be due to the analogical CLIL reasoning systems that are based on exactly the kinds of situations where the learner makes comparisons between two semantic systems of two languages and two or more underlying cultures" (Jäppinen, 2005, p.163).

Writing about the successful second language learning program in Finland, Jäppinen (2005) remarks, "the CLIL environment provides informal and natural language-learning opportunities because CLIL learners learn and acquire the foreign language in much the same way as they have learned their mother tongue" (Jäppinen, 2005, p.151).

The emphasis on teaching students vocabulary in Hong Kong, Malaysia and South Africa is an example of Moschkovich's (2002) vocabulary perspective of second language learning in mathematics. The negative trend in student achievement would suggest that this perspective does not benefit the students' learning of mathematics.

5.1.2 Ignore or embrace – Monolingual or bilingual

> "Teachers hold the power to determine what content will be offered for acquisition, what methods students will use to obtain it, what evidence will suffice to demonstrate successful acquisition, and what barriers to acquisition will be erected or removed." (Uribe-Florez et al., 2014, p.239)

The level of acceptance from the teachers in teaching mathematics in a second language environment varies in the selected literature. There are many ways that teachers can choose to ignore the issue of second language learners in their classroom. Mathematics teachers are encouraged to plan content standards in their lessons but are not required to consider any language strategies in their planning (Uribe-Florez et al., 2014, p.242) so it is easy to exclude or forget. Researchers found that one consequence regarding this omission was that weaker students were more affected by perceived teacher disinterest than stronger students (Riconscente, 2014, p.55).

As seen in the previous section, some authorities mandate a monolingual teaching environment but even in the USA, where there are no restrictions, many teachers ignore the existence of other languages in their mathematics classroom, even in classes with a large proportion of second language learners (Wildsmith-Cromarty & Gordon, 2009; Tan & Lan, 2010; Riconscente, 2014; Uribe-Florez et al., 2014; Tavares, 2015). Researchers noted the case of one teacher who used both Spanish and English with students in a reading class but, when teaching mathematics, spoke only English (Uribe-Florez et al., 2014, p.241).

> "We as educators failed in providing ELs with the experiences that value, respect, and support their heritage language. In all three experiences, we note that focusing on the mathematics content prevented the consideration of heritage language as a significant part of support for EL's emotional and cognitive needs." (Uribe-Florez et al., 2014, p.243)

In many countries where English is not the language of the majority, mathematics teachers, teaching mathematics in English in a second language learning environment, are now expected to be proficient English teachers as well. That may be a challenge to an English speaking mathematics teacher without training in the needs of second language learners but the situation is worse when English is not the mathematics teacher's native tongue (Tan & Lan, 2010; Planas, 2014).

Other countries embrace the opportunities for learning that a second language teaching environment can provide.

> "A shift is required from the language-as-resource approach to a more specific focus on the creation of learning opportunities in classroom situations in which some of the students experience language difficulties, which may be either real of presumed" (Planas, 2014, p.52).

By embracing the second language learners' native language, or L1, by, for example, making connections between the students' culture and the curriculum, using L1 as a resource in linguistic scaffolding and resisting any detrimental language policies in place, a wider view of the world opens up which affects learners in a personal way (Jäppinen, 2005; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015). Second

language learners, when given the opportunity to contribute, are also given the opportunity to create learning of the mathematics content through language use.

The approach of creating a positive second language learner environment is advantageous to learners of all abilities. Zevenbergen et al., 2004, Jäppinen, 2005, Wildsmith-Cromarty & Gordon, 2009,Planas, 2014, Uribe-Flores et al., 2014, and Tavares, 2015, all pointed out the benefits of a bilingual environment in the mathematics classroom. "In the flexible multiple approach students are encouraged to use their entire linguistic repertoire to make sense of the content" (Uribe-Florez et al., 2014, p.237).

5.2 Teaching strategies make a difference

This section answers the second question posed in the aims of this review.

- What characterises effective teaching strategies for teaching mathematics in a second language environment?

Many of the strategies appear in several of the selected studies which could be an indication of greater validity.

