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Teaching Mathematics in English to Swedish Speaking Students

The Use of Second Language Teaching Practices in the Teaching of Mathematics in English to Swedish Speaking Students in Lower Secondary School

Författare: Daniel Breton
Handledare: Jonas Jäder
Examinator: Magnus Jobs
Ämne/inriktning: Pedagogiskt arbete/ matematik
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Abstract: Over 20,000 Swedish lower high school students are currently learning mathematics in English but little research has been conducted in this area. This study looks into the question of how much second language learner training teachers teaching mathematics in English to Swedish speaking students have acquired and how many of those teachers are using effective teaching practices for second language learners. The study confirms earlier findings that report few teachers receive training in second language learning but indicates that some of the teaching practices shown to be effective with second language learners are being used in some Swedish schools.

Mer än 20 000 högstadiet elever i Sverige har valt att lära sig matematik på engelska, men det finns väldigt lite forskning inom området. Detta arbete granskar hur mycket utbildning i andraspråksinlärning har lärare som undervisar matematik på engelska till svensktalande elever och hur många av de undervisningsstrategier som tidigare forskning har visat att vara effektiva används numera under matematiklektionerna på engelska? Arbetet bekräftar tidigare forskning, som visar att få lärare som undervisar matematik på engelska till svensktalande elever har fått utbildning i andraspråksinlärning, men den här forskningen visar att några av de effektiva strategierna numera används under matematiklektioner i vissa skolor.

Keywords: Content and Language Integrated Learning (CLIL), teaching mathematics in English, second language learners, translanguage, code-switching, revoicing, open ended tasks, group work, sociocultural perspective on learning and development, språk- och innehållsintegrerad inlärning och undervisning.
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1. Introduction

In 2010, Skolverket, the Swedish school authority, released a report investigating Swedish schools that used English as a method of instruction (MOI) in some subjects. The report published the statistic that in the 2008/09 academic year, 9400 students in 42 elementary and lower secondary schools were participating in lessons where some subjects were taught using English as the MOI (Skolverket, 2010, p. 24). Of that total, about 4900 students attended 8 schools operated by one organisation, accounting for more than half of all the students attending schools with English as the MOI. Today, that one school concern now has 29 schools located throughout Sweden with a total of almost 20,000 students registered in its classes. This means that well over 2% of compulsory school aged children in Sweden have elected to attend schools where English is used as the MOI in some subjects and for the vast majority of those students (all students attending grades 4-9 in the aforementioned organisation), mathematics is one of the major subjects taught in English.

While studying to become a teacher, I had the opportunity to complete my practical training periods in a Swedish school that taught mathematics in English. I remember one of the first times I was teaching a lesson about probability to a class in grade 7. Knowing that the students were all Swedish speakers and this was their first year learning mathematics in English, I was very careful to look up all the translations for the keywords I was using from English into Swedish and I wrote them on the whiteboard as I explained the concepts for dependent and independent variables, while describing the act of pulling coloured marbles out of a bag. After speaking slowly and clearly for about 30 minutes, referring at all times to the mathematical keywords on the board and making sure that the students were taking copious amounts of notes, I asked the class if there were any questions. One student slowly raised their hand, looking around nervously and asked, “What’s a marble?” I hadn’t looked for that translation and I realised that I had made an incorrect assumption about their language abilities and/or cultural knowledge. After the class and I had worked out what a marble was in Swedish, I asked the class, out of curiosity, how many students did not know what a marble was in Swedish? More than 2/3 of the students raised their hands. Instead of taking that as a defeat or mistake, I clapped my hands in joy and said to the class, “Look around you and see how many people had the same question but only one person was brave enough to ask. Remember that if you have a question, it is almost certain that other people have the same question and if you take the chance to ask it, the other students will be so happy that you did!” From this lesson onwards, the class became more communicative and their opportunities for learning mathematics increased.

That anecdotal experience revealed to me that teaching mathematics in English to Swedish speaking students means more than just teaching mathematical terminology, concepts and methods. Teachers in this situation need to take language and culture into consideration in their lesson planning and execution. This is reflected in the Swedish curriculum for compulsory school requirement that “Teaching should be adapted to each pupil’s circumstances and needs” (Skolverket, 2011, p. 10) but language has a more important role in the learning of mathematics in general and more acutely so in the context of teaching mathematics in English to Swedish speaking students. Are the teaching practices for teaching mathematics in English as a second language, practices that have been shown to work in other countries, being used in Sweden or are teachers teaching mathematics in English to Swedish speaking students simply by teaching content in another language?
2. Background

Many explanations exist for the dramatic increase in the number of Swedish students choosing to learn mathematics (among other subjects) in English. According to Skolverket’s report on English instruction, most parents believe that increasing globalisation has increased the emphasis on the ability to communicate effectively in English (Skolverket, 2010, p. 30). Many students in Sweden choose to study in English because it is “fun” or will give them better opportunities for their future education and employment prospects (Skolverket, 2010, p. 32). This trend of increasing interest in bilingual education can be found in many other countries around the world and the advantages of subject and language integrated learning, commonly known in Europe as Content and Language Integrated Learning (CLIL) is that it provides “informal and natural language-learning opportunities because CLIL learners learn and acquire the foreign language in much the same way as they have learned their mother tongue” (Jäppinen, 2005, p.151).

Language itself has become more prominent in the teaching of mathematics over the last few decades. The latest Swedish curriculum for mathematics states that,

Teaching should help pupils to develop their ability to argue logically and apply mathematical reasoning. Pupils should through teaching be given the opportunity to develop familiarity with mathematical forms of expression and how these can be used to communicate about mathematics in daily life and mathematical contexts (Skolverket, 2011, p. 59).

This communicative aspect is a key component of many of the five abilities that students should be given the opportunity to develop through the teaching of mathematics,

• formulate and solve problems using mathematics and also assess selected strategies and methods,
• use and analyse mathematical concepts and their interrelationships,
• choose and use appropriate mathematical methods to perform calculations and solve routine tasks,
• apply and follow mathematical reasoning, and
• use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions (Skolverket, 2011, p. 59-60).

The teaching of mathematics today is much more than simply teaching methods, algorithms and procedures in order to solve word problems but also includes developing the students’ abilities to reason, analyse, present arguments and discuss selected mathematical strategies (Moschkovich, 2002, p. 192). Although previous teaching methods of reading mathematical textbooks and solving traditional word problems have been effective for some learners under the older national curriculums, they did not work for all learners and today are no longer up to the task of offering all students the opportunity to develop their communicative abilities in mathematics due to their inherent limitations of focusing on concept and method (Moschkovich, 2002, p. 193).

In reform-oriented mathematics classrooms, students are no longer grappling primarily with acquiring technical vocabulary, developing comprehension skills to
read and understand mathematics textbooks, or solving word problems. Students are now expected to communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments (Moschkovich, 2002, p. 190).

Researchers have found that mathematical learning opportunities can be generated when teachers encourage student participation through the uses of flexible language practices (Planas, 2014, p. 64). When confronted by a limited response from a second language learner in mathematics, a teacher who understands the cognitive challenges faced by second language learners can adapt their teaching practices to encourage student participation, often using bilingual strategies, thereby creating learning opportunities in mathematics. Teachers working in a second language environment when teaching mathematics must understand that the use of a student’s first language can have a positive impact on learning, particularly when the students are engaged in cognitively complex ideas (Tavares, 2015, p. 332).

Although many countries around the world are now encouraging the teaching of mathematics in English through content and language integrated learning programs, the level of teacher training for teaching mathematics in English to second language learners varies greatly from country to country. In Sweden, “CLIL is not specifically addressed in the new pre-service program. Presently, the availability of in-service CLIL training is severely limited” (Sylvén, 2013, p. 307). In Finland, where CLIL programs have been judged to be successful (Jäppinen, 2005), the only requirement for a CLIL teacher is proficiency in the target language (Sylvén, 2013).

3. Problem
Very little research has been conducted about the subject of teaching mathematics in English to Swedish speaking students in the last fifteen years. A recent systematic literature review by the author found no research dealing specifically with the teaching of mathematics in English to Swedish speaking students at the lower secondary level in the last ten years but there has been some research about CLIL in general. A recent study by Sylvén (2013) posits that CLIL is less effective in Sweden, when compared with other European countries, for several reasons. These include a lack of teacher training for subject teaching to second language learners and the fact that many CLIL programs were at the upper secondary level by which time students had already been exposed to so much English culturally, through music, film & television and the internet, that their English was almost on par with that of the Swedish speaking teachers assigned to teach in English (Sylvén, 2013). Both of these conclusions were also noted in Skolverket’s report from 2010.

If mathematics teachers, teaching mathematics in English to second language learners, are not aware of the cognitive and communicative challenges faced by second language learners, there is a risk that the students’ difficulties are misinterpreted as a lack of mathematical ability, that the students are seen by the teacher as not having the same learning potential as English speaking students (Campbell et al., 2007; Uribe-Flores et al., 2014). This misunderstanding of student abilities can lead to lower teacher expectations. Studies of CLIL in Malaysia have noted that students struggling with the language when learning mathematics in English were streamed into lower ability classes where they could focus on definitions
and keywords instead of reasoning, analysis and concept understanding (Tan & Lan, 2010). This resulted in declining results for the students in this group when compared to subject knowledge results before the introduction of English as the medium of instruction. The researchers noted that,

"many students cannot answer the questions given because they don’t understand the questions. They may be able to translate the words they don’t know with the help of a dictionary but often, the students fail to interpret (the meaning of) the question" (Tan & Lan, 2010, p.15).

Ineffective practices were also noted in Hong Kong where the government mandated unilingual, that is to say English only, teaching of mathematics. The net result was an increase in English proficiency among the students but a decline in mathematical content knowledge and ability (Tavares, 2015, p. 320).

Studies in other countries have noted the positive results achieved in CLIL programs when using effective teaching practices for the teaching of mathematics to second language learners (Jäppinen, 2005; Planas, 2014; Zevenbergen et. al, 2004). Opportunities for learning both mathematics and English together with effective teaching practices can result in learning that is deep and long-lasting because the student is so focused on the mathematical concepts being taught in English, that English language acquisition occurs covertly (Novotná & Moraová, 2005, p.109). Berger (2015) noted that students learning mathematics in English, their second language, also referred to as their L2, in being challenged by the language, were more engaged in analyzing the meaning of the mathematical content, leading to more reflection and, in the end, deeper understanding,

"learners using an L2 tend to use the text more profoundly for stepwise deduction of a mathematical model. The link between text reception and mathematically oriented processes also implies that translations of the problem text into the learners’ L1 triggered by the high linguistic demands during the test reception processes, i.e. establishing propositional meaning and building a mental model, encourage additional reflection of both linguistic and mathematical meaning." (Berger, 2015, p. 308)

The author’s earlier systematic literature review of teaching practices in the teaching of mathematics in English to non-English speaking students noted the following trend: instruction in a monolingual environment resulted in weaker mathematics knowledge, whereas mathematics instruction in a bilingual environment, where second language teaching practices were used, resulted in equal or better achievement in both content and language learning (Jäppinen, 2005; Novotná & Moraová, 2005; Tan & Lan, 2010; Tavares, 2015; Uribe-Flores et al., 2014; Zevenbergen, Mousley & Sullivan, 2004).

Due to scant research in the teaching of mathematics in English to Swedish speaking students, little is known about the actual practices being used when teaching mathematics in English to a significant number of Swedish students today.
4. Study Aims

One problem identified by Skolverket’s evaluation (Skolverket, 2010) of the use of English as a medium of instruction in Sweden conducted in 2010 was the lack of teacher training in second language learning. The report identified situations where language difficulties, caused either by English speaking subject teachers not being able to adequately support Swedish speaking students or by Swedish speaking subject teachers not having the necessary language competence to teach mathematics in English, resulted in difficulties in subject content learning (Skolverket, 2010, p. 72-73). This problem was identified again in a report that compared the effectiveness of subject and language integrated learning in Sweden and several other European countries (Sylvén, 2013). This lack of training existed both in pre-service and in-service training.

Focusing specifically on the teaching of mathematics in English in a second language environment, research has identified effective teaching practices that lead to deepened student content knowledge. The use of strategies such as code-switching, translanguaging, revoicing, visual aids, open-ended tasks and group work/student-student interaction have all been shown effective in increasing students’ content knowledge and understanding (Planas, 2014; Tavares, 2015; Uribe-Flores et al., 2014; Wildsmith-Cromarty et al., 2009; Zevenberg et al., 2004).

As stated earlier, very little research has been conducted in Sweden concerning the teaching of mathematics in English to Swedish speaking students. The aim of this study is to map out the current situation in order to have a clearer understanding of the situation that over 20,000 Swedish students are taking part in today. This study will attempt to answer the following questions within the context of teaching mathematics in English to Swedish speaking students;

1) What amount of training in second language learning have mathematics teachers teaching in English to Swedish speaking students received?
2) How common is the use of strategies that researchers have identified as being effective in the teaching of mathematics to second language learners in mathematics classrooms in Sweden?

5. Definitions and Limitations

Three key concepts used in this study, second language learner, mathematical language and CLIL, should be clearly defined as they can be interpreted in various ways.

5.1 Second language learner

As noted earlier, over 20,000 students currently enrolled in compulsory school in Sweden are learning mathematics in English. A very small number of those students have one or two English speaking parents at home. The majority are Swedish speaking but there is also a significant proportion of students whose mother tongue is neither Swedish nor English, that is to say, the students speak neither Swedish nor English regularly in their home environment. Although the number of students choosing to learn mathematics in English is a small percentage of the total number of students attending compulsory school in Sweden (2 – 3% of the total compulsory school population), as noted in the introduction, that number has risen dramatically in the last five years and will continue to rise as more bilingual schools are approved.
Among those students, the vast majority of the students learning mathematics in English have English as their second or even third language.

In the context of second language acquisition, Ellis defines second as referring to “any language other than the first language” (Ellis, 2008, p. 5). For the purposes of this study, any student whose mother tongue is not English, will be referred to as a second language learner, regardless of whether English is in fact their second, third or fourth language. Ellis argues that it can be difficult to distinguish between the conscious act of learning and the subconscious act of acquisition with regards to language as both are interwoven in a language-rich environment (Ellis, 2008, p. 7) so this article will use the term learning for consistency. Within the field of language acquisition, a learner’s first language is often referred to as L1, while the target language is referred to as L2.

5.2 Mathematical language

As evidenced in the Skolverket’s support material for mathematics teachers, language and communication are very important aspects of learning mathematics as noted in the Swedish school curriculum.

Mathematical language streamlines and supports mathematical reasoning. Language is thus functional for learning mathematics (Skolverket, 2015, 3(7)).

There exists mathematical vocabulary that is unique to mathematics, words such as hypotenuse and isosceles for example, words that are precise in meaning and rarely lend themselves to contextual interpretation. Students reading a new word in a novel can use the context to form some idea of what the word can mean but the vocabulary used in mathematical texts do not lend themselves to guesswork. Working out the sizes of the other two angles in an isosceles triangle bounded by a 90° angle is a simple calculation if one knows the meaning of isosceles but it is not possible to infer the meaning from the context of the problem.

Expanding on the list of vocabulary specific to mathematics, there are also a great number of words that are common to everyday vocabulary, words that can have a broader meaning in common use but which have a different or more exact meaning when used in mathematics. Words such as ‘evaluate’, ‘increase’ and ‘quarter’, for example, have precise meanings when used in mathematics but multiple or even different meanings in everyday use. In addition to the mathematical meaning of exactly ‘one of four equal parts’, the word ‘quarter’ can represent a coin in North America or a length of time in Swedish. Michal Halliday defined these words as being part of a mathematical register, “the mathematical use of natural language, that is not mathematics itself” (Chan, 2015, p.307). Skolverket states in their support material to mathematics teachers that “meaning-creating teaching opportunities can sometimes be about building students’ daily language, and at other times getting the students to be aware that words in daily use can have another meaning in mathematical language” (Skolverket, 2015, 4(7), author’s translation).

In addition to vocabulary, mathematical symbols can also be considered as a vital part of a mathematical vocabulary. By their nature, mathematical symbols and vocabulary convey specific meanings in compact and exact ways (Skolverket, 2015, 2(8)). It can be argued that even numbers can be considered part of a mathematical language. Examples of such are square numbers, such as 4, 9, 16, 25, 36 and so forth. Part of
teaching basic numeracy includes “real numbers and their properties and also their use in everyday and mathematical situations” (Skolverket, 2011, p.62). For a student to understand that the symbol ‘25’ also has another meaning as part of a family of square numbers is a demonstration of the student’s deeper understanding of mathematical language.

All students, whether second language learners or not, must learn both mathematical vocabulary and the mathematical register of natural language. As noted in Skolverket’s support material for mathematics teachers,

För att kunna abstrahera, resonera och bygga upp komplex matematik krävs att eleven kan och behärskar matematikspråket, de speciella ord, begrepp och symboler som ger betydelse och precision i matematiken (Skolverket, 2015, 2(8)).

In order to think abstractly, to reason and build up more complex mathematics requires that the student knows and masters mathematical language, vocabulary, concepts and symbols that give meaning and precision to mathematics (author’s translation).

For second language learners, there is an additional cognitive load just in the act of recognising the word and then trying to remember the mathematical meaning attached to it while trying to understand what needs to be done to solve the mathematical problem presented. Each new word adds to the level of complexity in a problem. “If the number of combined elements of information is more than the capacity of working memory, cognitive overload results” (Campbell et al., 2007, p. 6). Teachers teaching mathematics in English to Swedish speaking students must be aware of this additional challenge for their students.

For this study, the term mathematical language incorporates mathematical vocabulary, symbols and the mathematical register of everyday words.