5.2.1 Second language learning as an opportunity

The teachers who participated in the studies that demonstrated positive results of teaching strategies in second language learning environments all share the theme of creating a safe, language-rich environment where students were free to express themselves and explore the mathematical content in a bilingual setting, using both languages almost interchangeably. All the studies selected had English as the medium of instruction save for one where Catalonia was the majority language in the class, and the Medium of Instruction (MoI), and Spanish the minority language. The second language learners, instead of being forced to use one language in a monolingual environment, were given the opportunities to explore mathematics using all of the language tools available to them through open discussions, group discussions, writing, reading and watching videos in L1 and L2. In all cases, the teaching practices revolved "around the idea of immersing students in a language-rich environment so that mathematical language is signposted around the room" (Zevenbergen, Mousley & Sullivan, 2004, p.399).

This brings to mind Vygotsky's 'Zone of Proximal Development' where students use communicative tools to build meaning and achieve understanding of mathematical concepts. "CLIL learners need much support to reach the upper limit of their ZPD in terms of extra explanations and help from the teacher and fellow learners, in terms of special gesticulation and movement, in terms of special features of spoken language, and in terms of supportive materials" (Jäppinen, 2005, p.151). By creating a language-rich environment, both L1 and L2 become tools in the learner's thinking process (ibid, p.152).

5.2.2 Code-switching and translanguaging

Code-switching, where speakers change from one language to another, often within the same exchange (Ellis, 2008, p.956) was mentioned in some of the studies as a common practice encouraged by teachers. "In educational contexts where the majority of learners and teachers are speakers of African languages, strategic code-

switching is common. […] This includes code-switching for purposes of clarification and explanation" (Wildsmith-Cromarty & Gordon, 2009, p.362). Code-switching allowed students to continue an argument or discussion without the need to stop and look for the correct English word. Code-switching can even encourage further mathematical discussions as students strengthen their content knowledge in the search for correct mathematical language (Planas, 2014).

Translanguaging was another practice mentioned across research in many countries. Students in South Africa were offered books in their L1 while being taught mathematics in English. During discussions, students debated over the similarities and differences in the translations, creating an opportunity for concept and language learning together (Wildsmith-Cromarty & Gordon, 2009). Teachers in the USA, Australia and Hong Kong also practiced translanguaging in some form in their bilingual classroom environments (Zevenbergen, Mousley & Sullivan, 2004; Uribe-Florez et al., 2014; Tavares, 2015).

5.2.3 Revoicing

The strategy of revoicing by the teacher was seen by several of the researchers as an effective strategy (Zevenbergen, Mousley & Sullivan, 2004; Planas, 2014; Uribe-Florez et al., 2014). When a student in Hong Kong incorrectly said 'nominator' in her verbal solution to a given problem, the teacher allowed the student to finish her explanation and then repeated the solution to the class, revoicing the term *numerator*, while writing the term on the whiteboard and encouraging the class to write the terms from the board into their notebooks. Without overtly calling attention to the error and potentially causing the learner embarrassment, the teacher subtly reinforced all learners' knowledge of pronunciation, spelling and mathematical concept in an auditory and visual way (Tavares, 2015, p.329).

Researchers found examples of teachers using the practice of revoicing in studies from Spain, Australia and the USA (Zevenbergen, Mousley & Sullivan, 2004; Planas, 2014; Uribe-Florez et al., 2014).

> "Practices of revoicing are identified in the discourse of an English-dominant teacher who works with the use of informal mathematical vocabulary by Latino/a students, and models appropriate mathematical terms in her talk. Revoicing by students in small groups, to clarify meanings and words used by others, has also been documented in the field" (Planas, 2014, pp.53-54).

Even revoicing between students where students develop their knowledge of academic mathematical vocabulary by using the correct relevant terms amongst each other has been noted by researchers (Zevenbergen, Mousley & Sullivan, 2004; Planas, 2014; Uribe-Florez et al., 2014).

The practice of revoicing is an effective and efficient way to support the development of both language and content knowledge in a second language learner.

5.2.4 Using visual aids

The use of visual aids when teaching mathematics to second language learners was a teaching practice noted by researchers in Hong Kong, South Africa and the USA. Teachers used manipulatives, graphical and symbolic representations, often in the form of charts, diagrams and projected presentations, in order to clarify understanding of both mathematical language and content for the learners (Wildsmith-Cromarty & Gordon, 2009; Uribe-Florez et al., 2014; Tavares, 2015).