5.3 Content and Language Integrated Learning (CLIL) in Sweden

Content and language integrated learning programs, also known in Sweden are by the Swedish acronym SPRINT (Språk- och innehållsintegrerad inlärning och undervisning) are not clearly defined in Sweden as SPRINT schools and independent schools do not operate under common goals. SPRINT programs began in upper secondary schools in the 1980’s and the use of English as a medium of instruction was introduced on a trial basis to lower secondary municipal public schools in 2003. One of the goals of SPRINT teaching is that it will increase the communicative element of learning and will introduce specialised language concepts in a foreign language at an early age (Skolverket, 2010, p. 11, author’s translation). Bilingual schools as part of the compulsory lower secondary system have also existed since the introduction of independent schools in the 1990’s. According to one independent bilingual school,

English has become the world’s common language, “the key to the world”. Children should learn to command the English language, not just know it, at an early age. Fluency is best achieved through language immersion, instructed by native English-speaking teachers in an international atmosphere. (engelska.se/en/what-we-stand)
What is common in both systems is the teaching of some subject matter using a foreign language as the medium of instruction. This study will limit itself to the teaching of mathematics in English to Swedish speaking students.

6. Earlier Research

This chapter will lay out what is currently known about teaching mathematics in English to second language learners and how that applies to teaching mathematics in English to Swedish speaking learners. This first section will briefly describe how the selected reference literature was found. The later sections will outline the common themes that were found in the selected literature.

Much of the earlier research referred to in this chapter come from the author’s earlier systematic literature review into teaching practices for the teaching of mathematics in English to second language learners. A detailed description of the search methods and results can be found in that essay (Breton, 2016). Additional material was found with searches in the databases listed in table 1.

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</table>

Table 1 Search terms

All selected literature is referenced where necessary and publishing details are given at the end of this paper.

6.1 Monolingual second language teaching of mathematics

Some of the research into the teaching of mathematics in English to second language learners studied the effects of monolingual instruction, usually as a result of a government mandate (Tan & Lan, 2010; Tavares, 2015; Wildsmith-Cromarty & Gordon, 2009; Zevenbergen et al., 2004). The reports demonstrated that student content knowledge in mathematics declined after the introduction of monolingual English instruction of mathematics to students in Malaysia, Hong Kong, South Africa and to the aboriginal speakers of Australia. In some cases, students were overly focused on learning vocabulary to the neglect of deeper conceptual understanding (Tan & Lan, 2010; Tavares, 2015; Wildsmith-Cromarty & Gordon, 2009). Those students’ limited responses were often interpreted as a result of weak mathematical abilities and in many cases, the students were streamed into easier classes where less content would be covered (Tan & Lan, 2010). By limiting teaching to a simple delivery of contents and vocabulary, students’ creative abilities were not engaged and opportunities for learning were thereby limited (Novotná & Moraová, 2005, p. 110). One of the most damaging effects noted by other researchers, working in a mixed school environment of native speakers and second language learners was that “When heritage language ability is positioned as ‘language disability’ and students are marked as not having mathematical learning potential on equal footing with their English speaking peers, the mathematics classroom becomes oppressive” (Uribe-Flores et al., 2014, p. 236).
One of the biggest challenges to second language learners of mathematics as noted earlier is the increased risk of cognitive overload. The concrete data, mathematical terminology and even the use foreign words, such as English towns and names, become abstract terms (Novotná & Moraová, 2005, p. 113) for the second language learner to juggle in their working memory.

Recent research suggests that the most successful programs for teaching mathematics to second language learners incorporated the students’ existing language abilities through a judicious use of the students’ L1. The following section will describe some to the teaching strategies that have been found effective when teaching mathematics to second language learners.

6.2 Supportive Communicative Practices

Actively using language supports students’ understanding (Skolverket, 2015, 5(7)). Many teaching practices linked to increasing levels of student understanding and achievement in mathematics with regards to second language learners fall under the theme of communicative practices. Communicative ability is one of the requirements that must be met by all students following the Swedish curriculum, regardless of the language of instruction.

Pupils can account for and discuss their approaches in an appropriate and effective way and use symbols, algebraic expressions, formulae, graphs, functions and other mathematical forms of expression with good adaptation to purpose and context. In their accounts and discussions, pupils apply and follow mathematical reasoning by putting forward and responding to mathematical arguments in a way which takes the reasoning forward and deepens or broadens them (Skolverket, 2011, p. 68).

Instead of avoiding difficult words and expressions, students should be given the opportunity to develop their communicative abilities by interacting with new as well as with familiar words and expressions (Skolverket, 2015, 7(8)). Through interaction by means of a game, a puzzle or through group discussion and exploration, new vocabulary can be presented and practiced by students to allow them to become familiar with the terms. Even general terms, such as names and places, should be made familiar to second language learners so that they do not work with the unknown vocabulary as with variables (Novotná & Moraová, 2005, p.113). Repeated interaction with new mathematical vocabulary in a mathematical context (as opposed to a static review of wordlists), is crucial to knowledge acquisition (Chan, 2015, p.312). Researchers found that focusing on wordlists without discussing the mathematical concepts led to lower results on tests for conceptual knowledge (Tan & Lan, 2010).

In the case of second language learners of mathematics, translanguaging, from both the teacher and the students, has been shown to be an effective classroom practice. With translanguaging, students are presented a task in one language and asked to solve it in another. An example may be students working from a Swedish text book but being asked to discuss and present their solution in English. Studying translanguaging in a mathematics classroom in Hong Kong, Tavares described its effect as “using L1 as a rich resource in linguistic scaffolding to facilitate learners’ comprehension and understanding of the target language (L2)” (Tavares, 2015, p. 321). In code-switching, students or teachers switch from one language to another...
in order to allow the conversation flow more smoothly (Ellis, 2008, p.956). In bilingual environments, code-switching is a common practice used “for purposes of clarification and explanation” as shown in research from South Africa (Wildsmith-Cromarty & Gordon, 2009, p.362). **Revoicing**, the practice of covertly correcting students’ mistakes in vocabulary where a mistake is not pointed out but the correct word is used by the teacher in their summary of a student response has been shown to build second language learner confidence in the mathematics classroom (Tavares, 2015, p. 329). The revoicing is coupled with the visual aid of the teacher writing the correct terms on the board and asking all students to take notes in their books.

**Open ended tasks** create effective learning opportunities for all students in mathematics and are, by their nature, also easily accessible by students with knowledge or language limitations. Open ended tasks are tasks with numerous possible answers and which have the potential to be solved in different ways (Zevenbergen, Mousley & Sullivan, 2004, p.392). A simple example could be the question ‘Five children count up their money outside an ice cream stand and find that together, they have 18$. How much could each child have?’ From this question, the discussion could continue to cover mean, mode, median and range with the class.

> Whatever their responses, all students have been able to participate in the activity and all students can produce ‘correct’ results, which the teacher can use to gain deeper and more valid insights into students’ thinking than would be possible from closed questioning. The capacity for students to respond correctly and learn from tasks is important characteristics of mathematics classrooms. (Zevenbergen, Mousley & Sullivan, 2004, p.394)

Using open ended questions and allowing the students to explore with their own answers increases student interaction with mathematics.

**6.3 Visual Aids**
The use of visual aids was mentioned in several studies regarding teaching mathematics in English to second language learners. Graphs and diagrams display information more clearly which reduces the cognitive demands on second language learners who would otherwise have to deal with the additional demands of interpreting text-rich word problems (Chan, 2015, p. 314) as well as recalling the mathematical concepts needed to solve the problem. Graphic organisers such as mind-maps, tree-diagrams and flow-charts can also help unpack the layers of information found in many mathematics problems (Wildsmith-Cromarty & Gordon, 2009, p.365; Chan, 2015, p. 314). Posters and digital presentations offer extra scaffolding to the creation of understanding among all learners but particularly among second language learners (Uribe-Flores et al., 2014, p.242).

Writing offers the opportunity to collect thoughts and present them using appropriate mathematical language with appropriate use of concepts (Skolverket, 2015, 5(7)). Highlighting text, written and spoken has also been shown to be a strategy that facilitates learning (Tavares, 2015, p. 328).

Use of the students’ L1 in written documents, translated from the textbook into worksheets for example, can support the students’ acquisition of content knowledge. “The careful preparation of content materials in translated form is one of the main
problem-solving strategies employed in CLIL pedagogy in Queensland.” (Smala, 2013, p.201)

6.4 Student – Student Interaction

Skolverket’s support material for all mathematics teachers in compulsory school echoes a recurring theme in research into the teaching of mathematics in English to second language learners.

A simple way to produce verbal interaction is to allow students to work with mathematical tasks in groups, and then have the students present their solutions to the others (Skolverket, 2015, 5(7), author’s translation).

Many reports associated group work and student-student interaction as factors in higher achievements in both content and language learning among second language students.

Two different studies in Hong Kong (Chan, 2015; Tavares, 2015) both pointed out the benefits in building student confidence in both mathematics content and English language abilities as a result of group work or student-student interaction. Chan’s study focused more on learning mathematical vocabulary; “to maximise learners’ awareness of the language features in word problems and other mathematics texts, teachers may, drawing on principles of cooperative learning, design more pair work or small group activities” (Chan, 2015, p.314), while Tavares wrote about creating a “safe and supportive” atmosphere where “learners were encouraged to work cooperatively and supportively on the questions presented to them” (Tavares, 2015, p. 325).

Planas’ research of Catalanian and Spanish speaking students working together in Spain also highlighted the effectiveness of group work. “All the teachers prompted the students to work in small groups to solve the tasks and for the groups to report their decisions to the class in the final discussions” (Planas, 2014, p. 56).

Skolverket’s support material states that “Teaching must promote active language use in both verbal and written interaction and production and students must be offered varied language support” (Skolverket 2015, 1(10), author’s translation). Group work and class presentation offers a learning opportunity for students of all levels of mathematical and language abilities through active participation, with support from the teacher and from fellow learners being an important factor in the process.

6.5 Positive CLIL Outcomes

One decisive factor for students’ development of mathematical knowledge is that teaching includes communicative aspects of the subject (Skolverket 2015, 6(7), author’s translation).

Much research into CLIL outcomes focuses on increased language proficiency but some of the research also points out increased abilities in mathematics.

Trying to cope with the linguistic demands of expressing mathematical content adequately, bilingual learners often focus heavily on text production, particularly while creating a mathematical model and performing mathematical operations. This increased focus, in turn, encourages additional linguistic and mathematical reflection (Berger, 2015, p. 309).
Research in Finland suggests that CLIL students had developed stronger abilities in comparing concepts and meaning schemes with each other, possibly as a result of consistently working in two languages and comparing language as well as content features (Jäppinen, 2005). The researcher’s conclusion was that teaching through a foreign language promoted mathematical thinking and learning processes (Jäppinen, 2005, p.159).

The common conclusion in many reports was that a supportive bilingual learning environment leads to stronger student achievement in both mathematics and language knowledge as summarised this way by one researcher, “learners in a bilingual learning environment were found to be stronger in mathematical operations than their counterparts instructed in a monolingual classroom” (Tavares, 2015, p.321). Even in situations where the teacher is a monolingual speaker of the target language (in this case English), allowing the students to argue, discuss and explain mathematical problems in their first language gives the students an opportunity to “gain autonomy from the teacher and become active in communicating their mathematical thinking in either language” (Planas, 2014, p.54).

The use of teaching practices aimed at supporting the learning of mathematical vocabulary, including everyday language in a mathematical register and specific mathematical vocabulary and symbols, provides more resources and learning opportunities for all students.

7. Theoretical View
Cognitive theories define knowledge as the creation of mental models or schemes in long-term memory, in other words, knowledge is seen as an abstract version of the world around us which exists as a concept in our memory that can be recalled and used as needed (Säljö, 2012, p.161). Interaction with that model can result in adaptation to a better model or scheme. For a learner to know something involves the creation of that mental model through some active process.

Dewey’s pragmatic approach of ‘learning by doing’ had become a popular model for creating opportunities for learning since it could easily be used to transform a classroom from a teacher-centered environment to a student-centered learning experience which can support students of differing abilities (Säljö, 2012, p. 176). Although still referenced today, Dewey’s approach has been supplanted in many studies by a sociocultural perspective of learning. Some researchers describe the move from cognitive theories to a sociocultural perspective as a paradigm shift in educational theories (Berger, 2015, p. 287).

Evolving out of the work of Piaget and Vygotsky, the sociocultural perspective of learning and development proposes that meaning creation occurs through the process of perceiving connections and patterns, thereby creating insight and understanding (Säljö, 2012, p. 162). Through communication, meaning and understanding are created and these meanings, distinct from a cognitive view of knowledge, can be different from person to person. The sociocultural perspective of learning and development proposes that “participants bring multiple views to a situation; that representations have multiple meanings for participants; and that these multiple
meanings for representations and inscriptions are negotiated through conversation” (Moschkovich, 2002, p. 197).

Some of the words used by the previously noted researchers in the teaching of mathematics in English to second language learners, words such as ‘support’ and ‘mediate’, suggest the use of a sociocultural perspective of learning. Vygotsky proposed that language was a tool that we use to mediate or create understanding of a subject (Säljö, 2012, p. 187). Vygotsky later developed his theory of the ‘Zone of Proximal Development’ which posits that learning and development progress once a learner has mastered a concept. The learner is then ready to learn a new concept with the support of a teacher or of another, more experienced, learner (Säljö, 2012, p. 193). The sociocultural perspective of learning offers a way to understand how people are socially involved in knowledge and experience through interaction with each other in different activities (Säljö, 2012, p. 195), that “learning is inherently social and cultural” (Moschkovich, 2002, p. 197). This theoretical perspective underlines the value in student-teacher and student-student interaction and can be used when interpreting the effects of group work in mathematics classrooms because of the “significant role of communication in the construction of meaning” (Berger, 2015, p. 287).

When Moschkovich applied a sociocultural perspective to her study of second language learners of mathematics, she saw that it offered the opportunity to see resources from both the everyday and mathematical registers, along with from both languages, for students to use to communicate mathematically. “A situated-sociocultural perspective thus moves away from the description of obstacles and deficiencies to a description of resources and competencies and widens what counts as competence in mathematical communication” (Moschkovich, 2002, p. 197). She argues that communicating mathematically can be seen as “using social, linguistic, and material resources to participate in mathematical practices” (Moschkovich, 2002, p. 197).

In her study of CLIL in Australia, Smala also discussed Vygotsky’s ‘Zone of Proximal Development’ in relation to student learning. In a study conducted in Spain, Planas argues that “mathematical contents and the language used to communicate them are constructed in interaction by students in small groups in which the task is introduced as mathematical” (Planas, 2014, p.63). The recurring argument for the use of a sociocultural perspective when discussing the teaching of mathematics in English to Swedish speaking students is that language is part of the learner’s thinking process. It is the tool the learner uses to reflect on experiences and achieve understanding (Jäppinen, 2005, p.152).

8. Method

In this chapter, the method selected for this study will be explained in detail along with the advantages and disadvantages inherent in the selected method. The selection of study participants will also be described, followed by ethical considerations. Finally, the implementation, data processing and reliability of the study will be discussed.

8.1 Method Selection – A Quantitative Study

Eliasson argues that the choice of method used for a study is linked to the theoretical view associated with the study (Eliasson, 2006. p. 21). According to her, theoretical views that work best with quantitative studies are those where knowledge can be
measured ‘wide’ and can then be generalised from a small group to a large group. Quantitative studies are also best adapted to measurements that can be counted, such as ‘how often do you use the calculator function on your phone’. Larsen notes that a quantitative study is useful for obtaining a representative overview of a situation (Larsen, 2009, p. 23). These two views correlate well with the aims of this study, namely:

1) What amount of training in second language learning have mathematics teachers teaching in English to Swedish speaking students received?

2) How common is the use of strategies that researchers have identified as being effective in the teaching of mathematics to second language learners in mathematics classrooms in Sweden?

The intention of this study is to gather a general overview of the situation for teachers of mathematics in English to Swedish speaking students as it currently exists in Sweden through a survey of a limited number of teachers.

To this purpose, a questionnaire survey was prepared with questions covering the teacher’s educational background, use of practices shown to be effective in the teaching of mathematics in English to second language learners, and finally, the teacher’s opinions regarding the challenges of teaching to the Swedish curriculum for mathematics. Some of the questions were developed from previously published research articles into effective teaching practices in the teaching of mathematics in English to second language learners (see Planas, 2014; Tavares, 2015; Uribe-Flores et al., 2014; Wildsmith-Cromarty et al., 2009; Zevenberg et al., 2004). A pilot version of the questionnaire was prepared and sent out to a smaller number of teachers several weeks before the final version. Feedback from the pilot survey included removing the student’s gender on the explanation of the practice of revoicing and adding an option for a response of ‘other’ for some of the questions. The suggested changes were implemented and the survey was then sent out to a larger group of teachers.

One advantage to a survey questionnaire is that the resulting data is easily manageable as there is no need for transcription and data coding is easier to do. One disadvantage is that there is little room for elucidating a participant’s response. In order to accommodate for this limitation, the survey included the open ended question “Are there any other teaching practices that you would like to mention regarding the teaching of mathematics in English to Swedish speaking students?” This offered participants the opportunity to expand on their answers in ways that were unforeseen during the preparation of the questionnaire.

Another potential weakness in survey questionnaires is that participants may not answer truthfully. There exists the potential that participants answer in what they perceive to be a favourable way. To mitigate this potential as much as possible, the participants in this survey were informed that the information collected for study was completely confidential. This was highlighted in a letter of introduction, in an email link to the survey and again at the beginning of the survey, just before the questions on personal background.

Many of the questions posed in the survey offered the participants a chance to answer on a Likert-type scale. These five point scales were applied to questions concerning ‘Amount of use’ and ‘Level of Difficulty’ and were adapted from scales suggested by Vaggias (2006). An important consideration was to ensure that each question asked only one thing so as to cause difficulties for the respondents (Hartley, 2013, p.84). If
questions are repetitive in formulation, there is also the risk that “some respondents simply tick the same box for every item, perhaps without considering carefully enough the meaning of each one” (Hartley, 2013, p. 84). To avoid this, the formulation of the questions varies in the major part of the survey but this may be a concern for the last five questions concerning the five abilities assessed in the Swedish curriculum for mathematics where the same question is effectively repeated five times.