Several researches mentioned the scaffolding effects of using visual aids in supporting second language learners' development of content and language knowledge. "Different resources were used such as video and PowerPoint presentations that provided ELs with extra scaffolding" (Uribe-Florez et al., 2014, p.242). "With reference to the book's impact on their learners, teachers said that their use of the book actually stimulated discussion amongst the learners, especially when they were using their home language" (Wildsmith-Cromarty & Gordon, 2009, p.368).

Highlighting, verbally and visually, on the whiteboard and in students' notebooks was also noted as a teaching practice that promoted mathematics and language learning among the students in Hong Kong (Tavares, 2015). This teacher also used the next teaching practice which was the focus of the research study conducted in Australia, namely, the practice of using open-ended questions (Tavares, 2015).

5.2.5 Open-ended tasks

The studies from Australia, Hong Kong, Spain and the USA all noted the teaching practice of using open-ended questions as a significant factor in promoting subject and language learning in second language learners (Zevenbergen, Mousley & Sullivan, 2004; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015).

Open-ended questions are tasks with "multiple pathways and multiple responses" that are used "to stimulate high-order thinking, and, on the other hand, content specific to emphasize the mathematics being learned." (Zevenbergen, Mousley & Sullivan, 2004, p.392). Using open-ended questions challenges the notion that there is only one correct response to mathematical questions. The advantage is that they are inclusive as all students can answer at their level of language and mathematical understanding.

Examples of open-ended questions used by teachers in the study are as follows:

"Convert $2x + 1$ into Catalan"
In this case, the ensuing mathematical conversation between a group of students speaking Spanish (L1) and Catalan (L2), and the struggle to find the correct translation of *odd* into Catalan, resulted in students finding an arithmetical and a geometrical solution. $2x + 1$ can represent an odd number and it can represent the sum of two areas (Planas, 2014, p. 58).

"What three numbers have a mean of five?"
The infinite number of solutions, from simple whole number solutions such as *5, 5 and 5* or *4, 5 and 6* to solutions using fractional and negative numbers allows students of varying mathematical and language abilities to all participate and produce an answer (Zevenbergen, Mousley & Sullivan, 2004, p.394).

"Seven boys went fishing. The mean number of fish that each caught is five and the mode is four. How many fish might they have caught?"
There are many solutions to this problem even if the solutions are limited to whole number solutions. Giving learners the time to discuss the problem together creates the opportunity for mathematical content and language learning. Follow up questions can arise from the discussions or can be offered by the teacher for new discussions such as "What is the maximum number of fish one person can catch? How many combinations can be made?" A discussion can even focus on the question of what is an appropriate answer; when can negative numbers or fractions be considered (Zevenbergen, Mousley & Sullivan, 2004, p.393).

Because "the use of open-ended tasks challenges the status quo" (Zevenbergen, Mousley & Sullivan, 2004, p.393), challenging the common notion that there is only one correct solution in mathematics, it is important to consider the classroom environment which leads to the next teacher practice that was noted in much of the selected research.

5.2.6 A safe environment

Many of the selected studies have noted the importance of creating a safe environment for second language learners. (Jäppinen, 2005; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015).

Creating a safe learning environment for second language learners involves voicing norms for learner and classroom behavior and revoicing those norms repeatedly so that they are accepted by the students. This teaching practice was stressed as a major part of the success of bilingual teaching environments by researchers.

> "What was prominent in the learners' words was their increased level of confidence through working with their classmates and articulating their responses, and the notion of 'mistakes as learning opportunities'. Echoing the importance of a safe and supportive learning environment, Miss Sitt constantly reiterated, 'all successes come from failures. You make mistakes and then you learn and next time you become better'. In her view, '(i)f they have the confidence, they will just carry on'." (Tavares, 2015, p.325)

The teacher's role in creating a safe environment was noted in all the studies. Sometimes this role was covert, through the previously noted practice of revoicing, but in many cases, the teacher must be explicit in welcoming the role of all languages in learning. The teacher must establish the norms of a bilingual classroom, stating both what is expected and what is acceptable. Examples of this were seen in research from Hong Kong, South Africa, Spain and the USA.

> "The teaching and learning of norms like 'If there is a word that you do not usually use, try to understand its meaning' or 'You need time to shift into proper mathematical talk' refer to language and keep the focus on the goals of the mathematics classroom. Such norms cannot remain tacit in practice and need to be invoked while teaching. It is easier for students to choose language practices that have been made explicit to them as the teacher's preference" (Planas, 2014, p.64).