8.2 Subject Selection
The participants selected for this study were the mathematics teachers for grades 7-9 working for the largest independent bilingual school organisation in Sweden. Letters of introduction and emails were sent out to a little over one hundred teachers during the winter of 2016 with an invitation to participate in the online survey. Using the largest organisation of compulsory level bilingual schools in Sweden gave the researcher access to a large number of teachers which increases the possibility of generalising the results. When limiting the participants to those working within one organisation, it can be argued that the results apply only to that organisation and cannot be generalised to the larger population. That can certainly apply if the organisation is standardised in its working methods, where all teachers use the same textbooks and teach with the same lesson structure and organisation, or have the same educational and cultural background. In this case, the 28 schools participating in this survey operate independently from each other and each school is free to choose the textbooks and methods used for teaching. This can be seen in the varied responses offered to the questionnaire.

Out of 110 teachers invited to participate, 45 teachers offered a total of 48 responses (teachers teaching multiple grades were given the opportunity to respond once for each year grade taught). The response rate of 40.9% is higher than the average of 33% found by Nutley for online survey responses (Nutley, 2008) and should offer a good representation of the existing situation in the teaching of mathematics in English to Swedish speaking students.

8.3 Ethical Considerations
This study follows the guidelines established by the Swedish Research Council (Vetenskapsrådet) regarding ethical considerations in scientific research. These guidelines can be found in their publication, Good Research Practices (God forskningsråd, Vetenskapsrådet, 2011).

The Swedish Research Council outlines the following eight guidelines that should be part of every scientific investigation.
- Researchers should tell the truth about their research.
- Researchers must consciously examine and present the starting points for their studies.
- Researchers should disclose the methods and results used.
- Researchers must openly declare commercial interests and other interests.
- Researchers must not steal the research of others.
- Researchers should keep good order in their research including thorough documentation and archiving.
- Researchers should strive to conduct their research without harming people, animals or the environment.
- Researchers must be fair in the assessment of other’s research.
In addition to these general guidelines, Högskola Dalarna publishes a form entitled Ethical Self Evaluation for Student Projects involving Humans which this researcher used to judge if further ethical considerations needed to be taken. The conclusion was negative, no further action needed to be undertaken before the continuation of this research.

The two major ethical considerations in the development of this study were the study’s usefulness to scientific knowledge, and the confidentiality of participants. The results of this study will add to existing scientific knowledge as little is currently known about the learning environment for the 20,000 Swedish students who are currently learning mathematics in English so the results should be of use to later researchers. As to the second point, the online survey was designed to be anonymous for two reasons, to encourage truthfulness in responses and to protect the identity of the participants.

8.4 Implementation
After the conclusion of the pilot survey in the first week of February, the survey was further refined and was ready for submission by the first week of March. The final version of the survey was emailed to the Head of Mathematics of all 28 schools in the organisation along with an introduction letter and the request that the department heads forward the email to their mathematics teachers working with grades 7-9. This fell during the rotating period of Winter Break holidays with some schools being closed the first week of the study and other schools being closed the second week of the study so it was decided that the online study would remain open for ten days. After the first week, a reminder email was sent out to all mathematics department heads. As the returning results are confidential, there is no way to ascertain if all schools participated or not but, as noted earlier, 45 teachers out of an estimated 110 did respond so the response rate is acceptable. The data was collected in a spreadsheet and the results were analysed as proportions of total responses.

8.5 Reliability
As mentioned earlier, potential weaknesses of this survey include the possibility of the participants not answering truthfully, low participation and the fact that all the participants work for the same organisation. Other potential problems can occur if the questions asked are confusing or don’t actually measure the study’s aims.

The first few weaknesses have already been addressed in section 8.1 and 8.2. As to the potential for confusing or irrelevant questions, sending out a pilot study which included a question for feedback and discussions with local maths teachers, helped assure that the questions on the final survey were clear and relevant to this study’s aims. The survey in its entirety is appended at the end of this paper. To ensure that the participants’ responses are a reflection of current teaching practices, the participants were asked to reply in the affirmative only if they had used that teaching practice in the last two weeks of teaching.
9. Results

The material collected in this survey fall into three categories, Participant Background, Teaching Practices and Mathematical Abilities Assessed by Lgr-11 (the Swedish school curriculum).

9.1 Participant Background
Experience
As this school organisation has experienced a dramatic increase in number of schools over the last 7 years, expanding from 8 to 28 schools, it was thought that it might be relevant to determine the work experience of the participants in teaching mathematics in English to Swedish speaking students. The results show that there are not many years of experience to be found in the teaching of mathematics in English to Swedish speaking students. Less than one third of the respondents have been teaching mathematics in English to Swedish students for 5 years or longer. Almost half, 46.7% have been teaching for two years or less.

![Pie chart showing years of teaching experience](chart.png)

Teacher Certification
The question of teacher certification may give an insight into which teachers are using effective teaching practices when teaching mathematics in English. The responses to this question will be analysed along with the later responses to teaching practices to see if a correlation exists. Regardless of the Swedish school authority’s push to have only certified teachers in the classrooms, almost one quarter of the respondents lack teaching certification in mathematics for grades 7-9. The majority of teachers do have certification with 60% of the certification coming from outside Sweden.
Native Language
One of the key advantages of CLIL instruction is that students learn English through subject learning, in this case mathematics. Earlier studies revealed that CLIL instruction in Sweden was not as effective as in other countries because it generally started too late, in upper secondary, and that many of the teachers were not native English speakers (Sylvén, 2013). The results of this survey of lower secondary mathematics teachers in Sweden’s largest collection of bilingual schools show that almost three-quarters of the teachers are native English speakers. 18% are native Swedish speakers and the final 9%, or four teachers, are Arabic (2), Greek (1) and Polish (1) speakers.
Second Language Teacher Training

Skolverket’s early study of subject teaching in English in Sweden indicated that the majority of subject teachers teaching in English to Swedish speaking students did not have any formal training in teaching to second language learners (Skolverket 2010). It has also been noted that in-service training programs in Sweden are very limited (Sylvén, 2013). This survey appears to confirm that those findings are still true today. Almost three-quarters of the respondents have received no formal training at a college or university level in teaching to second language learners. Only three teachers out of 45, or 6.7%, have taken three or more courses for teaching to second language learners.

How much formal training (college or university) have you received in teaching to second language learners?

- No formal training: 33 (73%)
- 1 course: 8 (18%)
- 2 courses: 1 (2%)
- 3 or more courses: 3 (7%)

As for in-service training, 37 out of 45 respondents, 82%, answered that they have not received any in-service training for teaching to second language learners.

Have you received in-service training in teaching to second language learners?

- No: 37 (82%)
- Yes: 8 (18%)
9.2 Teaching Practices
Textbook Language

The question of textbook language is part of the subject of translanguaging but has been asked as a separate question in order to ensure a clearer result as some teachers may not be aware that using a Swedish textbook during an English lesson is translanguaging. In informal conversations with mathematics teachers over the years, the researcher has noted that teachers of Mathematics in English struggle to find an English language textbook that is easily adapted to the Swedish curriculum. That may explain in part why the majority of schools teaching mathematics in English are using Swedish textbooks. One point that will be analysed is whether the use of a Swedish textbook is grade dependant with the thought being that schools start with English textbooks in the lower years but switch to Swedish language mathematics textbooks in the later years as the students prepare for Swedish language high school.
**Code Switching**

The results show that code-switching, changing from one language to another during a exchange is in very common use by the teachers in this survey. Only 7 respondents, 14.6%, replied that they never or almost never use code-switching during their lessons.

When the question of the use of code-switching between students and teachers was asked, only three teachers out of 45, or 6.3%, discouraged or actively discouraged it, but half the teachers responded that they had not brought it up with their students. This appears to be a practice that is not actively discussed.
The responses regarding student-student code-switching revealed that no teachers discouraged or actively discouraged the practice yet two-thirds of the teachers replied that they had not addressed the issue with their students. This may be because teachers were asked to frame their answers to practices performed within the most recent two weeks before the survey. It may be something they have mentioned to the students previously, but not recently. Research into second language teaching of mathematics has shown that creating a communicative learning atmosphere in the classroom requires active encouragement from the teacher. Classroom norms of what is acceptable in discussing mathematics must be made clear by the teacher on a regular basis so that all students are encouraged to participate to the best of their abilities (Planas, 2014).

Translanguaging
As previously noted, using a Swedish language mathematics textbook while teaching the lesson in English is an example of translanguaging but the results of this question show that few teachers are aware of that. The example offered up in the survey before the question was, Translanguaging is the practice where the students receive instruction (verbal or written) in one language and produce work in another language. One example may be presenting a mathematical problem in Swedish and asking the students to solve it in English.

If the respondents based their answer on the example, forgetting that 87.5% of them are using a Swedish language textbook, the results show that one-third occasionally engage in translanguaging, a little over one-third frequently engage in translanguaging and under one-third almost never or never engage in translanguaging.
When asked if they tried to limit the use of Swedish by students during the mathematics lesson, only 27% or respondents replied yes. Looking at the answers given by the teachers who chose ‘Other’ as an answer, the majority explained that they only tried to limit interaction in Swedish with them as they were unilingual English speakers and could not understand if the student spoke Swedish. Skolverket noted the potential for difficulties in teachers supporting students in these situations and recommends that teachers coming to teach CLIL classes in Sweden learn Swedish as soon as possible (Skolverket, 2010).
Revoicing
When asked if they engaged in the practice of revoicing in the last two weeks, almost 65% of the participants responded that they frequently or almost every time engage in the practice with another 23 percent responding occasionally. Only 5 teachers, 10.4%, replied that they have never or almost never revoiced in the last two weeks. This practice seems to be very common among the teachers teaching mathematics in English to Swedish speaking students.

Although the majority of the responses are positive to revoicing among the students, the percentage of never or almost never triples to 31.2%. Again, this may be the result of teachers not actively defining the norms of what is acceptable in mathematical discussions on a regular basis.
Visual Aids
The majority of participants responded that they have used visual aids during their lessons in the last two weeks with three-quarters using charts, diagrams and projected presentations, half using manipulatives and one-third displaying posters. Three teachers wrote in that they also used videos as well as projected presentations although some of the other respondents may have included videos as part of their response of projected presentations. One teacher wrote in ‘Investigative group research projects’ in addition to selecting the other four responses.

<table>
<thead>
<tr>
<th>What visual aids have you used in your lessons during the last two weeks? Check all that apply.</th>
</tr>
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<tbody>
<tr>
<td>Other</td>
</tr>
<tr>
<td>Projected presentations</td>
</tr>
<tr>
<td>Posters</td>
</tr>
<tr>
<td>Charts or diagrams</td>
</tr>
<tr>
<td>Manipulatives</td>
</tr>
</tbody>
</table>

Open Ended Tasks
The practice of presenting open ended tasks to students is not prevalent among the participants of the survey if compared to other effective teaching practices. Only 8 teachers, or 16.7%, replied that they frequently or almost every time present open ended tasks to students in the last two weeks. Almost 44% replied that they occasionally used this practice but 18 teachers, 37.6%, replied that they never or almost never present open ended tasks to their students in the last two weeks.

<table>
<thead>
<tr>
<th>Have you engaged in the practice of Open-Ended tasks with your students in the last two weeks?</th>
</tr>
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<tbody>
<tr>
<td>Frequently use</td>
</tr>
<tr>
<td>Almost every time</td>
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<tr>
<td>Occasionally/sometimes</td>
</tr>
<tr>
<td>Almost never</td>
</tr>
<tr>
<td>Never use</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>
**Student-Student Interaction**

When asked if their students worked in groups to solve mathematical problems in the last two weeks, 35.4% of the participants responded that their students frequently or almost every time worked in groups. 21 teachers, 43.8% replied that students occasionally/sometimes worked in groups (one teacher replied that students worked in group every one of three lessons and so was placed in with the occasionally/sometimes responders). 10 teachers, 20.9%, responded that their students never or almost never work in groups to solve mathematical problems.

When asked what language the teachers would prefer student-student interaction to take place in, 37.5% of participating teachers replied that they would prefer student-student interaction to take place in English as much as possible. 58.4% replied that both English and Swedish were acceptable (the teacher who chose ‘Other’ wrote that they preferred English as they could not understand Swedish but they did let the students speak Swedish if the teacher was certain they were on task so their response was added to the ‘Both English and Swedish’ option). Two teachers, 4.2%, replied that student-student interaction should be in Swedish.
Other teaching practices
When offered the opportunity to write in their own teaching practices that were not mentioned in the survey, 10 teachers responded with comments. Among their answers of note (repetitions have been excluded for brevity) were the following comments;

“Parallel questions - questions that assess the same skill, but require different levels of understanding of the mathematics or the language so students can choose the one that best fits their level of challenge needed”

“I include key vocabulary translations. Key vocab for that particular lesson is given in English and Swedish with a definition (occasionally in both languages.)”

“50/50-practice for younger students. Saying one thing in English (always start with English) then repeat the same thing in Swedish.”

“A great deal can be achieved by facial and expression and using actions.”

“Peer tutoring, student translation, one to one teacher/student interaction during class.”

“As mentioned earlier, projects (both individual as well as group) that involve real life situations & then making presentations to the class in English.”

“I really like having the Swedish textbooks readily available so that the students can get the ideas a second time in their native language. A) I think it helps further their understanding and B) I think that it will also help them in gymnasium, when they are taking math in Swedish again.”

9.3 Mathematical Abilities Assessed by Lgr-11
In the final section of the survey, participants were asked to rank how easy or challenging they feel it was to teach the five abilities outlined by the Swedish curriculum in mathematics in English to their Swedish speaking students.

Problem Solving
Almost half of the participants responded that they felt it was difficult or very difficult to teach problem solving in English to their Swedish speaking students. Only 8 teachers out of 45, 16.7%, responded that they felt it was easy or very easy. A little over one-third of the participants felt neutral to the question.
Conceptual Abilities
The responses for the teaching of conceptual abilities were more positive in nature with 56.2% of participants responding that they felt it was easy or very easy to teach mathematical concepts in English to their Swedish speaking students. 12.5% felt that it was difficult while 31.3% felt it was neither easy nor difficult.

Methods
When asked how they felt about teaching the ability to choose and use appropriate mathematical abilities in English to their Swedish speaking students, 66.7% of the participants felt that it was easy or very easy. Only two teachers, 4.2%, responded that they felt it was difficult to teach methods while 29.2% felt neutral towards the ability.
**Reasoning**
When asked how they felt about teaching mathematical reasoning abilities in English to their Swedish speaking students, almost half of the participants, 47.9%, responded that they felt it was difficult or very difficult to teach reasoning to their students. Only 16.7% felt that it was easy or very easy while a little over one-third, 37.4%, felt that it was neither easy nor difficult to teach mathematical reasoning in English to their Swedish speaking students.

![Teaching Reasoning](image)

**Communication**
The last ability listed by the Swedish school curriculum in mathematics is the ability to use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions, the communication ability. Half of the participants felt that teaching mathematical communication was neither easy nor difficult. 31.3% felt that teaching communication was easy or very easy while 18.8% felt that it was difficult or very difficult.

![Teaching Communication](image)
10. Analysis
The data was broken down into groups defined by teaching experience in Sweden and then by teacher certification to compare the responses in the use of teaching practices such as translanguaging, code-switching and revoicing, the use of open ended tasks, the use of group work, language preferences and lastly, the difficulties in teaching the abilities listed in the Swedish curriculum.

10.1 Analysis by Teaching Experience
As this next chart shows, the more experienced teachers were, percentage wise, more favourable to the use of translanguaging during lessons.

The use of open ended tasks was not actively promoted by many teachers, though the newest teachers were proportionally less negative towards them. The teachers with the most experience were the least inclined to use open ended tasks in their teaching.
This next chart shows the participants’ inclination to having the students engage in group work sorted by the number of years of teaching experience in Sweden. The teachers who are least negative to group work are those with 1 to 4 years’ experience. The newest teachers and the most experienced teachers almost have an equal proportion positively inclined to group work as negatively inclined, although the newer teachers have the greatest proportion of neutral responses over all groupings.

The following chart analyses the participating teachers’ responses to the question of what language student-student interaction should be, once again, sorted by years of experience. The majority of all participating teachers accepted a mix of English and Swedish in their lessons although a notable proportion, particularly among the newest teachers but also among those with 3-4 years experience did prefer English only communication whenever possible. It was interesting to note that the two teachers who preferred Swedish for student-student interaction were native English speakers themselves.
This final chart of the data sorted by years of experience teaching in Sweden shows that all four groups of teachers feel that teaching problem solving is the most difficult of the five abilities when teaching mathematics in English to Swedish speaking students. Looking at the wealth of existing research into the difficulties learners face when undertaking problem solving in mathematics, this difficulty appears to be common among many mathematics learners and not just second language learners of mathematics.

Reasoning is also high on the list of difficulties for the teachers with the most experience as well as among the teachers with the least experience. Fewer of the teachers feel they have difficulties teaching the abilities of concept, method and communication. There is an exception for the teaching of the communication ability among the newer teachers. This may be a consequence of the fact that most of the newest teachers are in fact new to Sweden and have a very limited grasp of the Swedish language. This increases the challenge of teaching mathematical communication to second language learners.
10.2 Analysis by Teacher Certification
This next section will analyse the results in terms of teacher training with the results sorted into teachers with foreign certification, Swedish certification and no certification.

In this chart we see that proportionally, the teachers with foreign teacher certification are the most positive and least negative towards the use of translanguage, code-switching and revoicing. Teachers without certification follow the same pattern. It is interesting to note that the teachers with Swedish certification are the least positively inclined to use those teaching practices.

This next chart shows the participants inclination towards open ended tasks as organised by teacher certification. Although few respondents were positive to the use of open ended tasks, teachers with Swedish certification were proportionally the most negative towards that teaching practice while foreign certified teachers were neutral or positively inclined.
The following chart shows the distribution of responses about group work as sorted by teacher certification. It shows that the foreign trained teachers along with teachers with no certification are proportionally more positive or neutral towards the practice of group work than teachers with Swedish certification. This could be a result of a small number of responses but it is a question that could be explored further in later studies.