Student-student interaction will be taken up shortly but even there, the mathematics teacher has an important role in establishing the norms for language use that will create the opportunity for mathematical and language learning.

> "A good example of a group norm in line with the language-as-resource orientation would be the following: 'You may react to what has been said by any peer in group work.' This idea of direct reciprocity challenges power dynamics in that any student, independently of the dominant language, is allowed to react to the acts of any other student who has interacted with her/him before. Similarly to general classroom norms, group norms which recognize the voice of all potential speakers need to be invoked while teaching as part of the explicit contract in the multilingual classroom." (Planas, 2014, p.64)

Creating a safe learning environment where students are encourage to speak leads to the next teacher practice that has be shown effective in teaching mathematics to second-language learners.

5.2.7 Encouraging language talk

Creating a bilingual classroom environment where talk is encouraged in any language was another common practice shown to be effective in teaching mathematics to second language learners.

Participating in authentic conversation, real communication through discussions, encouraging students to reflect on the responses offered, using conversation as an opportunity for negotiating an alternative meaning were given as examples of teaching practices that promoted content and language learning in mathematics classes (Zevenbergen, Mousley & Sullivan, 2004; Planas, 2014; Uribe-Florez et al., 2014).

Even in situations where the teacher is monolingual, Planas posited that "Talking, arguing, and explaining are quality practices that let the students gain autonomy from the teacher and become active in communicating their mathematical thinking in either language" (Planas, 2014, p.54).

It is the conflict of two languages in the mathematics classroom itself that can create opportunities for learning. The "various and contradictory connections between the foreign language used and the learners' mother tongue which the learner discovers and makes use of in meaning making" (Jäppinen, 2005, p.151) were discussed by researchers from Finland, Spain and the USA (Jäppinen, 2005; Planas, 2014; Uribe-Florez et al., 2014).

By encouraging active discussions in the mathematics classroom, teachers are creating opportunities for students to learn both mathematics and language, whether by talking to the teacher or amongst themselves.

5.2.8 Student-student interaction

All of the studies selected for this review pointed out the significant advantages to second language learning in mathematics when teachers encouraged student-student interactions, in pairs or small groups. Uribes-Flores writes that incorporating student-student interaction in a mathematics lesson "helped to create an atmosphere in which

students' first language is seen as a resource rather than an obstacle" (Uribe-Florez et al., 2014, p.240).

Tavares writes that the teacher he observed in Hong Kong had the practice of creating "ample opportunities […] in a lesson for learners to engage in student-student interaction to collaboratively work out a solution or come up with a response" (Tavares, 2015, p.324). This created a safe atmosphere in the lesson where students could explore mathematical concepts and mathematical language. In many of the classrooms observed in the various studies, students were encouraged to work together in small groups to solve the tasks presented and then report back to the class (Zevenbergen, Mousley & Sullivan, 2004; Tan & Lan, 2010; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015).

5.3 Summary of results

The aim of this systematic review was to inventory and summarise recent knowledge about effective teaching practices for second language students of mathematics in early secondary school that other teachers of mathematics in second language environments can review. This section briefly summarises the findings from the selected literature that were deemed relevant to the aims of this review as a result of the content analysis.

Countries where the teaching of mathematics in English as a second language is mandated by the government have shown falling results in student achievement on tests of both language and mathematical content knowledge. Some countries have seen that the gap between the more able and the less able students is growing. The common factor in these cases is the government mandate insisting on monolingual education in English (L2) for mathematics education.

Countries that encourage the teaching of mathematics in English as a second language in a bilingual environment report that the second language learners participating in those lessons have shown equal or better mathematics and language knowledge and stronger cognitive abilities, even when prior ability and socio-economic situation are factored in.

Teacher practices that result in stronger achievements among second language learners of mathematics center on the creation of opportunities to use language for exploring and developing understanding in a bilingual learning environment. Seeing second language learning as an opportunity, effective teaching practices include the use of code-switching, translanguaging, revoicing, using visual aids and open-ended tasks.

Effective teaching practices also include creating a safe learning atmosphere where second language learners are encouraged to talk in class discussions. More importantly are the opportunities for learning that are created when students are given the chance to interact in paired or small-group activities where they can use and further develop their mathematical and language abilities.

6 Discussion of Results

6.1 Discussion of method

The method used for this study was a systematic review of literature selected from search words and criteria determined by the study's aim. The teaching of second language learners in general has been a focus of much research for many years but, for the purposes of this study, the search was limited to research looking into teaching practices for the teaching of mathematics to second language learners. The search was limited to research conducted in the last decade in order to find the most recent data available. This limitation means that older research may exist whose findings could contradict the conclusions of this paper and that is something for the reader to bear in mind.

Although unintentional, the search results and selection process for this review resulted in research that had been conducted in seven different countries over four continents. This indicates that the question of effective practices in teaching mathematics to second language learners is of worldwide interest. Although the database searches did not include any restrictions on language, all the articles selected under this review's criteria were published in English. This may be the result of English being the most common language used for the teaching of mathematics in a second language.

The majority of the studies selected for this systematic review were qualitative field observations. A few of the studies also included supporting quantitative data obtained through questionnaires distributed by the researchers and/or a review of quantitative data collect by the government such as national test results. As a result of the limited scope of qualitative research, the conclusions produced cannot be expanded to a larger population (Larsen, 2009, p.27). By noting similar trends across many studies of the same phenomenon through, for example, a systematic review, wider conclusions may be drawn.

> "En systematisk-kritisk genomgång och sammanställning av tidigare resultat på ett område kan också leda till att kunskapsnivån höjs och därmed sägas kunna utgöra ett exempel på forskning"
> (Vetenskapsrådet, 2011, p.29).

The value of a systematic review of previously published research, such as this one, is the opportunity for the creation of new insights, adding to the world's scientific knowledge.

6.2 Discussion of results

The starting point for this paper was the Swedish government's decision to allow schools, both municipal and independent, to teach mathematics in English to Swedish speaking students (Skolverket, 2010). As noted in the Swedish school authority's own report produced several years after that decision (Skolverket, 2010), most mathematics teachers teaching mathematics in English to Swedish speaking students lack training in teaching to second language learners. The lack of training in teaching to second language learners may make it more challenging for mathematics teachers to offer equal learning opportunities for all students as mandated in the Swedish

school curriculum. The new school law in Sweden states that teaching should be based on scientific research and proven experience (Sweden, 2010) and so, as a new teacher myself, teaching mathematics in English to Swedish students, I wanted to learn how to develop my skills and teaching practices in mathematics by reviewing the scientific literature produced by researchers working in this field.

6.2.1 What is known about teaching mathematics in a second language environment?

In answering the first question posed by this paper research conducted in countries from around the world indicates that second language learners are more successful in acquiring both mathematical content and language knowledge when given the opportunity to actively participate in a bilingual mathematical learning environment (Zevenbergen, Mousley & Sullivan, 2004; Jäppinen, 2005, Wildsmith-Cromarty & Gordon, 2009; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015). Research has also shown that use of a learners' first language has a significant impact on second language mathematic learning (Wildsmith-Cromarty & Gordon, 2009; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015).

The Swedish curriculum states that teaching should give the students the opportunity to develop their ability to "use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions" (Skolverket 2011, p.60). By creating opportunities for the learning of mathematics in a bilingual environment, teachers are using all the previous knowledge and abilities that the learners themselves bring to the classroom to provide equal learning opportunities for all students. This is also reflected in the Swedish curriculum,

> "Teaching should be adapted to each pupil's circumstances and needs. It should promote the pupils' further learning and acquisition of knowledge based on pupils' backgrounds, earlier experience, language and knowledge" (Skolverket 2011, p.10).

6.2.2 What characterises effective teaching strategies for teaching mathematics in a second language environment?

For the second question posed by this paper, this systematic review has summarised a number of teaching practices that provide opportunities for students to succeed in learning both the mathematic content and the English language.

Teachers need to create a bilingual environment in the mathematics classroom where all students, including second language learners, feel that they can participate and contribute regardless of their language or content knowledge ability. Using open-ended tasks to show that there is more than one correct response and using revoicing to covertly adjust learners' responses creates an atmosphere where all learners, regardless of ability, feel safe to respond (Zevenbergen, Mousley & Sullivan, 2004; Jäppinen, 2005, Wildsmith-Cromarty & Gordon, 2009; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015). Allowing learners to express themselves in their first language, through, for example, code-switching, shows the learners that their contribution, their language and culture are valued (Planas, 2014; Uribe-Florez et al., 2014).