![Group work chart](image)

In this chart the language preferences are sorted by teacher certification. Proportionally, the teachers with Swedish teaching certification are the most inclined to have the students interact in both English and Swedish. The teachers without certification are, proportionally, the ones most requesting a preference for English interaction. This may be a direct result of a lack of training in educational practices.

![Language preference chart](image)
When compared to the chart on page 33, where the analysis is by years of experience in Sweden, this chart shows a dramatically different result when the responses are categorised by teacher certification but the explanation is simple. There were 29 teachers with foreign certification, 7 with Swedish certification and 11 with no certification at all. The result is that there were significantly more responses from foreign certified teachers than the other two categories combined. Nonetheless, when analysed proportionally, the foreign certified teachers feel that teaching reasoning and communication are more difficult than the other abilities. Swedish certified teachers struggle most with problem solving and reasoning. The higher difficulty in teaching problem solving for Swedish certified teachers may be linked to their disinclination for group work and open ended tasks.

![Mathematical abilities according to the Swedish Curriculum (Lgr-11) - Difficulty](image)

Overall, the results show distinct trends in the teaching of mathematics in English to Swedish speaking students. Few teachers in general were positively inclined to use open ended tasks on a regular basis and none of the Swedish certified teacher were positively inclined to regularly use open ended tasks. When sorted by years of experience, the teaching of problem solving and reasoning received similar numbers of responses but when sorted by country of certification, reasoning was by far the most difficult to teach by the teachers with foreign certification.

### 11. Discussion
The aims of this study were to answer the following questions:

1) What amount of training in second language learning have mathematics teachers teaching in English to Swedish speaking students received?

2) How common is the use of strategies that researchers have identified as being effective in the teaching of mathematics to second language learners in mathematics classrooms in Sweden?

To answer these questions, a questionnaire survey was sent out to approximately 110 teachers teaching mathematics in English to Swedish speaking students in grades 7-9.
Together they teach the majority of the over 20,000 Swedish students who are learning mathematics in English. With a response rate of 41%, this study presents a moderately accurate picture of the current situation. As to the study’s first question, almost three-quarters of the teachers responded that they had received no pre-service training for the teaching to second language learners. Only 6.7% had received three or more college or university level course dealing with second language learners. These results confirm Skolverket’s results from 7 years earlier (Skolverket, 2010) and show that little has changed to improve the situation in the intervening years. One consequence of this lack of training is that mathematics teachers are at risk of not supporting each learner through their individual needs, a requirement written into the Swedish curriculum (Skolverket, 2011). As Moschkovich noted, mathematics teaching has shifted to a greater emphasis on students being able to discuss, explain and clarify their mathematical strategies and solutions (Moschkovich, 2002) and a mathematics teacher with limited understanding of teaching mathematics to second language learners runs the risk of not helping the learner achieve their full potential.

Regarding the second question of teaching practices, about one-third of respondents answered that they frequently used translanguaging during their lessons. This response, coupled with the fact that 87.5% of them used a Swedish mathematics textbook in their English language mathematics lesson, showed a poor understanding of translanguaging. Only about one-third regularly engaged in the practice of code-switching and very few teachers regularly used open ended tasks with their students. Group work was a regular feature in about one-third of the classes.

Of the teaching practices that were being used, three-quarters of respondents regularly used visual aids during their lessons; the textbook used for the majority of classes was in Swedish (with a response of 100% Swedish for grade 9 classes); revoicing was regularly used by two-thirds of the teachers; and a little over half of the teachers who responded accepted the use of both English and Swedish in student-student interaction.

Overall, some of the teaching practices that researchers have found to be effective in the teaching of mathematics in English to second language learners are being used in Sweden but there is a need for further teacher training in the use of these practices.

11.1 Method Discussion
This study has produced results that have allowed it to answer its stated aims and also opened avenues for further research. The use of a pilot study prior to the full survey was instrumental in ensuring that the questions were clear, easy to answer and focused on the study aims. The pilot study revealed that some teachers wanted an ‘Other’ response option, together with a text box to fill in their response, in order to feel they had correctly answered the question. That was included, where appropriate, in the final version of the survey. This answer option offered the participants the opportunity to give clear and accurate answers if they were unsure of the best choice. The addition of one additional “open answer” question allowed the participants a chance to expand further on their answers. The inclusion of an “Other” answer text box as well as an open ended question increases this study’s reliability and validity. This could have been covered more effectively in follow up interviews with any participants but the restricted time scale involved in the production of this paper prevented that as a viable option.
The last section of the survey, questioning how challenging the participants felt it was to teach the abilities listed in the Swedish mathematics curriculum in English may not have strictly been necessary to answer the study’s aims but they have offered a glimpse into an aspect which could be compared to the answers from teachers teaching the same curriculum in Swedish.

One weakness in the analysis section of the results is the small number of teachers with Swedish certification. Comparing the responses of 7 teachers with Swedish certification with the responses of 30 teachers with foreign certification leaves too much room for error. The overall conclusions can be generalised but the comparison between countries of certification cannot.

11.2 Result Discussion
In their support material for mathematics teachers, Skolverket points out that one of the best ways to promote mathematical communication is to allow the students to solve a problem working in groups before presenting their solution to the class (Skolverket, 2015, 5(7)). This can be viewed through a sociocultural perspective as a part of learning through communication. “Asserting that individuals create knowledge in interaction with their environment, such approaches emphasise the social dimension of learning and the significant role of communication in the construction of meaning” (Berger, 2015, p. 287). The results of this study show that while many teachers teaching mathematics in English to Swedish speaking students are aware of the advantages of group work, only one-third are regularly incorporating that teaching practice into their lessons.

The use of open ended questions in mathematics teaching encourages higher-order thinking and creates the opportunity for all students to participate regardless of the level of their mathematics or English abilities (Zevenbergen, Mousley & Sullivan, 2004, p.392). This practice also answers the Swedish school curriculum’s requirement that teaching “should take into account each individual’s needs” (Skolverket 2011, p. 16). The results of this study again show that while the majority of the responding teachers are aware of open ended tasks, only one out of eight teachers incorporate this teaching practice regularly in their lessons.

On the positive side, a bilingual environment is encouraged in most classrooms with the teaching practice of translanguaging used on a regular basis through the use of a Swedish mathematics textbook in an English mathematics class in the vast majority of the participating classes. The teachers participating in the survey were also likely to encourage bilingual communication with the aim of further supporting the student’s content and language development.

12. Conclusion
In answering the first question of this study, “What amount of training in second language learning have mathematics teachers teaching in English to Swedish speaking students received?”, it is clear that mathematics teachers need more pre-service and in-service training in effective teaching practices for the teaching of mathematics in English to Swedish speaking students. This result supports the earlier research conducted by Skolverket (Skolverket, 2010). Increased pre-service and in-service training in effective teaching practices for second language learners would give
mathematics teachers the opportunity to help more students to develop their mathematics knowledge and understanding.

The second question for this study looked at the use of effective teaching practices for the teaching of mathematics to second language learners. Although the results of this study show that many teachers encourage a bilingual environment in their classrooms, they are not clearly defining the norms for mathematical communication in their classrooms. Planas (2014) has shown that in order for methods such as translanguaging and code-switching to be most effective, mathematics teachers working in a bilingual learning environment must clearly state and actively promote the norms of discussion in the classroom, creating a safe and secure language and learning environment, and must do so on a regular and consistent basis. This will offer all students, but particularly second language learners, the opportunity to develop the mathematical communication skills that have now become a required ability in the Swedish school curriculum, as well as in many other mathematics curriculums internationally.

Group work and open ended tasks should be promoted both in pre-service and in-service training. This study has shown that many mathematics teachers are aware of these practices but that few teachers are regularly incorporating them into their mathematics lessons. One strategy for increasing the competency of mathematics teachers suggested by Skolverket (Skolverket 2010) is collegial learning. The creation of collegial learning centers in a school or within a group of schools allows working teachers to explore and employ effective teaching strategies, followed by an analysis of the results. This may be an effective way to promote the use of group work and open ended tasks within schools in Sweden.

Although this study focused uniquely on the teaching of mathematics in English to Swedish speaking students, many of the effective teaching practices discussed can be used in other subjects and in monolingual teaching environments.

13. Further research

This study leaves us with a few interesting areas that could be explored further.

Although many studies into the teaching of mathematics in Sweden focus on the subject of problem solving, this study has revealed that among the teachers teaching mathematics in English to Swedish speaking students, the issue of teaching reasoning is also very challenging to many teachers. Research into such questions as “What does it mean to teach ‘reasoning’” and “what are effective teaching practices that will create opportunities for learning ‘reasoning’” could be very useful in offering effective teaching practices in the future.

Further experimental research into the effectiveness of collegial learning programs that promote group work and open ended tasks in mathematics and any correlation with student achievement would be of great interest to the teaching profession.
References


Appendix 1.
Letter of Introduction and Informed Consent Form
(Modified from a generic consent form published by Concordia University College of Alberta)

Study Title
Current Teaching Practices in the Teaching of Mathematics in English to Swedish Speaking Students

Researcher: Daniel Breton

Before agreeing to participate in this research, I encourage you to thoroughly read the description of the study outlined below. This letter describes the purpose and procedures of the study. Note that you have the right to withdraw from the study at any time.