To teach mathematics to second language learners effectively, teachers need to teach mathematics using all the language resources available. Learners should have access to subject resources, books, videos and posters among other materials, in both languages, allowing the learners to see the links, the differences and similarities that can be found between languages themselves and between language and subject content (Zevenbergen, Mousley & Sullivan, 2004; Jäppinen, 2005, Wildsmith-Cromarty & Gordon, 2009; Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015). Translanguaging, where learners learn in one language and produce material in another language has been shown to be another effective practice (Wildsmith-Cromarty & Gordon, 2009; Uribe-Florez et al., 2014).

Opportunities for learning mathematics and language are created when learners are given activities to solve in small groups, where discussion is encouraged (Planas, 2014; Uribe-Florez et al., 2014; Tavares, 2015). Through support from the teacher and from other learners, the second language learner can build and develop their mathematics and language knowledge. When students work together, they often engage in revoicing each others' ideas and explore alternate means to voice their thoughts. This calls to mind the sociocultural perspective of learning and development. "Mathematical contents and the language used to communicate them are constructed in interaction by students in small groups in which the task is introduced as mathematical" (Planas, 2014, p.63).

Along the same vein, the support material for teachers prepared by the Swedish school authority noted that reading comprehension increases when students are given the opportunity to speak, read and write with mathematically relevant terms (Myndighet för skolutveckling, 2008, p.11). Boaler (2011) argues that students must be given the opportunity to discuss and reason through new concepts with each other to show that they actually understand the concept. A method explained by the teacher may sound logical but it is only when the student tries to explain it to another student that they develop a deeper understanding of the method (Boaler, 2011, p.48).

Moschkovich (2002) warns that mathematics teachers should not focus unduly on mistakes in terminology made by second language learners. "[I]f we focus on a students' failure to use a technical term, we might miss how a student constructs meaning for mathematical terms or uses multiple resources, such as gestures, objects, or everyday experiences" (Moschkovich, 2002, p.193).

Changing from a teacher-centered model, where the teacher lectures to the class, to a student-centered model where students are active and valued participants establishes the classroom as a learning community (Planas, 2014; Uribe-Florez et al., 2014). Together, these teaching practices give the second language learner the chance to feel like a valued and respected member of the learning community in the classroom.

The results of this systematic review will affect my future teaching as they have shown the importance of incorporating both languages when teaching mathematics in English to Swedish speaking students. To be more aware that when a student struggles to answer a mathematics problem, it may be a sign that they are struggling to express themselves through their second language rather struggling with the mathematical concept will alter my approach as a teacher. I look forward to developing these teaching practices over time.

7 Conclusion and Areas of Further Investigation

This systematic review has answered the questions posed in the paper's aims, namely, what is known about and what effective teaching practices exist for the teaching of mathematics in a second language environment. This paper has even attempted to draw general conclusions from a small number of selected studies in order to advance scientific knowledge. Nonetheless, there remain many questions than can be explored further. Here are but a few of those questions.

The review of four papers revealed a trend indicating that countries which had mandated a monolingual teaching environment for second language teaching suffered from falling achievement amongst its students. Countries offering bilingual environment for subject and language learning showed better rates of achievement for language and content knowledge. Larger international studies comparing the academic results in language and content knowledge conducted between second language learners in monolingual or bilingual environments could bring more validity to the argument for bilingual education.

The study from Finland (Jäppinen, 2005) was very thorough in showing that students in CLIL classes performed as well or better than their non-CLIL counterparts but did not go into much detail about how the CLIL lessons were taught. Research in the successful teaching methods would be interesting.

The Swedish school authority's survey of Swedish students learning mathematics in English in 2008 (Skolverket, 2010) showed that few mathematics teachers had received any training in teaching for second language learners. Since then, the number of Swedish-speaking students learning mathematics in English has more than doubled with many schools recruiting native English speaking teachers from abroad. How has the situation with the teachers changed? Have mathematics teachers teaching in English received training in teaching mathematics to second language learners? How are Swedish language resources being used in English mathematics classes? Are the teaching practices described in the reviewed literature used in schools teaching mathematics to Swedish-speaking students so as to ensure that all students learning mathematics in Sweden receive an equivalent education?

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