Study Purpose: The purpose of this study is to map out the teaching practices currently used by teachers in the teaching of mathematics in English to Swedish speaking-students in Sweden. Although there are over twenty-thousand students in Sweden currently attending schools where mathematics are taught in English, very little research has been conducted into the daily practices involved. Research into effective teaching practices when teaching mathematics in English in the context of English not being the language of the majority has been conducted in other countries, but not in Sweden. This research is being conducted as part of my degree thesis at Högskolan Dalarna.

Study Procedure: Participation in the study involves answering a questionnaire survey that asks you questions about any teacher training you have had and classroom practices you have employed in the last two weeks. If you choose, you may also participate in an individual interview conducted online which will delve further into teaching practices you employ that may not have been covered in the questionnaire. The interview should last between 30 and 60 minutes and be conducted sometime during week 7.

Risks: There are no risks anticipated from your participation in this study. All information collected in both the questionnaire and the interview stages will be completely confidential and it will not be possible to identify individual schools or teachers from the study’s results. The information collected in this study will only be used for the purposes outlined in the study’s purpose. The data will not be used for any other purposes.

Benefits: The potential benefit of participation in this study is the opportunity to add to the scientific knowledge base regarding educational practices in Sweden. This can lead to an improvement in educational practices for other students, in Sweden, and in other parts of the world.

Confidentiality: As stated earlier, the information gathered in the two stages of this study will remain confidential. Names and personal information will not be collected in the survey questionnaire nor interview transcripts. Names and other indentifying
details will not be revealed in the publication of this study. The results of this research will be published in the form of a thesis research paper which can be found online.

**Withdrawal without Prejudice**: Participation in this study is voluntary. You are free to withdraw consent and discontinue participation in this project at anytime without prejudice or penalty. You are also free to refuse to answer any question that is asked of you.

**Further Questions and Follow-Up**: You are welcome to ask the researcher any questions that occur to you during the survey or interview. If you have further questions after the interview is completed, you are welcome to contact the researcher using the contact information given on the next page.

**Consent**: By participating in the survey and/or interview stage of the study, I acknowledge having read the above information. I freely agree to participate in this study. I understand that I am free to refuse to answer any question and to withdraw from the study at any time. I understand that my responses will be kept anonymous.

Name:    Date:

If:
(a) you would like a copy of your interview transcript once it is available
(b) you are interested in information about the study results as a whole and/or,
(c) you would be willing to be contacted again in the future for a possible follow-up interview, please provide contact information below.

Check those that apply:
- [ ] I would like a copy of my interview transcript
- [ ] I would like information about the study results
- [ ] I would be willing to be contacted in the future for a possible follow-up interview

Provide your email address below.

Researcher contact information
Daniel Breton
v12danbr@du.se
Appendix 2. Survey questions

Teaching Mathematics in English to Swedish Speaking Students

Study Purpose: The purpose of this study is to map out the teaching practices currently used by teachers in the teaching of mathematics in English to Swedish speaking students in Sweden. Although there are over twenty-thousand students in Sweden currently attending schools where mathematics are taught in English, very little research has been conducted into the daily practices involved. Research into effective teaching practices when teaching mathematics in English in the context of English not being the language of the majority has been conducted in other countries, but not in Sweden. This research is being conducted as part of my degree thesis at Högskolan Dalarna.

By participating in this survey you acknowledge that you freely agree to participate in this study. You are free to refuse to answer any question and to withdraw from the study at any time. Your responses will be kept anonymous. All the data collected in this survey will only be used for the purpose outlined above.

Participant Background
A few questions about your general background. All answers are confidential. Your email will not be collected when you submit your answer.

How many years have you been teaching mathematics in English in Sweden?
- Less than 1 year
- 1 – 2 years
- 3 – 4 years
- 5 years or longer

Are you a certified teacher in mathematics for grades 7 - 9?
- Yes - Completed my studies/certification in Sweden
- Yes - Completed my studies/certification outside of Sweden
- No

What is your native language?
- English
- Swedish
- Other

What language do you primarily use when teaching mathematics in Sweden?
- English
- Swedish
- Other

How much formal training (college or university) have you received in teaching to second language learners?
- I have not received any formal training in teaching to second language learners.
- 1 college or university level course
- 2 college or university level courses
- 3 or more college or university level courses
- Other
Have you received in-service training in teaching to second language learners?
- Yes
- No
- Other

What grade level do you teach? If you teach more than one grade, frame the answers to this survey with regard to one grade only. You are very welcome to repeat the survey for additional grades at the end.
- Grade 7
- Grade 8
- Grade 9

**Teaching Practices**
*Base your answers to the following questions on any practices you have engaged in during the last TWO WEEKS of teaching.*

What language is the mathematics textbook you have primarily used in the last two weeks?
- English
- Swedish
- Other

**Code-switching**
*Code-switching is the practice of switching from one language to another during an exchange. For example "To add fractions, they have to have the same 'nämnare', so I..."*

Have you engaged in code-switching during a lesson in the last two weeks?
- Never use
- Almost never
- Occasionally/sometimes
- Almost every time
- Frequently use
- Other

Do you encourage students to code-switch in their communication with you?
- Actively discourage it
- Discourage it
- I have not addressed the issue with my students
- Encourage it
- Actively encourage it
- Other

Do you encourage students to code-switch in their communication with each other?
- Actively discourage it
- Discourage it
- I have not addressed the issue with my students
- Encourage it
- Actively encourage it
- Other
Translanguaging

Translanguaging is the practice where the students receive instruction (verbal or written) in one language and produce work in another language. One example may be presenting a mathematical problem in Swedish and asking the students to solve it in English.

Have you engaged in the practice of translanguaging in the last two weeks?
- Never use
- Almost never
- Occasionally/sometimes
- Almost every time
- Frequently use
- Other

Do you try to limit the use of Swedish by students during a mathematics lesson?
- No
- Yes
- Other

Revoicing

Revoicing is the subtle correction of a student error in terminology after they have finished speaking. For example: a student uses the word "nominator" in their explanation of the solution to a given problem. After the student has finished with their explanation, the teacher repeats the solution to the class, revoicing the term "numerator", while writing the term on the board along with other terms and encouraging all students to write the terms in their notebooks.

Have you engaged in the practice of revoicing in the last two weeks?
- Never use
- Almost never
- Occasionally/sometimes
- Almost every time
- Frequently use
- Other

Have you heard the students engaging in the practice of revoicing in the last two weeks?
- Never
- Almost never
- Occasionally/sometimes
- Almost every time
- Frequently
- Other
**Visual Aids**
Research has noted that the use of visual aids supports second language mathematics learners. This may include manipulatives, charts, diagrams, posters and videos.

What visual aids have you used in your lessons during the last two weeks? Check all that apply.
- Manipulatives
- Charts or diagrams
- Posters
- Projected presentations
- Other

**Open-Ended Tasks**
Open-ended questions are tasks with multiple pathways and multiple responses. They are inclusive as all students can answer at their level of language and mathematical understanding. Examples of open-ended questions include: "What three numbers have a mean of five?" or "Seven boys went fishing. The mean number of fish that each caught is five and the mode is four. How many fish might they have caught?"

Have you engaged in the practice of Open-Ended tasks with your students in the last two weeks?
- Never use
- Almost never
- Occasionally/sometimes
- Almost every time
- Frequently use
- Other

**Open-ended question**
Are there any other teaching practices that you would like to mention regarding the teaching of mathematics in English to Swedish speaking students?

**Student - Student Interaction**

Have your students worked in groups (four to six students) to solve mathematical problems in the last two weeks?
- Never
- Almost never
- Occasionally/sometimes
- Almost every time
- Frequently
- Other

Through what language would you prefer that student-student interaction take place?
- English as much as possible
- Swedish
- Both English and Swedish
- Other
**Mathematical Abilities - Lgr-11**

According to the Swedish school curriculum, Lgr-11, teaching in mathematics should essentially give pupils the opportunities to develop in 5 different abilities. In the context of teaching mathematics in English to Swedish speaking students, rank how easy or challenging you feel it is to teach each of those abilities to your current students.

Teaching Problem Solving: The ability to formulate and solve problems using mathematics and also assess selected strategies and methods.
- Very difficult
- Difficult
- Neutral
- Easy
- Very easy

Teaching Concepts: The ability to use and analyse mathematical concepts and their interrelationships.
- Very difficult
- Difficult
- Neutral
- Easy
- Very easy

Teaching Methods: The ability to choose and use appropriate mathematical methods to perform calculations and solve routine tasks.
- Very difficult
- Difficult
- Neutral
- Easy
- Very easy

Teaching Reasoning: The ability to apply and follow mathematical reasoning.
- Very difficult
- Difficult
- Neutral
- Easy
- Very easy

Teaching Communication: The ability to use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions.
- Very difficult
- Difficult
- Neutral
- Easy
- Very easy